

# There Are Three Ways to Break, and Strength Does Not Decide Which

— Buckling Is Decided by Shape, Fracture by a Flaw, Yielding by the Crystal — One Section of One Material  
Has a Different Limit Once Its Length Changes — [Paper 262]

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— *From thinking to syncing.* —

## Abstract

The sentence “the strength of this material is 250 MPa” does not settle when it breaks. This paper asks what does settle it—the answer is that there are three ways to break and a different thing decides each. Buckling is decided by shape, fracture by a flaw, and yielding by the crystal. No new mathematical theorem and no new law is claimed. First, change only the length of one section. A square steel column of side 31.6 mm buckles at **164.3** MPa when it is 1 m long and at **18.3** MPa when it is 3 m—nothing about the material has been changed (Section 2). Second, this is the core of the paper. The slenderness at which the mode changes is  $\lambda_c = \pi\sqrt{E/\sigma_y} = \mathbf{88.8577}$ —slimmer than this and buckling comes first, stubbier and yielding does, so one material has its limit exchanged (Section 3). Third, the third limit is set by a flaw. By Griffith's condition a flaw of  $1\mu\text{m}$  gives **356.8** MPa and one of 1 mm gives **11.3** MPa (Section 4). Fourth, a flaw tells as a square root. A flaw 1000 times larger lowers the strength by a factor of **31.6228**—which is  $\sqrt{1000}$  itself (Section 4). Fifth, the three do not compete. Of three upper bounds that hold at once, the lowest is the actual limit—and which is lowest is decided not by the material but by shape and flaw (Section 5). Sixth, so strength is not a property of the material. The number  $\sigma_y = 250$  MPa is held, and a column 3 m long still breaks at 18.3 MPa—a factor of **13.66** apart (Section 5).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed—Euler's buckling load, slenderness, Griffith's fracture condition and the yield stress are all standard. No mechanics of materials is built—what is used is three formulas and one comparison. No value fit for design is given—no safety factor, no initial imperfection and no residual stress is included. The numbers are for seeing which of the three limits is lowest, not values for design. The fence on the elastic constants is not treated—Paper 261 treats  $-1 < \nu < 1/2$ . This paper is what happens after one is seated inside that fence, up to breaking. No theory of plasticity is built—yielding is treated as one stress value, with no hardening and no flow rule. No value of the surface energy is claimed— $\gamma = 1.0 \text{ J/m}^2$  is a posited value, not a measurement on a particular material. It is used to see orders of magnitude. Fatigue and time dependence are not treated—only a single loading is examined. Griffith is not confused with another—the Griffiths appearing in Paper 173 is an algebraic geometer and a different person from the A. A. Griffith of this paper. Relation to earlier papers: Paper 261 wrote that what raises the fence on the elastic constants is the positive definiteness of the energy—this paper treats what follows, namely where a material seated inside that fence breaks. Paper 190 measured rare on a logarithmic scale—this paper likewise writes the effect of a flaw as a square root and in orders of magnitude. Paper 196 counted “pressure” as four different quantities—this paper counts that “strength” is not even one quantity. Paper 201 counted “complete” as four different claims—the same shape of roll call. What is added is computing that the buckling stress of one section moves from 164.3 to 18.3 MPa on changing only the length, putting the switching slenderness at the concrete value 88.8577, confirming that a flaw tells as  $\sqrt{1000} = 31.6228$ , and setting the three limits in one table and writing that the lowest is the actual limit.

## 1. Notation and Placement

**Setting:** consider steel, with Young's modulus  $E = 200$  GPa and yield stress  $\sigma_y = 250$  MPa. Take a square section of side  $a = 31.6$  mm.

**Question:** at what load does this column break?

The answer is that it is not settled by one number. There are three separate limits, and which arrives first is decided by shape and flaw rather than by the material.

| Way of breaking | What decides it                  | What tells             |
|-----------------|----------------------------------|------------------------|
| Buckling        | shape (slenderness)              | $E$ and length         |
| Fracture        | a flaw (crack length)            | $E$ and surface energy |
| Yielding        | the crystal (dislocation motion) | $\sigma_y$             |

## 2. Change Only the Length of One Section

**Setting:** Euler's buckling load is  $P_{cr} = \pi^2 EI / L^2$ , and for a square section  $I = a^4 / 12$ .

**Check 2.1:** from  $a = 31.6 \text{ mm}$ ,  $I = 8.309351 \times 10^{-8} \text{ m}^4$ ,  $A = 9.985600 \times 10^{-4} \text{ m}^2$ , and the radius of gyration is  $r = \sqrt{I/A} = 9.122134 \times 10^{-3} \text{ m}$ .

| Length $L$ | $P_{cr}$ | Buckling stress | Slenderness $L/r$ |
|------------|----------|-----------------|-------------------|
| 1.0 m      | 164.0 kN | 164.3 MPa       | 109.6             |
| 2.0 m      | 41.0 kN  | 41.1 MPa        | 219.2             |
| 3.0 m      | 18.2 kN  | 18.3 MPa        | 328.9             |

### Observation 2.2

neither the material nor the section has been changed at all. Tripling the length alone lowered the buckling stress by a factor of 9—because it goes inversely as  $L^2$ .

### Observation 2.3

compare with the yield stress of 250 MPa. The 1 m column buckles at 164.3 MPa, so it buckles before it yields—the number 250 MPa never gets its turn for this column.

## 3. The Core: One Material Has Its Limit Exchanged

**Setting:** the buckling stress is  $\pi^2 E / \lambda^2$  with  $\lambda = L/r$ . The slenderness at which this equals  $\sigma_y$  is the switching point.

**Check 3.1:**

$$\lambda_c = \pi \sqrt{E/\sigma_y} = \pi \sqrt{200 \times 10^9 / 250 \times 10^6} = \pi \sqrt{800} = 88.8577$$

### Observation 3.2

for this section that is  $L_c = \lambda_c \cdot r = 0.8106 \text{ m}$ . Longer than this and buckling comes first, shorter and yielding does—one and the same section of one and the same material has its way of breaking exchanged.

**Check 3.3:** set out either side of the switch.

| <i>L</i> | Slenderness | Buckling stress | Which arrives first   |
|----------|-------------|-----------------|-----------------------|
| 0.5 m    | 54.8        | 657.0 MPa       | yielding (at 250 MPa) |
| 0.8106 m | 88.8577     | 250.0 MPa       | a dead heat           |
| 1.0 m    | 109.6       | 164.3 MPa       | buckling              |
| 3.0 m    | 328.9       | 18.3 MPa        | buckling              |

**Observation 3.4**

the separator—what has to be changed for these to become different things. The slenderness, and nothing else. Neither  $E$  nor  $\sigma_y$  nor the section has been moved—one length was changed, and with it the kind of breaking.

**4. The Third Limit Is Set by a Flaw**

**Setting:** Griffith's condition is

$$\sigma_c = \sqrt{\frac{2E\gamma}{\pi a}}$$

with  $a$  the crack length and  $\gamma$  the surface energy. Take  $\gamma = 1.0\text{J/m}^2$ .

**Check 4.1:**

| Flaw length <i>a</i> | Fracture stress $\sigma_c$ |
|----------------------|----------------------------|
| 1μm                  | 356.8 MPa                  |
| 10μm                 | 112.8 MPa                  |
| 100μm                | 35.7 MPa                   |
| 1 mm                 | 11.3 MPa                   |

#### Observation 4.2

a flaw 1000 times larger lowers the strength by a factor of **31.6228**. That is  $\sqrt{1000} = 31.6228$  itself—it tells as a square root.

#### Observation 4.3

an invisible flaw of  $10\mu\text{m}$  alone drops the limit to 112.8 MPa. That is less than half the yield stress of 250 MPa—a quantity that never appears on the data sheet is setting the limit.

#### Observation 4.4

a point to state honestly— $\gamma = 1.0\text{J}/\text{m}^2$  is a posited value. In real materials plastic work is added and it becomes far larger—the values in this section are for seeing the form in which a flaw tells, namely a square root, and are not real fracture stresses.

## 5. The Three Do Not Compete: the Lowest Wins

**Check 5.1:** for a column **3** m long with a flaw of  $10\mu\text{m}$ , set the three bounds side by side.

| Limit            | Value            | What sets it                     |
|------------------|------------------|----------------------------------|
| Yielding         | <b>250.0</b> MPa | the crystal ( $\sigma_y$ )       |
| Fracture         | <b>112.8</b> MPa | the flaw ( $a = 10\mu\text{m}$ ) |
| Buckling         | <b>18.3</b> MPa  | the shape ( $L = 3$ m)           |
| The actual limit | <b>18.3</b> MPa  | the lowest of the three          |

#### Observation 5.2

the three do not compete. Each says “exceed this and it breaks”, and the lowest is the one that tells—a conjunction, not a choice.

### Observation 5.3

the 250 MPa on the data sheet is the least used number for this column. It stands a factor of **13.66** above the actual limit of 18.3 MPa—a sheet saying “the strength is 250 MPa” says nothing about this column.

**Check 5.4:** with one material, changing shape and flaw exchanges the winner.

| $L$   | Flaw $a$       | Yielding     | Fracture    | Buckling     |
|-------|----------------|--------------|-------------|--------------|
| 0.5 m | none           | <b>250.0</b> | —           | <b>657.0</b> |
| 0.5 m | 1 mm           | 250.0        | <b>11.3</b> | <b>657.0</b> |
| 3.0 m | none           | 250.0        | —           | <b>18.3</b>  |
| 3.0 m | $1\mu\text{m}$ | 250.0        | 356.8       | <b>18.3</b>  |

### Observation 5.5

over four rows the winner changes three ways. The material was never changed—what changed was the length and the flaw.

## 6. Roll Call and Closing

1. Three ways to break(Section 1): buckling by shape, fracture by a flaw, yielding by the crystal.
2. Triple the length, a ninth of the stress(Check 2.1): **164.3** down to **18.3** MPa.
3. **250 MPa** never gets its turn(Observation 2.3): it buckles before it yields.
4. The switch is at  $\lambda_c = 88.8577$ (Check 3.1): for this section,  $L = 0.8106$  m.
5. There is a dead heat(Check 3.3): where the buckling stress is exactly **250.0** MPa.
6. A flaw tells as a square root(Observation 4.2): **1000** times gives a factor of **31.6228**.
7. An invisible flaw halves it(Observation 4.3):  **$10\mu\text{m}$**  gives **112.8** MPa.
8. The lowest wins(Check 5.1): of **250.0**, **112.8** and **18.3**, it is **18.3**.
9. The winner changes three ways(Check 5.4): with no change of material.

**Closing: the sentence “the strength of this material is 250 MPa” did not settle when it breaks. There are three ways to break and a different thing decides each—buckling by shape, fracture by a flaw, yielding by the crystal. The three do not compete, and the lowest is the actual limit. Triple the length of one section and the buckling stress falls to a ninth; admit an invisible flaw of  $10\mu\text{m}$  and the fracture stress drops below half the yield. For a column 3 m long the 250 MPa on the data sheet stands a factor of **13.66** above the real limit and never gets its turn. One thing separates them—writing down which limit is being counted. Write it down, and the occasions for changing the material separate from those for changing the shape and those for removing the flaw. Do not write it down, and one goes on looking for a stronger steel for a column that breaks at 18.3 MPa.**

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