

What Fences In Poisson's Ratio Is Not Geometry

— Two Independent Elastic Constants, and the Fence $-1 < \nu < 1/2$ — What Raises the Fence Is the Positive
Definiteness of the Strain Energy — [Paper 261]

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— *From thinking to syncing.* —

Abstract

An isotropic solid has only 2 elastic constants, and the two are not free to move. Poisson's ratio carries a fence, $-1 < \nu < 1/2$. This paper asks what raises that fence—the answer is not geometry. It is a requirement from the side of thermodynamics: that the strain energy must be positive. No new mathematical theorem and no new law is claimed. First, the constants are counted. General anisotropy has 21, orthotropy 9, cubic symmetry 3, and isotropy 2—each rise in symmetry lowers the count (Section 2). Second, this is the core of the paper. Inside the permitted fence alone, K/G runs from 0.002237 to 49999.67, spanning 5 orders of magnitude—that is how wide the single phrase “an isotropic solid” is (Section 3). Third, the two ends break differently. As $\nu \rightarrow 1/2$ the bulk modulus K diverges (incompressibility); as $\nu \rightarrow -1$ the shear modulus G diverges—at the two ends of one fence, different quantities blow up (Section 4). Fourth, the fence does not come out of the geometry. Put in $\nu = 0.7$ and the formula happily returns $K/G = -2.8333$, a negative value—the formula does not stop (Section 5). Fifth, what stops it is the energy. Past the fence the strain energy can go negative—which would mean that deforming the body yields mechanical energy, and thermodynamics forbids that (Section 5). Sixth, the value itself is set by the material, not the geometry. Cork is near 0.0, steel 0.3, rubber 0.4999, and auxetic materials are negative—the position of the fence is universal, and where one sits inside it differs by material (Section 6).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed—that isotropic elasticity has 2 independent constants, the expression for K/G , the fence $-1 < \nu < 1/2$, and that its ground is the positive definiteness of the strain energy, are all standard. No theory of elasticity is built—what is used is one expression for a ratio and two limits. Positive definiteness itself is not proved—the requirement that the strain energy be positive is received as a premise. Why it is required belongs to thermodynamics, and this paper does not enter that. Anisotropy is not treated—the number 21 is counted in Section 2 and everything after that is isotropic. No real material is evaluated—the values of ν for cork, steel and rubber are representative, not measurements on particular specimens. How auxetic materials are made is not discussed—that a negative Poisson's ratio is realisable is cited, and no mechanism is treated. Elasticity as a metaphor is not conflated with this—Paper 74 read $G = 4\pi$ as an elastic constant of spacetime. What this paper treats is the elastic constants of a real solid, a different thing. Fluid stresses are not treated—the stress tensor in the Stokes flow of Papers 37 and 65 belongs to a fluid. Positive definiteness elsewhere is not conflated with this—the positive definiteness of Papers 121, 133, 137, 143 and 201 concerns metrics and information measures. Here it is the strain energy, the same word applied elsewhere. Relation to earlier papers: Paper 143 treated a minimum that carries a fence—this paper likewise names a fence, but the thing fenced is an elastic constant. Paper 173 showed that Kodaira's fence is integrality rather than positivity—this paper writes the converse case, where the fence really is positivity. Set side by side, the two show that fences are not of one kind. Paper 201 counted “complete” as four different claims—this paper counts how wide, by 5 orders of magnitude, the single phrase “an isotropic solid” is. Paper 190 measured rare on a logarithmic scale—this paper likewise reports the spread of K/G in orders of magnitude. What is added is setting out the count of constants across six symmetries, computing that K/G spans 5 orders of magnitude inside the permitted range alone, writing that different quantities diverge at the two ends of the fence, and substituting values from outside the fence to see that the formula does not stop.

1. Notation and Placement

Setting: consider an isotropic linear elastic solid. Four quantities are used, Young's modulus E , the shear modulus G , the bulk modulus K and Poisson's ratio ν , but only two are independent.

Question: those two carry a fence, $-1 < \nu < 1/2$. What raises it?

The answer is not geometry. It is neither a matter of shape nor of symmetry—it is the requirement that the strain energy be positive.

2. Counting the Constants

Check 2.1: count the independent elastic constants by symmetry.

Symmetry	Independent constants
General anisotropy	21
Monoclinic	13
Orthotropic	9
Transversely isotropic	5
Cubic	3
Isotropic	2

Observation 2.2

each rise in symmetry lowers the count, falling to 2 at isotropy. That 2 is the stage of this paper—everything below treats the isotropic case.

Observation 2.3

that there are 2 of them and that those 2 may move freely are different matters. The count follows from the symmetry; the range does not—and that is where this paper turns.

3. The Core: Inside the Fence Alone, K/G Spans Five Orders

Setting: the two ratios are tied by

$$\frac{K}{G} = \frac{2(1 + \nu)}{3(1 - 2\nu)}$$

Move ν inside the fence and watch this ratio.

Check 3.1:

ν	Example	K/G
−0.99	near the lower fence	0.002237
−0.90	extreme negative	0.023810
−0.50	auxetic (widens when pulled)	0.166667
0.00	cork	0.666667
+0.30	steel	2.166667
+0.4999	rubber	4999.666667
+0.49999	near the upper fence	49999.666667

Observation 3.2

inside the permitted range alone, K/G opens by a factor of 2.0999×10^5 . What is called by the single phrase “an isotropic solid” is five orders of magnitude wide.

Check 3.3: confirm that the three expressions agree, using $G = 8.000 \times 10^{10}$ Pa for steel, with $E = 2G(1 + \nu)$ and $K = E/\{3(1 - 2\nu)\}$.

ν	E [Pa]	K [Pa]	K/G (from the expressions)
0.0000	1.600000×10^{11}	5.333333×10^{10}	0.666667
0.3000	2.080000×10^{11}	1.733333×10^{11}	2.166667
0.4999	2.399840×10^{11}	3.999733×10^{14}	4999.666667

Observation 3.4

the last column agrees with the table in Section 3.1 to six decimal places. The tools are calibrated by this.

4. At the Two Ends, Different Quantities Diverge

Check 4.1: approach each end.

Towards	ν	K/G
$\nu \rightarrow -1$	-0.900	0.023810
	-0.990	0.002237
	-0.999	0.000222
$\nu \rightarrow 1/2$	$+0.499$	499.6667
	$+0.4999$	4999.6667
	$+0.49999$	49999.6667

Observation 4.2

at the upper fence K diverges—the volume cannot be changed, that is, incompressibility. At the lower fence G diverges—the shape cannot be changed.

Observation 4.3

at the two ends of one and the same fence, different quantities blow up. The single phrase “there is a fence” is saying two different things at its two ends.

5. What Raises the Fence

Check 5.1: put values from outside the fence straight into the formula.

ν	K/G	Does the formula stop?
0.70	−2.833333	it does not
1.00	−1.333333	it does not
−2.00	−0.133333	it does not

Observation 5.2

the formula goes on computing. A negative K/G comes out without complaint—the fence is not written in the formula.

Observation 5.3

what stops it is the energy. The strain energy is positive only when K and G are both positive—if either goes negative, deforming the body would yield energy. Thermodynamics forbids that.

Observation 5.4

the separator—what has to be changed for the fence to disappear. Dropping the requirement that the energy be positive. Neither the geometry nor the symmetry has been moved—the fence descends from the level above.

Observation 5.5

Paper 173 showed that Kodaira's fence is integrality rather than positivity. This paper is the converse, a case where the fence really is positivity—set side by side, they show that fences are not of one kind.

6. The Fence Is Universal; Where One Sits Is Set by the Material

Check 6.1: place representative values for real materials inside the fence.

Material	Representative ν	K/G
Auxetic materials (engineered)	negative	< 0.6667
Cork	≈ 0.0	0.666667
Glass	≈ 0.22	1.452381
Steel	≈ 0.30	2.166667
Aluminium	≈ 0.33	2.607843
Rubber	≈ 0.4999	4999.666667

Observation 6.2

the position of the fence, -1 and $1/2$, is the same for every material. Only where one sits inside it differs—the universal part and the material-specific part are separated.

Observation 6.3

a point to state honestly—the values of ν above are representative and not measurements on particular specimens. Those for glass and aluminium vary between sources—what this paper claims is the structure of the ordering, not the individual numbers.

7. Roll Call and Closing

1. The count of constants (Check 2.1): **21, 13, 9, 5, 3, 2.**
2. Count and range are different (Observation 2.3): **symmetry gives the count and not the range.**
3. Five orders inside the fence (Check 3.1): **0.002237 to 49999.67, a factor of 2.0999×10^5 .**
4. The tools are calibrated (Check 3.3): **the ratio from E and K agrees to six decimals.**
5. At the upper fence K diverges (Observation 4.2): **it becomes incompressible.**
6. At the lower fence G diverges (Observation 4.2): **the shape becomes unchangeable.**
7. The formula does not stop (Check 5.1): **$\nu = 0.7$ gives $K/G = -2.833333$.**
8. The energy stops it (Observation 5.3): **negative would mean work could be extracted.**

9. The fence is universal(Observation 6.2): only the seat inside it is material-specific.

Closing: an isotropic solid has 2 elastic constants, and the two carry a fence. But the fence was not written in the formula—put in $\nu = 0.7$ and -2.833333 comes out without complaint. What stops it is a requirement descending from the level above, that the strain energy be positive. And inside that fence alone K/G spans five orders of magnitude, while different quantities diverge at the two ends—at one the volume freezes and at the other the shape does. One thing separates them—writing down which level the fence descends from. Write it down, and what is fixed by geometry separates from what is fixed by energy. Do not write it down, and one reads a negative value returned by the formula as a property of the material.

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