

# A Solid Angle Is Hidden inside the 0.161 of the Reverberation Time

— The 4 in the Mean Free Path  $4V/S$  Is  $4\pi$  Divided by  $\pi$  — And the 0.161 Is Not a Constant — [Paper 277]

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— *From thinking to syncing.* —

## Abstract

Sabine's formula  $T_{60} = 0.161V/(S\alpha)$  appears in every textbook of acoustics. This paper asks where that 0.161 came from—the answer is a solid angle. The 4 in the mean free path  $4V/S$  turned out to be  $4\pi$  divided by  $\pi$ . No new mathematical theorem and no new law is claimed. First, take the 0.161 apart. The number of reflections needed to fall 60 dB is  $6 \ln 10 / \alpha$  and each takes  $4V/(cS)$ —their product is  $24 \ln 10 \cdot V/(cS\alpha)$  (Section 2). Second,  $24 \ln 10 = 55.262042$ . Divided by a sound speed of 343 this gives **0.161114**—the 0.161 of the textbooks (Section 2). Third, this is the core of the paper. The 4 in  $4V/S$  is the whole solid angle  $4\pi$  divided by the  $\pi$  obtained by integrating  $\cos$  over a hemisphere—numerical integration returns **4.0000000000** (Section 3). Fourth, the same 4 stands in the kinetic theory of gases. It is the same thing as the 4 in the wall collision rate  $nv/4$  (Section 3). Fifth, drop the 4 and the formula breaks. In a room  $10 \times 8 \times 3$  m the  $T_{60}$  falls from **0.7214** s to 0.1804 s—a factor of 4 (Section 4). Sixth, the 0.161 is not a constant. At 0°C it is **0.166804** and at 40°C it is **0.155712**—because the sound speed sits inside it (Section 5).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed—Sabine’s reverberation formula, Cauchy’s mean chord formula ( $4V/S$  for a convex body), and the  $nv/4$  of the kinetic theory of gases are all standard. No acoustics is built—what is used is one integral over solid angle and one multiplication. Sabine’s formula is not derived—the assumption of a diffuse field, that sound arrives equally from every direction, is received as a premise. Whether it holds in a real room is not treated. Cauchy’s formula is not proved—that the mean chord of a convex body is  $4V/S$  is cited. No absorption coefficient is claimed—every  $\alpha$  is a posited value, not a measurement on a material. Eyring’s formula is not treated—in rooms with large  $\alpha$  Sabine’s formula fails and Eyring’s is used. This paper looks only inside Sabine’s formula. The effect of room shape is not treated—convexity is assumed, and concave or coupled rooms are not treated. The  $4\pi$  is not called a discovery—Paper 02 treated the  $4\pi$  of an inverse-square field, Paper 258 the conditions for it, and Paper 265 the  $4\pi$  inside a density of states. This paper only adds one to that family. Relation to earlier papers: Paper 02 showed that a pure solid angle  $4\pi$  appears in an inverse-square field—this paper writes a case where the  $4\pi$  appears as the thing being divided. Paper 258 wrote the condition for  $4\pi$  as point source, three dimensions and isotropy—the 4 here uses isotropy alone, and neither a point source nor an inverse square. Paper 265 found the  $4\pi$  inside a density of states—this is the third of the same family. Paper 190 measured rare on a logarithmic scale—the  $6 \ln 10$  here likewise comes from the logarithmic scale of 60 dB. What is added is taking the 0.161 apart into  $24 \ln 10/c$ , confirming by numerical integration that the 4 of  $4V/S$  is the solid-angle ratio  $4\pi/\pi$ , equal to 4.0000000000, writing that it shares a root with the  $nv/4$  of kinetic theory, and obtaining that the 0.161 runs from 0.1668 to 0.1557 with temperature.

## 1. Notation and Placement

**Setting:** let a room have volume  $V$ , surface area  $S$  and mean absorption coefficient  $\alpha$ . The time for the sound pressure level to fall 60 dB after the source stops is the reverberation time  $T_{60}$ .

**Sabine’s formula:**

$$T_{60} = 0.161 \frac{V}{S\alpha}$$

**Question:** where did that 0.161 come from?

The answer is a solid angle. More exactly, three things are multiplied into it: a logarithmic scale (60 dB), a sound speed, and a ratio of solid angles.

**A caution:** this paper assumes a diffuse field, sound arriving equally from every direction. Whether that holds in a real room is not treated.

## 2. Taking the 0.161 Apart

**Setting:** at each reflection a fraction  $\alpha$  of the sound is absorbed. After  $n$  reflections the intensity is  $(1 - \alpha)^n$ .

**Check 2.1:** 60 dB is  $10^{-6}$  in intensity. For small  $\alpha$ ,  $(1 - \alpha)^n \approx e^{-\alpha n}$ , so  $e^{-\alpha n} = 10^{-6}$  gives  $n = 6 \ln 10 / \alpha$ .

| Absorption $\alpha$ | Reflections to fall 60 dB |
|---------------------|---------------------------|
| 0.1                 | 138.16                    |
| 0.2                 | <b>69.08</b>              |
| 0.5                 | 27.63                     |

**Check 2.2:** the time for one reflection is the mean free path divided by the sound speed. In a convex room the mean free path is  $4V/S$  (Cauchy’s formula), so each reflection takes  $4V/(cS)$  seconds.

**Check 2.3:** multiply the two.

| Quantity                                                                               | Value            |
|----------------------------------------------------------------------------------------|------------------|
| $T_{60} = \frac{6 \ln 10}{\alpha} \cdot \frac{4V}{cS} = \frac{24 \ln 10 V}{cS \alpha}$ | —                |
| $24 \ln 10$                                                                            | <b>55.262042</b> |
| $24 \ln 10 / 343$                                                                      | <b>0.161114</b>  |

### Observation 2.4

the 0.161 of the textbooks has come out. Its contents are three factors—the  $6 \ln 10$  from the 60 dB, the 4 from the mean free path, and the sound speed  $c$ .

### 3. The Core: the 4 Is a Ratio of Solid Angles

**Observation 3.1**

where does the 4 in  $4V/S$  come from? It appears on counting the rate at which isotropically moving particles strike a wall—one integrates the cosine of the angle between the direction of travel and the wall normal over a hemisphere in solid angle.

**Check 3.2:** compare the two solid angles by numerical integration.

| Quantity                                     | Value                           |
|----------------------------------------------|---------------------------------|
| $\int \cos \theta d\Omega$ over a hemisphere | <b>3.1415926536</b> ( $= \pi$ ) |
| The whole solid angle $4\pi$                 | <b>12.5663706144</b>            |
| The ratio $4\pi/\pi$                         | <b>4.0000000000</b>             |

**Observation 3.3**

the 4 in  $4V/S$  was the whole solid angle divided by the cosine-weighted solid angle of a hemisphere. Divide  $4\pi$  by  $\pi$  and one gets exactly 4—not a coincidental 4, but a ratio of solid angles.

**Observation 3.4**

the same 4 stands in the kinetic theory of gases. It is the 4 of the wall collision rate  $nv/4$ —the same number out of the same integral, and not a distinct root.

**Observation 3.5**

Paper 258 wrote the condition for  $4\pi$  as point source, three dimensions and isotropy. The 4 here uses isotropy alone—neither a point source nor the inverse-square law enters. The same  $4\pi$  appearing in another form under other conditions.

### Observation 3.6

the separator—what has to be changed for the 4 to change. Isotropy. If sound arrives preferentially from certain directions, the average of the cosine changes and so does the 4.

## 4. Drop the 4 and the Formula Breaks

Check 4.1: compute for a room  $10 \times 8 \times 3$  m ( $V = 240\text{m}^3$ ,  $S = 268\text{m}^2$ ).

| Quantity                           | Value           |
|------------------------------------|-----------------|
| Mean free path $4V/S$              | <b>3.5821</b> m |
| Reflections per second, $c/(4V/S)$ | <b>95.75</b>    |
| $T_{60}$ at $\alpha = 0.2$         | <b>0.7214</b> s |
| $T_{60}$ with the 4 dropped        | 0.1804 s        |
| Ratio                              | <b>4.0000</b>   |

### Observation 4.2

dropping the 4 makes the reverberation time a quarter of what it is. 0.72 s, a natural value for a meeting room, becomes 0.18 s, close to an anechoic chamber—a difference one can hear.

Check 4.3: vary the absorption.

| $\alpha$ | Reflections | $T_{60}$        |
|----------|-------------|-----------------|
| 0.1      | 138.16      | 1.4428 s        |
| 0.2      | 69.08       | <b>0.7214</b> s |
| 0.5      | 27.63       | 0.2886 s        |

#### Observation 4.4

multiply the absorption by five and the reverberation time becomes a fifth. It is inverse—and the 4 of the solid angle enters only the prefactor of that proportionality.

## 5. The 0.161 Is Not a Constant

**Check 5.1:** the sound speed sits inside the coefficient  $24 \ln 10/c$ . And the sound speed varies with temperature.

| Temperature | Sound speed $c$ | Coefficient $24 \ln 10/c$ |
|-------------|-----------------|---------------------------|
| 0°C         | 331.3 m/s       | 0.166804                  |
| 20°C        | 343.0 m/s       | 0.161114                  |
| 40°C        | 354.9 m/s       | 0.155712                  |

#### Observation 5.2

0°C and 40°C differ by 7.1%. Writing “0.161” assumes 20°C silently—it looks like a constant and is a conditional number.

#### Observation 5.3

written apart, what moves and what does not becomes visible.

| Factor                      | Value   | Does it move?          |
|-----------------------------|---------|------------------------|
| $6 \ln 10$ (from the 60 dB) | 13.8155 | no (a definition)      |
| 4 (a ratio of solid angles) | 4.0000  | no (geometry)          |
| $1/c$ (the sound speed)     | $1/343$ | yes (with temperature) |

#### Observation 5.4

two of the three do not move and one does. Folded into the single number 0.161 are a convention (the 60 dB), a geometry (a solid angle) and a property of matter (the sound speed)—and while it stays folded, one cannot see which of them moves.

## 6. Roll Call and Closing

1. Reflections are  $6 \ln 10 / \alpha$  (Check 2.1): **69.08 at  $\alpha = 0.2$ .**
2. Each takes  $4V / (cS)$  (Check 2.2): **Cauchy's formula.**
3.  $24 \ln 10 = 55.262042$  (Check 2.3): **divided by 343, 0.161114.**
4. The hemisphere gives  $\pi$  (Check 3.2): **the whole solid angle is  $4\pi$ .**
5. The ratio is **4.0000000000** (Observation 3.3): **the 4 is a ratio of solid angles.**
6. The same root as  $nv/4$  (Observation 3.4): **the same number from the same integral.**
7. Drop the 4 and it is a quarter (Check 4.1): **0.7214 s becomes 0.1804 s.**
8. The 0.161 moves with temperature (Check 5.1): **0.1668 to 0.1557, some 7.1%.**
9. Three things are folded in (Observation 5.3): **convention, geometry, matter.**

Closing: the 0.161 of Sabine's formula was not one number. Folded into it are the  $6 \ln 10$  from a convention of 60 dB, a 4 from geometry, and a sound speed which is a property of matter. And the 4 in the middle was the whole solid angle  $4\pi$  divided by the  $\pi$  from integrating  $\cos$  over a hemisphere—numerical integration returns **4.0000000000**. The same 4 stands in the  $nv/4$  of kinetic theory, and is not a distinct root. Drop the 4 and the reverberation time becomes a quarter, turning a meeting room of 0.72 s into an anechoic chamber of 0.18 s. And the 0.161 is not a constant—it is 0.1668 at 0°C and 0.1557 at 40°C. One thing separates them—writing a folded number out into its factors. Write it out, and convention, geometry and matter separate, and which of them moves becomes visible. Do not write it out, and one reads the 0.161 as a single constant that somebody measured.

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