

One and the Same “Twice as Strong” Opens by a Factor of 6.70 on the Stimulus Side

— Fechner Stacked Discriminations and Stevens Asked for a Number — The Two Laws Measure Different Operations — [Paper 272]

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— *From thinking to syncing.* —

Abstract

The sentence “the sensation has doubled” does not settle how much the stimulus was raised. This paper asks how wide the range is—the answer is a factor of 6.70. And the two famous laws are not measuring the same thing to begin with. No new mathematical theorem and no new law is claimed. First, count the stimulus ratio required. Under the power law $S = kI^a$, doubling the sensation takes a stimulus ratio of $2^{1/a}$ (Section 2). Second, this is the core of the paper. For brightness ($a = 0.33$) it is **8.1698** and for electric shock ($a = 3.50$) it is **1.2190**—the same “twice” opens by a factor of **6.7020** on the stimulus side (Section 2). Third, only length gives exactly 2. At $a = 1.00$, $2^{1/1} = 2$ —the naive expectation holds in one row alone (Section 2). Fourth, the other law has a different shape. Fechner's is $S = k \ln(I/I_0)$, a logarithm—a power and a logarithm are two different shapes out of one phenomenon (Section 3). Fifth, what separates them is the operation. Fechner stacks just-noticeable differences (indirect); Stevens has the subject say a number (direct) (Section 4). Sixth, neither is measuring the magnitude of a sensation. With a Weber fraction of 0.10, doubling takes **7.2725** steps of discrimination—counting steps and asking for a number are different operations (Section 4).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed—Weber's law, Fechner's logarithmic law, Stevens's power law and the representative values of its exponents are all standard.No psychology is built—what is used is one exponentiation and one logarithm.Neither law is called correct—it is written only that they measure different operations, and no merit is judged between them.No exponent is measured—every value of a is a cited representative value. They vary between studies and between individuals—what this paper claims is the structure of the ordering, not the individual numbers.The reality of sensation is not discussed—whether “the magnitude of a sensation” is independently observable is not entered.Neural mechanism is not treated—neither receptors nor coding are treated.Weber is not confused with another—the 3 occurrences of Weber in the English earlier papers (11 and 55) are the Ghirardi-Rimini-Weber collapse model, a different person from the E. H. Weber of this paper.Applications are not discussed—neither volume scales nor display brightness are treated.Relation to earlier papers: Paper 190 measured rare on a logarithmic scale—this paper treats a case where a logarithm and a power both stand on one phenomenon.Paper 232 counted four different claims written with the same equals sign—this paper writes that one and the same “twice” carries a range of 6.70 on the stimulus side.Paper 201 counted “complete” as four different claims—the same shape of division.Paper 255 treated accuracy as two things—this paper stands on the same figure, that the operation of measuring settles the answer.What is added is computing $2^{1/a}$ for five senses and obtaining a range of 6.7020, observing that only length returns the naive 2, obtaining 7.2725 as the number of discriminations needed to double under a Weber fraction, and writing the two laws as a difference of operation, stacking against asking.

1. Notation and Placement

Setting: write the strength of a stimulus as I and the magnitude felt as S . There are two famous laws.

Law	Shape	How it is measured
Fechner (1860)	$S = k \ln(I/I_0)$	stacking discriminations (indirect)
Stevens (1957)	$S = kI^a$	asking the subject for a number (direct)

Question: when one says “the sensation has doubled”, by how much has the stimulus risen?

The answer is not settled by one number. There is a range of 6.70 across the senses, and it changes again with which law one measures by.

A caution: Weber in this paper means E. H. Weber. The 3 occurrences in earlier papers are the Ghirardi-Rimini-Weber model, a different person—the two are not conflated.

2. The Core: One “Twice” Opens by a Factor of 6.70

Setting: under the power law $S = kI^a$, doubling S takes a factor of $2^{1/a}$ in I .

Check 2.1: computed with the representative exponents.

Sense	Exponent a	Stimulus ratio to double
Brightness	0.33	8.1698
Loudness	0.67	2.8138
Length	1.00	2.0000
Heaviness	1.45	1.6129
Electric shock	3.50	1.2190

Observation 2.2

widest over narrowest is **6.7020**.One and the same phrase “twice as strong” means eight times the light for brightness and 1.2 times the current for shock.

Observation 2.3

only length returns 2.0000.Because $a = 1$ —the naive expectation holds in this row alone.

Observation 2.4

the direction reverses across $a = 1$.Brightness and loudness blunt (a large ratio is needed), while shock sharpens (a slight rise feels twice as strong)— $a = 1$ is the boundary.

Observation 2.5

the separator—what has to be changed for the ratio to change.The exponent a alone.Neither the target of doubling nor the shape of the law has been moved.

3. The Other Law Has a Different Shape

Observation 3.1

Fechner's shape is a logarithm.Under $S = k \ln(I/I_0)$, doubling S takes squaring I/I_0 —so the ratio required depends on how far one already stands from the reference I_0 .

Check 3.2: under Fechner's law, the stimulus ratio needed to double S , by starting point.

Starting I/I_0	After doubling S	Stimulus ratio
2	4	2.0000
10	100	10.0000
100	10000	100.0000

Observation 3.3

under Fechner's law the required ratio depends on the starting point.Under the power law it is the same $2^{1/a}$ wherever one starts—here the two laws differ decisively.

Observation 3.4

so the two do not stand as an approximation and an exact form.Their structures differ—the power law needs no reference point and the logarithmic law does.

4. What Separates Them Is the Operation

Observation 4.1

Fechner integrated Weber's law, that the just-noticeable difference is proportional to the stimulus, and obtained a logarithm.What is measured is the smallest change that can be told apart, and not the magnitude of a sensation.

Check 4.2: with a Weber fraction of **0.10**, stacking ***n*** discriminations multiplies the stimulus by **1.1^n** .

Steps <i>n</i>	Stimulus factor 1.1^n
1	1.1000
7	1.9487
10	2.5937
24	9.8497

Observation 4.3

doubling the stimulus takes $\ln 2 / \ln 1.1 = \mathbf{7.2725}$ steps.Fechner identified that count of steps with the magnitude of the sensation—and that is the assumption.

Observation 4.4

Stevens performed a different operation.He asked the subject to say by what number one tone exceeded another—what is measured is an assignment of numbers, which is also not the magnitude of a sensation.

Observation 4.5

the separator—what has to be changed for a different law to come out.Which operation one measures by.Count the steps and a logarithm comes out; ask for a number and a power does—measuring the same sense in the same person, a different operation gives a different shape.

Observation 4.6

a point to state honestly—neither measures the magnitude of a sensation directly.Whether sensation is an independently observable quantity is not entered here—what can be said is only that a different operation gives a different answer.

5. Roll Call and Closing

1. The ratio is $2^{1/a}$ (Section 2): **under the power law.**
2. **8.1698** for brightness(Check 2.1): **at $a = 0.33$.**
3. **1.2190** for electric shock(Check 2.1): **at $a = 3.50$.**
4. A range of **6.7020**(Observation 2.2): **for the same “twice”.**
5. Only length gives **2.0000**(Observation 2.3): **the row at $a = 1$.**
6. **$a = 1$** is the boundary(Observation 2.4): **blunting or sharpening.**
7. Fechner depends on the start(Check 3.2): **ratios of 2, 10 and 100.**
8. The power law needs no reference(Observation 3.4): **the structures differ.**
9. **7.2725** steps to double(Observation 4.3): **at a Weber fraction of 0.10.**
10. The separator is the operation(Observation 4.5): **stacking, or asking.**

Closing: the sentence “the sensation has doubled” did not settle how much the stimulus was raised. For brightness it is **8.1698** and for electric shock **1.2190**, a range of **6.7020**—and the naive 2 holds in the single row of length. Further, there are two laws. Under Fechner's logarithm the required ratio depends on the starting point and under Stevens's power it does not—their structures differ, so neither is an approximation to the other. And what separates the two is the operation of measuring—stack the discriminations and a logarithm comes out; ask for a number and a power does. Neither measures the magnitude of a sensation directly. One thing separates them—writing down which operation the measurement was. Write it down, and the two laws appear as different ways of measuring rather than as rivals. Do not write it down, and one sets 8.1698 and 1.2190 side by side as the same “twice”.

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