

Inside One Phenomenon Live a Number That Moves with the Lattice and a Number That Does Not

— The Threshold Belongs to the Lattice and Opens by a Factor of **2.8**; the Exponent Belongs to the Dimension and Does Not Move at All — [Paper 264]

Yuuki YAMAGISHI (山岸 勇輝)
Yamagishi Complexity Torus Lab, Japan
August 2026

— *From thinking to syncing.* —

Abstract

Papers 117 and 120 treated the universality of exponents. This paper asks how far that universality reaches—the answer is that a boundary runs through the middle of one phenomenon. In percolation the threshold opens by a factor of **2.8** between lattices, while the critical exponents do not move at all. No new mathematical theorem and no new law is claimed. First, set out the thresholds. Site percolation on the triangular lattice and bond percolation on the square lattice are exactly $1/2$; site on the square is **0.592746**, site on the honeycomb **0.6970**, and bond on the simple cubic **0.2488** (Section 2). Second, the threshold moves a great deal. From the smallest **0.2488** to the largest **0.6970** it opens by a factor of **2.8014**—change the lattice alone and the concentration at which things start to connect changes almost threefold (Section 2). Third, this is the core of the paper. Within two dimensions, triangular, square and honeycomb all give $\beta = 5/36 = \mathbf{0.138889}$ and $\nu = 4/3$, agreeing exactly (Section 3). Fourth, change the dimension and the exponents do move. In three dimensions $\beta \approx 0.4181$ and $\nu \approx 0.8751$ —unmoved by the lattice and moved only by the dimension (Section 3). Fifth, the two numbers hold different offices. The threshold settles where it happens; the exponents settle how it happens (Section 4). Sixth, above six dimensions even that stops. For $d \geq 6$ the values are fixed at $\beta = 1$ and $\nu = 1/2$ —the number that moved only with dimension stops moving there too (Section 5).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed—that the thresholds for triangular site and square bond percolation are exactly $1/2$, that the two-dimensional exponents are $\beta = 5/36$ and $\nu = 4/3$, that the upper critical dimension is 6, and the notion of universality itself, are all standard. No theory of percolation is built—what is used is a list of published values and the computation of ratios. No threshold is computed—the numerical thresholds such as 0.592746 are cited values and were not obtained here. No exponent is computed—both the exact two-dimensional values and the three-dimensional numerical ones are cited. Universality is not proved—why the exponents do not depend on the lattice is not treated. The fact that they do not is simply placed beside a quantity that does. The renormalisation group is not entered—Papers 117 and 120 treat it. No general theory of phase transitions is built—the 20 occurrences of critical points in Papers 08, 117, 120, 203–205, 209 and 212 concern the criticality of phase transitions, whereas this paper concerns a threshold of connectivity. No precision is claimed for the three-dimensional values—0.4181 and 0.8751 are estimates from numerical work, and no claim is made that their digits are settled. Relation to earlier papers: Paper 117 separated the four ways in which scale invariance fixes an exponent and wrote the condition that there be exactly one Π group—this paper stands on the side where that exponent is universal and places beside it a quantity in the same phenomenon that is not. Paper 120 showed that dimensional analysis fails in two ways—that was a classification of failures; this is the cohabitation of what fails and what does not. Paper 94 showed that dimension is not one thing—the dimension here is the dimension of space, not a fractal dimension. Paper 201 counted “complete” as four different claims—this paper divides one phenomenon into two possessions. Paper 190 measured rare on a logarithmic scale—this paper conversely sets out the digits that do not move. What is added is putting the spread of thresholds at the concrete factor 2.8014, setting three lattices of one dimension side by side to show the exponents agreeing exactly, dividing the two into the offices of “where” and “how”, and writing that at the upper critical dimension the exponents stop moving too.

1. Notation and Placement

Setting: let each site, or each bond, of a lattice be open with probability p . Raise p and at some point an infinite connected component appears—that p is the threshold p_c .

What is received: Papers 117 and 120 treated the universality of critical exponents. This paper receives that as a premise—why they are universal is not treated here.

Question: how far does that universality reach?

The answer is that a boundary runs through the middle of one phenomenon. The threshold belongs to the lattice and the exponents belong to the dimension.

2. The Threshold Moves with the Lattice

Check 2.1: set out the published values.

Lattice	Kind	p_c
Two-dimensional triangular	site	1/2 (exact)
Two-dimensional square	bond	1/2 (exact)
Two-dimensional square	site	0.592746
Two-dimensional honeycomb	site	0.6970
Three-dimensional simple cubic	bond	0.2488
Three-dimensional simple cubic	site	0.3116

Observation 2.2

the smallest is **0.2488** and the largest **0.6970**, a factor of **2.8014**. It is one and the same phenomenon called percolation, and the concentration at which it begins differs almost threefold.

Observation 2.3

even within two dimensions it runs from 1/2 to 0.6970. It moves without any change of dimension—the threshold is looking at the detail of the lattice.

Observation 2.4

a point to state honestly—0.592746 and 0.3116 are values from numerical work, and exact values are known only for the two rows reading 1/2. This paper cites these and did not obtain them.

3. The Core: the Exponents Are Not Looking at the Lattice

Setting: near the threshold the probability of belonging to the infinite cluster behaves as $P_\infty \sim (p - p_c)^\beta$ and the correlation length as $\xi \sim |p - p_c|^{-\nu}$. These β and ν are the critical exponents.

Check 3.1: set three two-dimensional lattices side by side.

Lattice	p_c	β	ν
Triangular, site	0.5	0.138889	1.333333
Square, site	0.592746	0.138889	1.333333
Honeycomb, site	0.6970	0.138889	1.333333

Observation 3.2

the second column moves from 0.5 to 0.6970, a factor of **1.394**. The two on the right do not move by a single digit— $\beta = 5/36$ and $\nu = 4/3$, the same fractions for all three.

Check 3.3: change the dimension.

Dimension d	β	ν
2	$5/36 = 0.138889$	$4/3 = 1.333333$
3	≈ 0.4181	≈ 0.8751
6 and above	1	$1/2$

Observation 3.4

the number that did not move when the lattice changed moves when the dimension does. β goes from 0.138889 to 0.4181, a factor of **3.01**—it is looking at something different from the threshold.

Observation 3.5

the separator—what has to be changed for each to move. Change the lattice and only the threshold moves; change the dimension and only the exponents do. The two axes of rotation are orthogonal.

4. The Two Numbers Hold Different Offices

Check 4.1: divide the two by office.

Quantity	What it settles	Moves with the lattice?	Moves with the dimension?
Threshold p_c	where it happens	yes (factor 2.8014)	yes
Exponents β, ν	how it happens	no	yes

Observation 4.2

the threshold settles when it begins; the exponents settle how it rises once begun. Inside one phenomenon live a quantity that sees the detail and a quantity that does not.

Observation 4.3

what this implies in practice—to know the threshold one must specify the lattice exactly, and to know the exponents it suffices to name the dimension. Which of the two is being measured changes how much information is required.

5. Above Six Dimensions the Exponents Stop Moving Too

Check 5.1: raise the dimension.

Dimension d	β	State
2	0.138889	differs by dimension
3	≈ 0.4181	differs by dimension
6	1	the upper critical dimension
7	1	no longer moves
100	1	no longer moves

Observation 5.2

for $d \geq 6$ the values are fixed at those of mean field. The threshold still moves with the lattice, and the exponents stop depending even on the dimension—the staircase of universality rises another step there.

Observation 5.3

there are three steps—the threshold is per lattice, the exponents are per dimension for $d < 6$, and one single value for $d \geq 6$. Inside one and the same phenomenon, dependence on detail comes in three grades.

6. Roll Call and Closing

1. The list of thresholds (Check 2.1): $1/2$, **0.592746**, **0.6970**, **0.2488**, **0.3116**.
2. The threshold opens by **2.8014** (Observation 2.2): from **0.2488** to **0.6970**.
3. It moves within two dimensions alone (Observation 2.3): from $1/2$ to **0.6970**.
4. The exponents agree exactly over three lattices (Check 3.1): **0.138889** and **1.333333**.
5. The threshold moved by **1.394** (Observation 3.2): inside the same table.
6. Change the dimension and the exponents move (Check 3.3): β by a factor of **3.01**.
7. The axes are orthogonal (Observation 3.5): lattice to threshold, dimension to exponent.
8. Different offices (Check 4.1): where, and how.
9. It stops at six (Check 5.1): fixed at $\beta = 1$ and $\nu = 1/2$.
10. Three grades of dependence (Observation 5.3): per lattice, per dimension, one value.

Closing: the universality treated by Papers 117 and 120 was not a property of a whole phenomenon. A boundary runs through the middle of percolation itself—the threshold belongs to the lattice and opens by a factor of **2.8014**, from **0.2488** for simple cubic bond to **0.6970** for honeycomb site. The critical exponents, meanwhile, belong to the dimension and do not shift a single digit from $\beta = 5/36$ and $\nu = 4/3$ across triangular, square and honeycomb. And above six dimensions the exponents stop depending even on the dimension—dependence on detail comes in three grades. One thing separates them—writing down which quantity belongs to what. Write it down, and the occasions requiring an exact lattice separate from those where naming the dimension suffices. Do not write it down, and one reads **0.2488** and **0.6970** as measurement error on one number.

References

1. Broadbent, S. R. and Hammersley, J. M. (1957). Percolation Processes. *Mathematical Proceedings of the Cambridge Philosophical Society*, 53(3).
2. Stauffer, D. and Aharony, A. (1994). *Introduction to Percolation Theory*, 2nd edition. London: Taylor and Francis.
3. Grimmett, G. (1999). *Percolation*, 2nd edition. Berlin: Springer.
4. Kesten, H. (1980). The Critical Probability of Bond Percolation on the Square Lattice Equals $1/2$. *Communications in Mathematical Physics*, 74(1).
5. den Nijs, M. P. M. (1979). A Relation between the Temperature Exponents of the Eight-Vertex and q-State Potts Model. *Journal of Physics A*, 12(10).
6. Wang, J., Zhou, Z., Zhang, W., Garoni, T. M. and Deng, Y. (2013). Bond and Site Percolation in Three Dimensions. *Physical Review E*, 87(5).
7. Yamagishi, Y. (2026). “Dimension” Is Not One Thing [Paper 94]. Zenodo. 10.5281/zenodo.21758581
8. Yamagishi, Y. (2026). Scale Invariance Fixes an Exponent — in Four Different Ways [Paper 117]. Zenodo. 10.5281/zenodo.22005515
9. Yamagishi, Y. (2026). Dimensional Analysis Fails in Two Ways [Paper 120]. Zenodo. 10.5281/zenodo.22036433
10. Yamagishi, Y. (2026). “Rare” Is Measured on a Logarithmic Scale [Paper 190]. Zenodo. 10.5281/zenodo.22106182
11. Yamagishi, Y. (2026). “Complete” Is Four Separate Statements, and What Ties Them Into One Is That the Metric Is Positive Definite [Paper 201]. Zenodo. 10.5281/zenodo.22109052

— From thinking to syncing. —

Yuuki YAMAGISHI (山岸 勇輝)

© 2026 Yamagishi Complexity Torus Lab, Japan · Licensed under Creative Commons Attribution 4.0 International (CC BY 4.0)