

# $4\pi$ Appears Only When the Source Is a Point

— Raise the Dimension of the Source by One and  $4\pi$  Becomes  $2\pi$  While  $1/r$  Becomes a Logarithm — Sources in Living Tissue Are Often Not Points — [Paper 258]

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— *From thinking to syncing.* —

## Abstract

Paper 02 showed that a pure solid angle  $4\pi$  appears in an inverse-square field. This paper asks when that  $4\pi$  does not appear—the answer is the moment the source stops being a point. Raising the dimension of the source by one alone turns  $4\pi$  into  $2\pi$  and turns the  $1/r$  fall into a logarithm. No new mathematical theorem and no new law is claimed. First, for a point source the  $4\pi$  stands in the denominator. Putting  $I = 1\mu\text{A}$ ,  $\sigma = 0.33\text{ S/m}$  and  $r = 1\text{ cm}$  into  $\phi = I/(4\pi\sigma r)$  gives **24.1144** $\mu\text{V}$  (Section 2). Second, this is the core of the paper. Make the source a line and the  $4\pi$  disappears, leaving  $2\pi$  and a logarithm. The prefactor of  $\phi = (I'/2\pi\sigma) \ln(r_2/r_1)$  is **4.822877**  $\times 10^{-5}\text{ V}$ , and there is no  $4\pi$  anywhere in it (Section 3). Third, the fall reverses direction. Going from 2 mm to 50 mm, the point source drops from 120.5719 to 4.8229 $\mu\text{V}$ , a factor of **25.0**, while the line source rises from 33.4296 to 188.6721 $\mu\text{V}$  (Section 3). Fourth, three sources are called over. Point  $4\pi = 12.566371$ , line  $2\pi = 6.283185$ , plane  $2\pi$ —each rise in the dimension of the source flattens the fall one step, from  $1/r$  to  $\ln$  to constant (Section 4). Fifth, sources in living tissue are often cylinders. The reach of oxygen around a capillary is  $L = \sqrt{2DC_0/M}$ , which at rest is **316.8** $\mu\text{m}$  (Section 5). Sixth, that length is not set by the resting demand. At maximal exercise it shrinks to **44.8** $\mu\text{m}$ , which matches the observed capillary spacing of 20 to 80 $\mu\text{m}$ —since  $L \propto M^{-1/2}$ , a demand 50 times larger shrinks the reach by a factor of **7.0711** (Section 6).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed—the potentials of point, line and plane sources, the volume-conductor approximation, and Krogh's cylinder model are all standard. No electrophysiology is built—what is used is three solutions, one square root, and a conversion of units. The origin of the  $4\pi$  is not discussed—Paper 02 treats that. This paper treats only the converse, the condition under which that  $4\pi$  disappears. Inverse problems are not entered—Paper 99 asks whether the interior conductivity can be recovered from boundary data, and Paper 159 separated the senses of ill-posedness. This paper looks in the forward direction alone, asking which solid angle appears once the shape of the source is given. Capture rates are not treated—Paper 249 treated the Smoluchowski capture rate for a sphere, in steady state, with an absorbing boundary. This paper takes a cylinder, includes a consumption term, and asks not for a capture rate but for a reach. No physiological claim is made—no account is offered of why capillaries are arranged as they are. An agreement is reported, and no cause is asserted. Tissue is treated as isotropic and homogeneous—real tissue is neither. The numbers are values for seeing orders of magnitude, not measurements on a particular tissue. The source is treated as an infinitely long cylinder—at finite length the ends return towards  $4\pi$ . What is claimed is that in the infinitely long limit the  $4\pi$  does not appear. Relation to earlier papers: Paper 02 showed that a pure solid angle  $4\pi$  appears in an inverse-square field—this paper writes the domain of that  $4\pi$ : it appears only when point source, three dimensions and isotropy hold together. Paper 249 showed that the  $4\pi$  that catches molecules and the  $4\pi$  that spreads a field share a root—this paper sets out the cases where the same  $4\pi$  does not appear. Confirming a shared root and confirming a domain are two halves of one thing. Paper 01 showed from four directions that  $n = 2$  is unique—what this paper moves is the dimension of the source rather than the exponent, a different axis. Paper 199 wrote that what decides whether you come home is the exponent in the denominator—Section 4 here likewise sets out the form of the denominator, but asks about the fall rather than about recurrence. Paper 112 counted resolution as one of six distinct roots—measurement resolution is not treated here. What is added is writing the condition for  $4\pi$  to appear as the three items point source, three dimensions and isotropy, making a roll call of the correspondence by which raising the dimension of the source halves the solid angle and flattens the fall, seeing in the concrete number 25.0 that the distance dependence of a point source and a line source point in opposite directions, and computing the capillary reach both at rest,  $316.8\mu\text{m}$ , and at maximum,  $44.8\mu\text{m}$ , and stating that it is the latter that matches the observed spacing.

## 1. Notation and Placement

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**Setting:** place a current source in a uniform medium of conductivity  $\sigma$  (a volume conductor) and look at the potential  $\phi$  at a distance  $r$ .

**What is received:** Paper 02 showed that a pure solid angle  $4\pi$  appears in an inverse-square field. This paper receives that as a premise—why the  $4\pi$  appears is not treated here.

**Question:** when does that  $4\pi$  fail to appear?

The answer is the moment the source stops being a point. The  $4\pi$  appears only when point source, three dimensions and isotropy hold together, and raising the dimension of the source by one alone removes it.

## 2. The Point Source: the $4\pi$ Stands in the Denominator

**Setting:** a point current source  $I$  makes the potential  $\phi = I/(4\pi\sigma r)$ . The  $4\pi = 12.566371$  stands in the denominator.

**Check 2.1:** computed with  $I = 1\mu\text{A}$  and  $\sigma = 0.33\text{ S/m}$ .

| Distance $r$ | Potential $\phi$     |
|--------------|----------------------|
| 5 mm         | $48.2288\mu\text{V}$ |
| 10 mm        | $24.1144\mu\text{V}$ |
| 20 mm        | $12.0572\mu\text{V}$ |
| 50 mm        | $4.8229\mu\text{V}$  |

### Observation 2.2

doubling the distance halves the potential exactly. This is  $1/r$ —and the  $4\pi$  in the denominator is the whole solid angle of the sphere itself (Paper 02).

## 3. The Core: Make the Source a Line and the $4\pi$ Disappears

**Setting:** place an infinitely long straight source. With a current  $I'$  per unit length the potential difference is

$$\phi = \frac{I'}{2\pi\sigma} \ln \frac{r_2}{r_1}$$

The denominator carries  $2\pi$ , and there is no  $4\pi$  anywhere. And the fall is a logarithm rather than  $1/r$ .

**Check 3.1:**  $I' = 10^{-4}\text{ A/m}$  and  $\sigma = 0.33\text{ S/m}$ . The prefactor is  $I'/(2\pi\sigma) = 4.822877 \times 10^{-5}\text{ V}$ .

| $r_2/r_1$ | $\ln(r_2/r_1)$ | Potential difference             |
|-----------|----------------|----------------------------------|
| 2         | 0.693147       | <b>33.4296<math>\mu</math>V</b>  |
| 10        | 2.302585       | <b>111.0508<math>\mu</math>V</b> |
| 100       | 4.605170       | <b>222.1017<math>\mu</math>V</b> |

### Observation 3.2

the  $4\pi$  has gone. Raising the dimension of the source from 0 to 1 alone halved the solid angle in the denominator, from 12.566371 to 6.283185.

**Check 3.3: the fall itself reverses direction.** The two are set side by side over the same distances, taking  $r_1 = 1$  mm as the reference for the line source.

| Distance | Point source                     | Line source                      |
|----------|----------------------------------|----------------------------------|
| 2 mm     | <b>120.5719<math>\mu</math>V</b> | <b>33.4296<math>\mu</math>V</b>  |
| 5 mm     | <b>48.2288<math>\mu</math>V</b>  | <b>77.6212<math>\mu</math>V</b>  |
| 10 mm    | <b>24.1144<math>\mu</math>V</b>  | <b>111.0508<math>\mu</math>V</b> |
| 20 mm    | <b>12.0572<math>\mu</math>V</b>  | <b>144.4805<math>\mu</math>V</b> |
| 50 mm    | <b>4.8229<math>\mu</math>V</b>   | <b>188.6721<math>\mu</math>V</b> |

### Observation 3.4

the point source falls by a factor of **25.0** and the line source rises by a factor of **5.64**. The same distance is being travelled and the directions are opposite—one cannot say that a field weakens with distance without first writing down the shape of the source.

**Observation 3.5**

the separator—what has to be changed for the  $4\pi$  to disappear.Raising the dimension of the source by one, and nothing else.Neither the medium, nor the current, nor the number of dimensions of the space has been moved.

**4. A Roll Call of Solid Angles over Three Sources**

**Check 4.1:** place sources of dimension **0**, **1** and **2** in three-dimensional space.

| Source          | Its dimension | Solid angle        | Fall       |
|-----------------|---------------|--------------------|------------|
| Point           | 0             | $4\pi = 12.566371$ | $1/r$      |
| Line (cylinder) | 1             | $2\pi = 6.283185$  | $\ln(1/r)$ |
| Plane (layer)   | 2             | $2\pi = 6.283185$  | constant   |

**Observation 4.2**

each rise in the dimension of the source flattens the fall by one step— $1/r$ , then a logarithm, then a constant.For a plane source the potential stops depending on distance, and the solid angle saturates at  $2\pi$ .

**Observation 4.3**

the  $4\pi$  appears in the first row alone.Written out, the condition has three items, point source, three dimensions and isotropy, and losing any one of them removes the  $4\pi$ .

**Observation 4.4**

Paper 249 showed that the  $4\pi$  that catches molecules and the  $4\pi$  that spreads a field share a root.This paper is the other side of that—confirming a shared root and confirming a domain are two halves of one thing.To say how far two things agree, one must say where they begin to differ.

### 5. An Instance: a Capillary Is a Cylinder, Not a Point

**Setting:** oxygen diffuses out of a capillary into the tissue and is consumed on the way. **The reach is**

$$L = \sqrt{\frac{2DC_0}{M}}$$

where  $D$  is the diffusion coefficient,  $C_0$  the concentration at the vessel wall, and  $M$  the consumption per unit volume.

**Check 5.1:**  $D = 2.0 \times 10^{-9} \text{m}^2/\text{s}$ ,  $\alpha = 1.4 \times 10^{-3} \text{mol}/(\text{m}^3 \cdot \text{mmHg})$  and  $P = 40 \text{ mmHg}$ . Hence  $C_0 = \alpha P = 0.056000 \text{mol}/\text{m}^3$ .

| State                                             | Consumption $M$ [mol/(m <sup>3</sup> · s)] | Reach $L$       |
|---------------------------------------------------|--------------------------------------------|-----------------|
| Skeletal muscle at rest ( <b>0.3</b> mL/100g/min) | <b>2.232143</b> $\times 10^{-3}$           | <b>316.8</b> μm |
| Moderate ( <b>3.0</b> mL/100g/min)                | <b>2.232143</b> $\times 10^{-2}$           | <b>100.2</b> μm |
| Maximal exercise ( <b>15.0</b> mL/100g/min)       | <b>1.116071</b> $\times 10^{-1}$           | <b>44.8</b> μm  |

**Observation 5.2**

this is a cylinder, so it belongs to the second row of the table in Section 3. It is a world of  $2\pi$  rather than  $4\pi$ , the place where a potential would be logarithmic—what this section computes is a reach rather than a potential, but the shape of the source is the same cylinder.

### 6. The Length Is Not Set by the Average

**Check 6.1:** the observed capillary spacing (the Krogh radius) is **20** to **80**μm. Set the three rows against it.

| State            | $L$             | Against the observed 20–80μm        |
|------------------|-----------------|-------------------------------------|
| At rest          | <b>316.8</b> μm | four times too far; looks excessive |
| Moderate         | <b>100.2</b> μm | somewhat too far                    |
| Maximal exercise | <b>44.8</b> μm  | it matches                          |

### Observation 6.2

at the resting demand the reach extends to  $316.8\mu\text{m}$ , so a spacing of 20 to  $80\mu\text{m}$  looks excessive. What it matches is the demand at maximal exercise—the length is set by the maximum rather than the average.

### Observation 6.3

$L \propto M^{-1/2}$ . A demand 50.0 times larger shrinks the distance by  $\sqrt{50} = 7.0711$ —and  $316.8/7.0711 = 44.8$ , so the first and third rows of the table correspond exactly.

### Observation 6.4

a point to state honestly—this paper reports an agreement and no more. It does not claim that the arrangement of capillaries is set by the maximal demand—that would require evidence from the developmental side, which this paper does not have. Treating the tissue as isotropic and homogeneous is also a departure from the real thing.

## 7. Roll Call and Closing

1. For a point source the  $4\pi$  stands (Check 2.1):  $24.1144\mu\text{V}$  at  $r = 1\text{ cm}$ .
2. For a line source it disappears (Check 3.1): prefactor  $4.822877 \times 10^{-5}\text{ V}$ , denominator  $2\pi$ .
3. The solid angle halves (Observation 3.2):  $12.566371$  to  $6.283185$ .
4. The fall reverses (Check 3.3): the point falls by 25.0, the line rises by 5.64.
5. A roll call of three sources (Check 4.1):  $1/r$ , logarithm, constant.
6. The condition has three items (Observation 4.3): point source, three dimensions, isotropy.
7. The reach at rest (Check 5.1):  $316.8\mu\text{m}$ .
8. The reach at maximum (Check 5.1):  $44.8\mu\text{m}$ , matching the observed 20– $80\mu\text{m}$ .
9. It shrinks as a square root (Observation 6.3): a demand 50 times larger shrinks the distance by 7.0711.

Closing: the  $4\pi$  that Paper 02 found is not something that appears everywhere. It appears only when point source, three dimensions and isotropy hold together, and raising the dimension of the source by one alone removes it. What appears after it has gone is  $2\pi$  and a logarithm, and the fall points the other way—over the same distance the point source becomes 25.0 times weaker while the line source becomes 5.64 times stronger. And sources in living tissue are

often not points. A capillary is a cylinder, and the reach around it is  $316.8\mu\text{m}$  at rest and  $44.8\mu\text{m}$  at maximal exercise, the latter being what matches the observed spacing. One thing separates them—writing down the dimension of the source. Write it down, and the occasions where  $4\pi$  may be used separate from the occasions where using it is wrong by a factor of two. Do not write it down, and one fits a  $1/r$  to a field that grows stronger with distance.

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