

“We Measured the Curvature” — of Which Manifold?

— Gauss's Triangle Saw the Surface of the Earth, Not Space — [Paper 27]

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— *From thinking to syncing.* —

ABSTRACT

A story is widely repeated: that Gauss, in his geodetic survey, measured the angles of a triangle to test whether space is flat. This paper passes no judgement on historical intent. It judges numbers. **NO NEW MATHEMATICS AND NO NEW PHYSICS IS CLAIMED.** First, the exact statement is Gauss–Bonnet. For a geodesic triangle, the angle sum minus π equals the integral of K over the area. On an octant of the unit sphere both sides come to $\pi/2$ (checked here) (Section 2). Second, the triangle does measure a curvature. With sides **69, 85, 107** km, Heron's formula gives an area of **2929.43** km², and against an earth radius of **6371** km the spherical excess is **14.8865 arcseconds** (computed here). That is comfortably visible to a nineteenth-century instrument (Section 3). Third, the brake. That is the curvature of the surface, not of space. The spatial slice around the earth has $|K| \sim (GM/c^2)/r^3 = 1.71503 \times 10^{-23} \text{ m}^{-2}$, giving over the same triangle a defect of only **1.03628×10^{-8} arcseconds**. The two differ by nine orders of magnitude (computed here) (Section 4). Fourth, the cosmological curvature is thirty-one orders below that again (**6.042×10^{-40} arcseconds**). And a perfect null result at **0.01 arcseconds** would bound the radius of curvature only above **38.58 earth radii** — as a test of the geometry of space the triangle is far too small (Section 5). Fifth, the sharpest fence is not precision at all. A survey's horizontal angles are reduced to the reference surface as part of the procedure. So the object being measured is, by construction, a triangle on that surface, before any question about space is even asked (Section 6). The separator is which manifold the triangle lies in (Section 7).

Scope of this paper: NO NEW MATHEMATICAL THEOREM AND NO NEW PHYSICAL LAW IS CLAIMED.

Gauss–Bonnet, spherical excess and the curvature of the Schwarzschild spatial slice are **all standard**. On what Gauss historically intended this paper decides nothing — reading the sources is the historians' work, and what is treated here is only whether the story holds up as numbers. What is added is **addresses and orders of magnitude for four curvatures**, an actual computation of the bound a null result would give, and the observation that the sharpest fence is the reduction procedure rather than the precision.

1. Notation and position

Write the Gaussian curvature of a surface as ***K***, the radius of a sphere as ***R*** and the area of a triangle as ***A***. One arcsecond is $\pi/(180 \times 3600)$ radians.

Position: Paper 91 treated intrinsic curvature (the Theorema Egregium) and Paper 128 restricted Poincaré–Hopf to one direction. **This paper returns to Gauss's side and shows, in orders of magnitude, that one phrase can name four different quantities.**

2. The exact statement is Gauss–Bonnet

Fact 2.1: For a geodesic triangle, $\sum \theta_i - \pi = \int K dA$. At constant curvature the right side is ***K**A***, and on a sphere ***A*/*R*²**.

Verification 2.2: On an octant of the unit sphere (three right angles, area **$4\pi/8$**)

angle sum $-\pi$	1.57079632679
A/R^2	1.57079632679
difference	0.0

(QED)

3. The triangle does measure a curvature

Verification 3.1: With sides **69, 85, 107** km,

area (Heron)	2929.43 km ²
$K = 1/R^2$ of the surface	$2.46368 \times 10^{-14} \text{ m}^{-2}$
spherical excess	$7.21717 \times 10^{-5} \text{ rad} = 14.8865 \text{ arcsec}$

(QED)

Observation 3.2

This is not a hidden small quantity. Fifteen arcseconds is a size triangulation dealt with routinely. Surveys correct for spherical excess precisely because it is visible.

4. The brake: that is the surface's curvature, not space's

Verification 4.1: Using $GM/c^2 = 4.4350 \times 10^{-3}$ m for the earth,

$ K \sim (GM/c^2)/r^3$ of the spatial slice	$1.71503 \times 10^{-23} \text{ m}^{-2}$
defect over the same triangle	$5.02404 \times 10^{-14} \text{ rad} = 1.03628 \times 10^{-8} \text{ arcsec}$
ratio to the surface term	1.43653×10^9

(QED)

Observation 4.2

The two differ by nine orders of magnitude. Neither a nineteenth-century instrument nor a twenty-first-century one separates them with this triangle. What the survey saw was the ground, not space.

Observation 4.3

The confusion arises because the Theorema Egregium (Paper 91) is true. The surface's curvature is intrinsic and can be measured from inside the surface alone. That is exactly why “measured from the inside” sounds like “measured from inside space” — but being intrinsic does not say whose intrinsic quantity it is.

5. The cosmological curvature, and what a null result bounds

Verification 5.1: Taking a conservative radius of curvature of 10^{27} m gives $|K| \lesssim 10^{-54} \text{ m}^{-2}$ and a defect over the same triangle of $2.929 \times 10^{-45} \text{ rad} = 6.042 \times 10^{-40} \text{ arcsec}$, **thirty-one orders below Section 4.** (QED)

Verification 5.2: Conversely, a null result at precision σ bounds $|K| < \sigma/A$.

precision	bound on $ K $ (m^{-2})	radius of curvature (m)	in earth radii
1 arcsec	1.655×10^{-15}	2.458×10^7	3.858
0.5 arcsec	8.275×10^{-16}	3.476×10^7	5.456
0.1 arcsec	1.655×10^{-16}	7.773×10^7	12.2
0.01 arcsec	1.655×10^{-17}	2.458×10^8	38.58

(QED)

Observation 5.3

Even a perfect null result at 0.01 arcseconds says only that the radius of curvature exceeds 38.58 earth radii. As a test of the geometry of space the triangle is far too small — and “too small” is a matter of area, not of instruments.

6. The sharpest fence is not precision at all

Observation 6.1

A survey's horizontal angles are reduced to the reference surface as part of the procedure. However good the instrument becomes, the “angles of the triangle” it outputs are quantities on that surface.

Observation 6.2

So the object being measured is, by construction, a triangle on that surface, before any question about space is asked. This is a sharper fence than Sections 4 and 5 — those say “too small to see”, this one says “a different object is being looked at”.

Observation 6.3

This paper passes no judgement on historical intent. What Gauss had in mind is a question about sources, and the only tool used here is arithmetic. What the arithmetic says is that, whatever the intent, this triangle could not have decided the curvature of space.

7. The census: four curvatures

Observation 7.1

name	which manifold	order of K (m ⁻²)	over this triangle
surface curvature	the earth's surface (2D)	2.5×10^{-14}	15 arcsec: measured
spatial curvature	the slice around the earth	1.7×10^{-23}	10^{-8} arcsec: not measurable
cosmological curvature	the cosmological slice	$< 10^{-54}$	10^{-40} arcsec: not measurable
spacetime curvature	tidal (4D)	—	not an angle at all: another quantity

Observation 7.2

The separator is which manifold the triangle lies in. Not difficulty, and not precision — the same word, the same figure and the same angle sum name four different quantities.

8. Conclusion

1. The exact statement is Gauss–Bonnet (Ver. 2.2): on an octant the difference is zero.
2. The survey did measure a curvature (Ver. 3.1): 14.8865 arcsec.
3. But it is the surface's (Ver. 4.1): the spatial term is 10^{-8} arcsec, nine orders below.
4. A null result bounds little (Ver. 5.2): even at 0.01 arcsec, only 38.58 earth radii.
5. The sharpest fence is the reduction procedure (Obs. 6.2).
6. The separator is which manifold (Obs. 7.1).

**Closing: The sentence “we measured the curvature” does not yet name a quantity until the manifold is named.
Gauss's triangle did lie on something curved. It was the ground we stand on.**

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