

The Decibel Is a Ratio and Not a Quantity

— One and the Same “Twice” Becomes **6.0206** dB, **3.0103** dB, or **+10** Phon — Without a Reference Written
Down, a Decibel Is Not Even a Number — [Paper 276]

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— *From thinking to syncing.* —

Abstract

The sentence “it went up by 3 dB” does not settle what became how many times larger. This paper asks how many ways there are—the answer is that three different numbers answer to one and the same “twice”. And all three rest on different grounds. No new mathematical theorem and no new law is claimed. First, write one “twice” in three ways. For sound pressure it is **6.0206** dB, for sound power **3.0103** dB, and for loudness **+10** phon (Section 2). Second, this is the core of the paper. The difference between 20 and 10 is that power goes as the square of amplitude, and the +10 phon comes from a power law—all three rest on different grounds (Section 3). Third, the references are three separate things too. Sound pressure level takes $20\mu\text{Pa}$, sound power level 1 pW, and loudness takes 1 kHz at 40 dB as 1 sone (Section 4). Fourth, “3 dB” is not exactly twice. A power ratio of 3 dB is **1.9953**—out by exactly as much as rounding 3.0103 to 3 (Section 2). Fifth, one decibel figure means different factors for different quantities. 20 dB is **10** times in pressure and **100** times in power (Section 2). Sixth, this is the converse of Paper 273. There, one unit carried four quantities—here, one symbol, dB, carries no quantity at all until a reference is written (Section 5).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed—the definition of the decibel, the references for sound pressure and sound power level, and the sone and phon scales are all standard. No acoustics is built—what is used is a logarithm and the comparison of three references. Hearing is not discussed—neither the origin of the equal-loudness contours nor individual variation is entered. The sone scale is not derived—that a doubling of the sone corresponds to +10 phon is cited as a convention. Acoustic measurement in practice is not treated—neither frequency weightings (A-weighting and the like) nor time constants are treated. The psychophysical laws are not re-examined—Paper 272 treated Fechner and Stevens. This paper is its acoustic counterpart and does not enter the question of which law is right. The decibel of electrical engineering is not treated—the same difference of 20 and 10 arises for voltage and power ratios, but this paper is confined to sound. Relation to earlier papers: Paper 272 showed that one “twice as strong” opens by a factor of 6.70 on the stimulus side—this paper is its acoustic counterpart, and the +10 phon comes out of Stevens’s power law. The two make a pair. Paper 273 counted four different quantities written in the same J/kg—this paper runs the other way, writing that one symbol carries no quantity until a reference is written. Paper 190 measured rare on a logarithmic scale—this paper writes that the logarithmic scale itself comes in three kinds. Paper 255 treated accuracy as two things—the same figure. Paper 201 counted “complete” as four different claims—the same shape of roll call. What is added is setting 6.0206, 3.0103 and +10 against one “twice”, distinguishing the three grounds, obtaining that 3 dB is 1.9953 rather than exactly twice, and setting out that 20 dB is 10 times in pressure and 100 times in power.

1. Notation and Placement

Setting: the decibel is the logarithm of a ratio of two quantities. For power-like quantities one uses $10 \log_{10}$ and for amplitude-like quantities $20 \log_{10}$.

Question: when one says “it went up by 3 dB”, what became how many times larger?

The answer is that it is not settled. Without writing which quantity’s ratio it is and what the reference is, a decibel is not even a number.

A caution: this paper is confined to sound. The same difference of 20 and 10 arises in electrical engineering and is not treated.

2. The Core: One “Twice” Becomes Three Numbers

Check 2.1: set out how a doubling is written in decibels.

What doubled	Expression	Value
Sound pressure	$20 \log_{10} 2$	6.0206 dB
Sound power	$10 \log_{10} 2$	3.0103 dB
Loudness (1 → 2 sone)	by convention	+10 phon

Observation 2.2

three different numbers answer to one and the same “twice”. All are correct, and all say different things.

Check 2.3: conversely, set out what one decibel figure means for each quantity.

Decibels	Factor in pressure	Factor in power
3 dB	1.4125	1.9953
6 dB	1.9953	3.9811
10 dB	3.1623	10.0000
20 dB	10.0000	100.0000

Observation 2.4

20 dB is **10** times in pressure and **100** times in power. The same figure means factors differing by ten, according to the quantity.

Observation 2.5

and “**3 dB** is exactly twice” is not accurate. The power ratio is **1.9953**—out by exactly as much as rounding **3.0103** to **3**. The difference is **0.24%**, and it is a rounding and not a definition.

Observation 2.6

a pleasant detail: the pressure ratio at 6 dB is also 1.9953. It is the same number as the power ratio at 3 dB—both being $10^{3/10}$, and no coincidence.

3. The Three Grounds Are Separate

Observation 3.1

the first ground: the difference between 20 and 10 is that power goes as the square of amplitude. Since $P \propto p^2$, $10 \log_{10}(p^2/p_0^2) = 20 \log_{10}(p/p_0)$ —a consequence of physics.

Check 3.2: 20 divided by 10 is 2.0000. That is the exponent of the square itself.

Ground	Where it comes from	Standing
20 against 10	$P \propto p^2$	a consequence of physics
+10 phon = 2 sone	subjects assigning numbers	a convention (from a power law)
The references (20 μ Pa and the rest)	agreement	a convention

Observation 3.3

+10 phon = 2 sone does not come out of physics. The exponent of the loudness power law is about 0.6, so doubling the sensation takes roughly ten times the sound power—Stevens’s power law, treated in Paper 272, is embedded here as a convention.

Observation 3.4

the separator—what has to be changed for the number to change. Which quantity’s ratio is being taken. Neither the sound itself nor the listener has been moved at all.

4. The References Are Three Separate Things Too

Check 4.1: set out the reference of each scale.

Scale	Reference	Unit
Sound pressure level	20μPa	dB
Sound power level	1 pW	dB
Loudness	1 kHz at 40 dB SPL = 1 sone	sone / phon

Observation 4.2

all three references differ. So a reading of “80 dB” has no settled meaning until one writes which reference the 80 is from.

Observation 4.3

the reference 20μPa is itself a convention. It was chosen as roughly the threshold of hearing at 1 kHz—an agreement, not a measurement.

Observation 4.4

a point to state honestly—in practice a frequency weighting such as A-weighting is applied, giving yet another number. This paper does not enter that—including weightings would only increase what must be distinguished.

5. This Is the Converse of Paper 273

Observation 5.1

Paper 273 counted four different quantities written in the same unit J/kg. This paper runs the other way—the symbol dB carries no quantity until a reference is written.

Check 5.2: set the two side by side.

Paper 273		This paper
Symbol	J/kg	dB
The trouble	one unit, four quantities	one symbol, no quantity
What is missing	a name	a reference

Observation 5.3

J/kg has a dimension, and the decibel is dimensionless. Being the logarithm of a ratio, it points at nothing until one writes what it was divided by.

Observation 5.4

Paper 272 showed that “twice as strong” opens by a factor of 6.70 on the stimulus side. This paper is its acoustic counterpart, in which one “twice” becomes 6.0206, 3.0103 and +10—the two make a pair.

6. Roll Call and Closing

- 1. One twice, three numbers(Check 2.1): 6.0206, 3.0103, +10.
- 2. 20 dB is 10 and 100(Check 2.3): factors differing by ten.
- 3. “3 dB is twice” is a rounding(Observation 2.5): it is 1.9953.
- 4. 6 dB in pressure is also 1.9953(Observation 2.6): both are 10^{3/10}.
- 5. 20 against 10 is physics(Observation 3.1): from $P \propto p^2$.
- 6. +10 phon is a convention(Observation 3.3): embedded from a power law.
- 7. Three separate references(Check 4.1): 20μPa, 1 pW, 1 sone.
- 8. 20μPa is a convention too(Observation 4.3): not a measurement.
- 9. The converse of Paper 273(Check 5.2): one symbol with no quantity.

Closing: the decibel was a ratio and not a quantity. One and the same “twice” becomes 6.0206 dB in sound pressure, 3.0103 dB in sound power, and +10 phon in loudness—and all three rest on different grounds. The 20 against 10 is a consequence of physics (power goes as the square of amplitude), the +10 phon is a convention drawn from a power

law, and the references are agreements. Conversely one and the same 20 dB means 10 times in pressure and 100 times in power. And “3 dB is exactly twice” is itself a rounding: it is 1.9953. One thing separates them—writing down which quantity’s ratio, against what reference. Write it down, and 6.0206 and 3.0103 are seen to be one thing written two ways. Do not write it down, and one writes a sentence in which 20 dB can be read as ten times or as a hundred.

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