

That the Triple Point Is a Point Is Not an Experimental Fact

— It Follows from the Gibbs Phase Rule $F = C - P + 2$ — Dry Ice Sublimes and Ice Melts by the Size of One
and the Same Number — [Paper 268]

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— *From thinking to syncing.* —

Abstract

On a phase diagram the triple point is a point. The coexistence of two phases is a line, and a single-phase region is an area. This paper asks why it is a point—the answer is not that measurement found it so. The Gibbs phase rule $F = C - P + 2$ settles it before any measurement. No new mathematical theorem and no new law is claimed. First, count the degrees of freedom. For a pure substance ($C = 1$), one phase gives $F = 2$, two phases $F = 1$, and three phases $F = 0$ (Section 2). Second, this is the core of the paper. $F = 0$ means not one variable may be moved—so the place where three phases coexist can only be a single point (Section 3). Third, the number becomes the figure. $F = 2$ is an area, $F = 1$ a line, $F = 0$ a point—the appearance of a phase diagram is the result of a count (Section 3). Fourth, one number settles subliming against melting. Compare the triple-point pressure with one atmosphere—water gives **0.0060** and carbon dioxide **5.1172** (Section 4). Fifth, that comparison divides everyday life. Water, whose ratio is below **1**, melts; carbon dioxide, whose ratio is above **1**, sublims without ever becoming liquid (Section 4). Sixth, the two triple-point pressures differ by a factor of **847.70.611657** kPa against 518.5 kPa—the difference in behaviour between ice and dry ice comes out of this one number (Section 4).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed—the Gibbs phase rule, the triple-point values, and the distinction between subliming and melting are all standard. No thermodynamics is built—what is used is one subtraction and one division. The phase rule is not derived— $F = C - P + 2$ is received as a premise, and the counting of equalities of chemical potential is not entered. The criticality of phase transitions is not treated—the 20 occurrences of critical points in Papers 08, 117, 120, 203–205, 209 and 212 lie on the side of critical phenomena, whereas this paper counts only phases and degrees of freedom. Mechanical degrees of freedom are not conflated with these—the 129 occurrences of “degrees of freedom” in earlier papers are those of dynamics and statistics. The F here is the number of intensive variables that may be moved independently, a different quantity. No triple point is measured—all values are cited. The standing of the triple point of water is not entered—until the SI revision of 2019, 273.16 K was a defined value, and it became a measured one after. That change of standing is not a subject here. Metastable states and supercooling are not treated—only equilibrium is examined. Relation to earlier papers: Paper 257 treated that a catalyst cannot move an equilibrium and that Le Chatelier is not a theorem—that concerned rate and equilibrium, while this paper concerns the number of phases and of degrees of freedom. Siblings with different questions. Paper 201 counted “complete” as four different claims—this paper writes that the figures of a phase diagram are the result of a count. Paper 240 counted “mass” as seven things—the same shape of roll call. Paper 190 measured rare on a logarithmic scale—this paper likewise writes the difference of triple-point pressures as the ratio 847.7. What is added is setting the correspondence between degrees of freedom and the dimension of the figure in a table of three rows, reducing subliming against melting to the single number “triple-point pressure over one atmosphere”, computing that ratio for four substances and seeing that only carbon dioxide crosses 1, and obtaining the factor 847.7 between the triple-point pressures of water and carbon dioxide.

1. Notation and Placement

Setting: the Gibbs phase rule reads

$$F = C - P + 2$$

with C the number of components, P the number of coexisting phases, and F the number of intensive variables that may be moved independently. The trailing 2 accounts for temperature and pressure.

Question: on a phase diagram, why is the triple point a point?

The answer is not because it was measured. Because $F = 0$ —it is settled before any measurement.

A caution: the degrees of freedom here are a count of intensive variables. The 129 occurrences in earlier papers are those of dynamics and statistics, a different quantity—the two are not conflated.

2. Counting the Degrees of Freedom

Check 2.1: count for a pure substance ($C = 1$).

Phases P	$F = 1 - P + 2$	What may be moved	Figure
1 (one phase)	2	both temperature and pressure	an area
2 (two coexisting)	1	one of the two only	a line
3 (three coexisting)	0	neither	a point

Observation 2.2

each added phase lowers the freedom by one. It reaches 0 at three phases and stops there—for a pure substance four phases cannot coexist (that would give $F = -1$).

Observation 2.3

F comes out of a subtraction of integers. Not the name of the substance, nor the shape of its molecule, nor the strength of its bonds enters—the same table holds for every pure substance.

3. The Core: F Becomes the Dimension of the Figure

Observation 3.1

if $F = 2$, temperature and pressure may be moved independently without changing the phase—hence an area. If $F = 1$, fixing one fixes the other—hence a line. If $F = 0$, both are fixed—hence a point.

Observation 3.2

to look at a diagram and say “the triple point is a point” has the order backwards. It is not $F = 0$ because it is a point; it is drawn as a point because $F = 0$.

Check 3.3: with two components ($C = 2$) the same count raises the figure by one.

C	F at $P = 3$	Figure	Example
1	0	a point	the triple point
2	1	a line	a eutectic line
3	2	an area	—

Observation 3.4

adding a component raises the locus of three-phase coexistence from a point to a line. The separator—what has to be changed for the figure to change. The number of components or of phases, and not the substance itself.

4. Subliming or Melting Is the Size of One Number

Setting: below the triple-point pressure a solid passes to gas without a liquid stage. So what happens at one atmosphere (101.325 kPa) is settled by whether the triple-point pressure is above or below it.

Check 4.1: compare the triple-point pressure with one atmosphere for four substances.

Substance	Triple-point T	Triple-point p	Ratio to 1 atm	At 1 atm
Oxygen	54.36 K	0.146300 kPa	0.0014	melts
Water	273.16 K	0.611657 kPa	0.0060	melts
Nitrogen	63.15 K	12.520000 kPa	0.1236	melts
Carbon dioxide	216.59 K	518.500000 kPa	5.1172	sublimes

Observation 4.2

of the four, only carbon dioxide has a ratio above 1. So dry ice never becomes liquid and ice melts—the difference is only the size of this one number.

Observation 4.3

the triple-point pressures of water and carbon dioxide differ by a factor of **847.7**.0.611657 kPa against 518.5 kPa—the difference visible in a kitchen and in a laboratory comes out of this ratio.

Observation 4.4

the separator—what has to be changed to turn subliming into melting. The surrounding pressure. Apply 5.2 atmospheres and carbon dioxide too becomes liquid; drop below 0.006 and ice too sublimates—a matter of where a thing is placed, not of what it is.

5. Roll Call and Closing

1. Freedom is a subtraction(Check 2.1): $F = 1 - P + 2$ gives 2, 1, 0.
2. Four phases cannot coexist(Observation 2.2): that would give $F = -1$.
3. No name of a substance enters(Observation 2.3): the same table for every pure substance.
4. F is the dimension of the figure(Observation 3.1): area, line, point.
5. The order is backwards(Observation 3.2): it is drawn as a point because $F = 0$.
6. A component raises it a step(Check 3.3): from a point to a eutectic line.
7. One ratio decides subliming(Check 4.1): triple-point pressure over one atmosphere.
8. Only carbon dioxide exceeds 1(Observation 4.2): at **5.1172**.
9. A factor of **847.7**(Observation 4.3): **0.611657** kPa against 518.5 kPa.

Closing: that the triple point is a point was not a matter of how it looked in experiment. The phase rule $F = C - P + 2$ settles it before any measurement— $F = 2$ gives an area, $F = 1$ a line, $F = 0$ a point, and the dimension of the figure is the freedom itself. And the same count settles what is seen in everyday life. The triple-point pressure divided by one atmosphere is **0.0060** for water and **5.1172** for carbon dioxide—and merely by crossing 1, ice melts and dry ice sublimates. The two pressures differ by a factor of **847.7**, but what tells is the single point of which side of 1 the ratio falls on. One thing separates them—counting the phases and the components. Count them, and one knows whether the figure came first or the count did. Do not count them, and one reads the ratio 5.1172 as a peculiarity of a substance.

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