

The Sampling Theorem Is Correct and Its Premise Is Never Met

— A Band-Limited Signal of Finite Duration Does Not Exist — Infinitely Many Inputs Give the Same Samples,
and the Samples Hold No Record of Which — [Paper 266]

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— *From thinking to syncing.* —

Abstract

The sampling theorem is a theorem. This paper asks whether its premise is ever met—the answer is never. The premise of being band-limited is incompatible with lasting only a finite time. That a theorem is correct and that it applies are different matters. No new mathematical theorem and no new law is claimed. First, count the aliases. At a sampling rate of 1000 Hz, inputs of 100, 900 and 1100 Hz all appear at 100 Hz (Section 2). Second, this is the core of the paper. The inputs giving the same samples are not three but infinitely many. They run 100, 900, 1100, 1900, 2100, 2900, 3100, 3900, . . . without end (Section 3). Third, the samples hold no record. What decides which it was is not the samples but the premise of being band-limited (Section 3). Fourth, that premise is not met. A band-limited signal is an entire function of exponential type and cannot vanish identically on any interval—so a signal that ends in finite time cannot be band-limited (Section 4). Fifth, a 4π sits here too. The time-bandwidth uncertainty reads $\sigma_t \sigma_f \geq 1/(4\pi) = 0.079577$ —at $\sigma_t = 1$ ms, $\sigma_f \geq 79.5775$ Hz (Section 5). Sixth, compressed sensing is not a relaxation. In place of band-limitedness it posits sparsity, another premise—the premise was exchanged, not removed (Section 6).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed—the sampling theorem, aliasing, the Paley-Wiener theorem and the time-bandwidth uncertainty relation are all standard. No signal processing is built—what is used is one modular arithmetic and one inequality. The sampling theorem is not denied—the theorem is correct. What is asked here is whether the premise holds, not whether the theorem does. The Paley-Wiener theorem is not proved—that band-limitedness implies an entire function of exponential type is cited. Practical remedies are not discussed—neither anti-aliasing filters, nor window functions, nor oversampling are treated. Each is a procedure for meeting the premise approximately, and this paper stops short of them. Compressed sensing is not treated—Paper 188 treats recovery from fewer measurements than unknowns. This paper writes only that the premise has been exchanged and does not enter the conditions for recovery. The Nyquist of thermal noise is not conflated with this—the Nyquist of Paper 78 is Johnson-Nyquist thermal noise, a different work by the same man. The sampling of Paper 243 is not conflated with this—it occurs once there, in another context. The treatment of the delta is not entered—Paper 178 showed that the delta is a map. This paper uses sampling only as a multiplication. Relation to earlier papers: Paper 02 showed that a pure solid angle 4π appears in an inverse-square field—Section 5 here writes that the same 4π stands in the denominator of the uncertainty relation. Paper 258 wrote the condition for 4π to appear as three items and Paper 265 saw it inside the density of states—this paper adds the pair of time and frequency to that family. Paper 188 showed exact recovery from fewer measurements than unknowns—this paper writes only that a premise has changed places there. Paper 201 counted “complete” as four different claims—this paper separates a correct theorem from a usable one. Paper 190 measured rare on a logarithmic scale—this paper counts an infinite family. What is added is listing explicitly that the inputs giving the same samples form an infinite family, writing in one line, from Paley-Wiener, why the premise is not met, connecting the 4π in the denominator of the uncertainty relation to the family of earlier 4π s, and writing that compressed sensing is an exchange of premise rather than a relaxation.

1. Notation and Placement

Setting: sample a continuous signal at a fixed interval. Call the sampling rate f_s and its half $f_s/2$ the Nyquist frequency.

The sampling theorem: if the signal has no component above $f_s/2$ (if it is band-limited), the original can be recovered exactly from the samples.

Question: is that premise ever met?

The answer is never. The theorem is correct—but the premise itself is incompatible with real signals.

2. Counting the Aliases

Check 2.1: take $f_s = 1000$ Hz, with a Nyquist frequency of 500 Hz.

Input frequency	Frequency seen in the samples
100 Hz	100 Hz
400 Hz	400 Hz
500 Hz	500 Hz
600 Hz	400 Hz
900 Hz	100 Hz
1100 Hz	100 Hz

Observation 2.2

600 Hz appears at 400 Hz.900 Hz and 1100 Hz both appear at 100 Hz—and looking at the samples alone they cannot be told apart.

3. The Core: Infinitely Many Inputs Give the Same Samples

Check 3.1: count upwards the inputs that appear at 100 Hz.

Input frequency	Frequency seen in the samples
100	100
900	100
1100	100
1900	100
2100	100
2900	100
3100	100
3900	100
⋮	without end

Observation 3.2

the candidates are not three but an infinite family, $kf_s \pm 100$. However far up one goes, another input giving the same samples is found.

Observation 3.3

the samples hold no record at all of which it was. What chooses is not the samples but the premise that there is nothing above 500 Hz—what is being recovered is not a measurement but an assumption.

Observation 3.4

the separator—what has to be changed for recovery to fail. Dropping the premise of band-limitedness. Neither the number of samples, nor the precision, nor the computing power has been moved.

4. That Premise Is Never Met

Observation 4.1

a band-limited signal has a Fourier transform with bounded support.By the Paley-Wiener theorem such a signal is an entire function of exponential type.

Observation 4.2

an entire function cannot vanish identically on any interval (accumulating zeros would make it vanish everywhere). So a signal that starts at a finite time and ends at a finite time cannot be band-limited.

Check 4.3: set real signals out and see which side each falls on.

Signal	Duration	Band
An ideal sine	unending	band-limited
A recorded note	finite	not band-limited
A single impact	finite	not band-limited
Measurement data	finite	not band-limited

Observation 4.4

everything actually measured lasts a finite time.So nothing whatever meets the premise—the theorem is correct and no object satisfying its condition exists in reality.

Observation 4.5

this is not a defect of the theorem.Whether a theorem is correct and whether it applies are different questions—practice meets the premise approximately with anti-aliasing filters, which is another matter and is not entered here.

5. A 4π Stands in the Denominator of the Uncertainty

Setting: the spread in time σ_t and the spread in frequency σ_f obey

$$\sigma_t \sigma_f \geq \frac{1}{4\pi}$$

with equality only for a Gaussian.

Check 5.1: $1/(4\pi) = 0.079577$.

σ_t	Lower bound on σ_f
1 ms	79.5775 Hz
100 μ s	795.7747 Hz
1 μ s	79577.4715 Hz

Observation 5.2

the shorter the signal is cut, the wider the band. Confining it to 1 ms alone forces a spread of at least 79.5775 Hz—the qualitative statement of Section 4 has become a number.

Observation 5.3

the $4\pi = 12.566371$ in the denominator is the same solid angle Paper 02 saw in an inverse-square field. Paper 258 wrote the three conditions for it to appear and Paper 265 found it inside the density of states —what this section adds is the pair of time and frequency.

Observation 5.4

a point to state honestly—the form $1/(4\pi)$ is the value when σ is a standard deviation and frequency is measured in Hz. Written in angular frequency it becomes $1/2$ —the 4π appears through the choice of units and is not itself a discovery.

6. Compressed Sensing Is an Exchange of Premise

Observation 6.1

compressed sensing is said to recover from fewer samples than Nyquist requires. Reading this as the theorem having relaxed is a mistake.

Check 6.2: set the two side by side by the premise each posits.

	Premise posited	What decides recovery
The sampling theorem	band-limitedness	the upper edge of the band
Compressed sensing	sparsity	the number of non-zero components

Observation 6.3

the premise was not removed but replaced by another. Both require information from outside the samples, and only the kind of information differs.

Observation 6.4

Paper 188 treated exact recovery from fewer measurements than unknowns—what this paper adds is the single point that a premise changes places there. The conditions for recovery themselves are not entered.

7. Roll Call and Closing

1. Aliasing occurs(Check 2.1): both 900 and 1100 appear at 100 Hz.
2. The candidates form an infinite family(Check 3.1): $k f_s \pm 100$ without end.
3. The samples hold no record(Observation 3.3): the premise is what chooses.
4. Band-limited means entire(Observation 4.1): Paley-Wiener.
5. An entire function cannot vanish on an interval(Observation 4.2): incompatible with finite duration.
6. Everything measured is finite(Check 4.3): nothing meets the premise.

7. **The uncertainty bound**(Check 5.1): $1/(4\pi) = 0.079577$, and **79.5775 Hz at 1 ms.**
8. **The 4π depends on units**(Observation 5.4): **in angular frequency it is $1/2$.**
9. **Compressed sensing exchanges**(Check 6.2): **band-limitedness for sparsity.**

Closing: the sampling theorem is correct. And no signal whatever meets its premise. To be band-limited is to be an entire function of exponential type, and such a function cannot vanish on any interval—so a signal ending in finite time cannot, in principle, be band-limited. The samples hold no record at all of whether the input was 100, 900 or 1100 Hz. What chooses is not the measurement but a premise that is not satisfied. And compressed sensing did not remove that premise but swapped it for another. One thing separates them—writing down separately that the theorem is correct and that the premise holds. Write it down, and the occasions for doubting the theorem separate from those for doubting the premise. Do not write it down, and one reads an answer chosen by a premise out of infinitely many candidates as an answer settled by measurement.

References

1. Shannon, C. E. (1949). Communication in the Presence of Noise. *Proceedings of the IRE*, 37(1).
2. Whittaker, E. T. (1915). On the Functions Which Are Represented by the Expansions of the Interpolation Theory. *Proceedings of the Royal Society of Edinburgh*, 35.
3. Paley, R. E. A. C. and Wiener, N. (1934). *Fourier Transforms in the Complex Domain*. Providence: American Mathematical Society.
4. Slepian, D. (1976). On Bandwidth. *Proceedings of the IEEE*, 64(3).
5. Gabor, D. (1946). Theory of Communication. *Journal of the Institution of Electrical Engineers*, 93(26).
6. Unser, M. (2000). Sampling—50 Years after Shannon. *Proceedings of the IEEE*, 88(4).
7. Yamagishi, Y. (2026). *The 4π Mass Cancellation Theorem* [Paper 02]. Zenodo. 10.5281/zenodo.21403931
8. Yamagishi, Y. (2026). *Friction and Dissipation as Coarse-Graining* [Paper 78]. Zenodo. 10.5281/zenodo.21731937
9. Yamagishi, Y. (2026). *The Delta Is Not a Function but a Map* [Paper 178]. Zenodo. 10.5281/zenodo.22098261
10. Yamagishi, Y. (2026). *Fewer Measurements Than Unknowns, and Still Exact* [Paper 188]. Zenodo. 10.5281/zenodo.22105691
11. Yamagishi, Y. (2026). *“Rare” Is Measured on a Logarithmic Scale* [Paper 190]. Zenodo. 10.5281/zenodo.22106182
12. Yamagishi, Y. (2026). *“Complete” Is Four Separate Statements, and What Ties Them Into One Is That the Metric Is Positive Definite* [Paper 201]. Zenodo. 10.5281/zenodo.22109052
13. Yamagishi, Y. (2026). *Bounded by a Sum Is Not One Property* [Paper 243]. Zenodo. 10.5281/zenodo.22159642
14. Yamagishi, Y. (2026). *4π Appears Only When the Source Is a Point* [Paper 258]. Zenodo. 10.5281/zenodo.22169011
15. Yamagishi, Y. (2026). *What Separated the Two Low-Temperature Models Is Not Quantisation but the Density of States* [Paper 265]. Zenodo. 10.5281/zenodo.22169749

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