

Drude Was Right Because Two Errors Cancelled

— The Heat Capacity Too Large by **82.68** and the Squared Speed Too Small by **180.70** — Their Product Is **2.1856**, Against the Actual Discrepancy of **2.1932** — [Paper 274]

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— *From thinking to syncing.* —

Abstract

Drude (1900) obtained the ratio of thermal to electrical conductivity in a metal from classical theory alone, and got a value close to measurement. This paper asks why he was right—the answer is not that the premises were correct. Two premises erred largely in opposite directions and very nearly cancelled on taking a ratio. No new mathematical theorem and no new law is claimed. First, set the two answers out. Drude gives $L = \frac{3}{2}(k/e)^2 = 1.11388 \times 10^{-8}$ and Sommerfeld $L = \frac{\pi^2}{3}(k/e)^2 = 2.44300 \times 10^{-8}$ (Section 2). Second, measurement lies near the latter. Copper 2.23×10^{-8} and silver 2.31×10^{-8} —Drude is about half (Section 2). Third, this is the core of the paper. For the heat capacity, classical theory is **82.68** times too large—because only the fraction kT/E_F of the electrons contributes, and it took all of them to (Section 3). Fourth, the other errs the other way. For the squared speed, classical theory is **180.70** times too small—because what carries the heat is not the thermal speed but the Fermi speed (Section 3). Fifth, the product remains. $180.70/82.68 = 2.1856$ —agreeing with the actual ratio of the two Lorenz numbers, **2.1932**, to **0.35%** (Section 4). Sixth, the separator is taking a ratio. Look at κ alone or at σ alone and classical theory is out by orders of magnitude—only on taking the ratio do the errors cancel down to a factor of about **2** (Section 5).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed—the Wiedemann-Franz law, the Drude model, Sommerfeld’s Lorenz number, and the expressions for the Fermi speed and electronic heat capacity are all standard. No solid-state physics is built—what is used is two ratios and one product. The Drude model is not re-derived— $L = \frac{3}{2}(k/e)^2$ is cited only, and the relaxation-time approximation is not entered. The Sommerfeld expansion is not treated—the origin of the $\pi^2/3$ is cited. Failures of the Wiedemann-Franz law are not treated—that it fails at low temperature and under inelastic scattering is not treated. This paper looks at one point near room temperature. No precision is claimed for the measured values—the values for copper and silver are representative and vary with temperature and purity. The 82.68 and 180.70 are not claimed as precise—they are numbers for seeing orders of magnitude, obtained by inserting representative values of the Fermi temperature and speed. That the agreement fell within 0.35% is partly because those representative values happened to sit well. Sommerfeld is not confused with another work—the 4 occurrences in earlier papers are Bohr-Sommerfeld quantisation (Paper 180), signal velocity (Paper 210) and a biographical mention (Paper 125), not the free-electron model. Relation to earlier papers: Paper 265 showed that what separated the two low-temperature models is the density of states—this paper also treats an electronic heat capacity, but asks after the cancellation of errors. Paper 255 showed that accuracy is two things and calibration removes only one—this paper treats a case where two errors survive as a product. Paper 190 measured rare on a logarithmic scale—this paper likewise writes the error as a factor. Paper 271 counted how far mean field errs by dimension—this paper counts two errors that cancel. Paper 232 counted the standings of one equals sign—this paper asks after the standing of “he was right”. What is added is putting the excess of the heat capacity at 82.68 and the shortfall of the squared speed at 180.70, confirming that their product 2.1856 agrees with the actual discrepancy 2.1932 to 0.35%, and writing that without taking a ratio classical theory is out by orders of magnitude.

1. Notation and Placement

Setting: in a metal the ratio of the thermal conductivity κ to the electrical conductivity σ is proportional to the temperature (the Wiedemann-Franz law). The constant of proportionality, $L = \kappa/(\sigma T)$, is the Lorenz number.

Question: Drude obtained this L from classical theory alone and got a value close to measurement. Why was he right?

The answer is not that the premises were correct. It is that two premises erred in opposite directions and cancelled in the ratio.

A caution: Sommerfeld in this paper means the free-electron model. The 4 occurrences in earlier papers are Bohr-Sommerfeld quantisation, signal velocity and a biography, different work—the two are not conflated.

2. Two Answers, and Measurement

Setting: both theories start from $\kappa = \frac{1}{3}c_v v^2 \tau$ and $\sigma = ne^2 \tau / m$. On taking the ratio the τ cancels, leaving only c_v and v^2 .

Check 2.1: with $(k/e)^2 = 7.425843 \times 10^{-9}$.

	Expression	L [W Ω K $^{-2}$]
Drude (1900)	$\frac{3}{2}(k/e)^2$	1.11388 $\times 10^{-8}$
Sommerfeld (1928)	$\frac{\pi^2}{3}(k/e)^2$	2.44300 $\times 10^{-8}$
Measured (copper)	—	2.23 $\times 10^{-8}$
Measured (silver)	—	2.31 $\times 10^{-8}$

Observation 2.2

Drude’s value is about half of measurement. In 1900 the measurements were still coarse, and getting the order of magnitude right looked like a great success—and indeed the order is right.

Observation 2.3

the ratio of the two theories is $(\pi^2/3)/(3/2) = \mathbf{2.1932}$. This 2.1932 is the number this paper must account for.

3. The Core: Counting the Two Errors Separately

Setting: take copper, with a Fermi temperature $T_F = 8.16 \times 10^4$ K, a Fermi speed $v_F = 1.57 \times 10^6$ m/s, and room temperature $T = 300$ K.

Check 3.1: the first error, the heat capacity. Classical theory has every electron contribute $\frac{3}{2}k$, whereas only the fraction kT/E_F of them does.

Quantity	Value
Classical electronic heat capacity	$\frac{3}{2}nk$
Actual (Sommerfeld)	$\frac{\pi^2}{2}nk(T/T_F)$
T/T_F	3.6765×10^{-3}
Factor by which the classical is too large	82.68

Observation 3.2

classical theory overestimates the electronic heat capacity by **82.68**. This was a known difficulty at the time, that no electronic contribution to the heat capacity of a metal could be seen.

Check 3.3: the second error, the squared speed. Classical theory uses the thermal speed $v^2 = 3kT/m$, whereas what actually carries the heat are the electrons at the Fermi surface, moving at v_F .

Quantity	Value [m^2/s^2]
Classical $v^2 = 3kT/m$	1.3641×10^{10}
Actual v_F^2	2.4649×10^{12}
Factor by which the classical is too small	180.70

Observation 3.4

the directions are opposite. Too large by 82.68 in the heat capacity and too small by 180.70 in the squared speed—both errors of two orders of magnitude.

Observation 3.5

both errors come from one and the same cause. Classical theory did not know that electrons obey Fermi statistics (the Pauli principle)—one oversight moved two quantities in opposite directions.

4. The Product Remains

Check 4.1: since $\kappa \propto c_v v^2$, the two errors tell **as a product**.

Quantity	Value
Excess in the heat capacity	$\div 82.68$
Shortfall in the squared speed	$\times 180.70$
Factor remaining, $180.70/82.68$	2.1856
Actual discrepancy $(\pi^2/3)/(3/2)$	2.1932
Disagreement	0.35%

Observation 4.2

two errors of two orders of magnitude, and their product leaves only **2.19**. Most of both the 82.68 and the 180.70 has vanished inside the ratio.

Observation 4.3

the disagreement between 2.1856 and 2.1932 is 0.35%. That lies within the rounding of the representative values, so the account by cancelling errors holds up as a number.

Observation 4.4

a point to state honestly—representative values were inserted for T_F and v_F , so the closeness of the 0.35% agreement is partly fortuitous. What is claimed is the structure of the orders of magnitude, not this decimal.

5. The Separator Is Taking a Ratio

Check 5.1: look at κ and σ separately, without taking a ratio.

Quantity	How classical theory errs	Does it cancel?
Electrical conductivity σ	depends on the estimate of τ	—
Thermal conductivity κ	errs in both c_v and v^2	no
Electronic heat capacity c_v alone	too large by 82.68	no
The ratio $\kappa/(\sigma T)$	a factor of 2.19	yes

Observation 5.2

the separator—what has to be changed for the error to become visible. Ceasing to take a ratio. With the same theory and the same premises, merely changing which quantity one looks at makes the error appear as a factor of 82.68.

Observation 5.3

so one cannot say in a word that Drude was right, nor that he was wrong. The answer changes with the quantity one is speaking of—right about the ratio, out by two orders about the heat capacity.

Observation 5.4

Paper 255 wrote that repetition and calibration are each connected to one quantity alone. This paper is the converse, a case where two errors meet inside one quantity and cancel—errors are not always independent.

6. Roll Call and Closing

1. Two answers(Check 2.1): 1.11388×10^{-8} and 2.44300×10^{-8} .
2. Measurement lies near the latter(Check 2.1): copper 2.23, silver 2.31.
3. The heat capacity is too large by 82.68(Check 3.1): $T/T_F = 3.6765 \times 10^{-3}$.
4. The squared speed is too small by 180.70(Check 3.3): thermal against Fermi speed.
5. The directions are opposite(Observation 3.4): both of two orders.
6. One cause(Observation 3.5): Fermi statistics were not known.

7. The product is **2.1856**(Check 4.1): **against an actual 2.1932**.
8. A disagreement of **0.35%**(Observation 4.3): **the account holds as a number**.
9. Without a ratio it shows(Check 5.1): **a factor of 82.68 in the heat capacity alone**.

Closing: Drude was right not because the premises were correct. Two premises erred by two orders of magnitude each, in opposite directions, and cancelled on taking a ratio—the heat capacity too large by **82.68**, the squared speed too small by **180.70**, and their product **2.1856**. That agrees with the actual ratio of the two Lorenz numbers, **2.1932**, to **0.35%**. And both errors came from one and the same oversight—that electrons obey Fermi statistics. One oversight moved two quantities in opposite directions, and they met inside the ratio and vanished. The separator is taking a ratio—cease to take it and, with the same theory, the error appears as a factor of 82.68. One thing separates them—writing down which quantity one means when one says he was right. Write it down, and being right about the ratio separates from being out by two orders about the heat capacity. Do not write it down, and one reads a fortunate cancellation as evidence that the premises were sound.

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