

What Changed the Exponent of a Chain Was Self-Avoidance Alone, and above Four Dimensions That Cost Disappears

— An Error of **2.11%** in the Exponent Becomes **29.33%** in the Length at $N = 10^9$ — And $2 + 2 = 4$ Empties

Avoidance of Its Meaning — [Paper 279]

Yuuki YAMAGISHI (山岸 勇輝)

Yamagishi Complexity Torus Lab, Japan

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— *From thinking to syncing.* —

Abstract

The spread of a polymer chain is fixed by a power of the number of units N . For a Gaussian chain it is $N^{1/2}$, and for a self-avoiding chain $N^{0.588}$. This paper asks where that difference comes from and where it disappears—the answer is the count $2 + 2 = 4$. No new mathematical theorem and no new law is claimed. First, set the two chains side by side. At $N = 10^6$ the Gaussian chain gives 1000.0 and the self-avoiding chain **3353.8**—a factor of **3.3538** (Section 2). Second, the gap keeps opening with N . At $N = 10^{12}$ it is **11.2481**, and the ratio reaches 10 at $N = 10^{11.42}$ (Section 2). Third, this is the core of the paper. Flory's formula gives $\nu = 0.6$ against the exact 0.58759—an error of **2.11%** in the exponent, which at $N = 10^9$ becomes **29.33%** in the length (Section 3). Fourth, the two coincide in four dimensions. Flory's $3/(d + 2)$ is exactly **0.5000** at $d = 4$ —a difference of zero from the Gaussian chain (Section 4). Fifth, and there the formula ends its office. At $d = 5$ it returns **0.4286**, which falls below 0.5 and is physically impossible (Section 4). Sixth, the **4** comes out of a count. The images of two d -dimensional walks have dimensions summing to $2 + 2 = 4$ —above $d = 4$ they do not meet in general position, so there is nothing to avoid (Section 5).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed—the $N^{1/2}$ of a Gaussian chain, Flory’s $\nu = 3/(d + 2)$, the exact three-dimensional value 0.58759, and that the upper critical dimension is 4 are all standard.No polymer physics is built—what is used is one power and a count of dimensions.Flory’s formula is not derived— $3/(d + 2)$ is cited only, and the balance of free energies from which it comes is not entered.The 0.58759 is not computed—it is a cited value from numerical work and the renormalisation group.The renormalisation group is not entered—Papers 117 and 120 treat it.Rubber elasticity is not treated—an earlier candidate on forces holds the entropic force of a rubber band. This paper is confined to the exponent, not elasticity.Real polymers are not treated—neither solvent quality, nor stiffness, nor branching is treated. Only an idealised chain is examined.Flory’s formula is not used at $d \geq 4$ —it returns values below 0.5 and is outside its range. This paper writes that honestly.Relation to earlier papers: Paper 271 treated the upper critical dimension 4 of mean field—the 4 here is also an upper critical dimension, but in a different phenomenon (an Ising transition against the self-avoidance of a chain) at the same dimension.Paper 256 counted the range needed to tell two exponents apart—this paper counts the converse, how far a small error in an exponent is amplified in the length.Paper 144 read the exponent as the signature of what is conserved—the signature here is the constraint of self-avoidance.Paper 117 separated the four ways in which scale invariance fixes an exponent—the exponent here belongs to one of them, the fixed point.Paper 190 measured rare on a logarithmic scale—this paper likewise writes ratios in orders of magnitude.What is added is computing that an error of 2.11% in the exponent becomes 29.33% in the length at $N = 10^9$, obtaining $10^{11.42}$ as the N at which the ratio reaches 10, writing honestly that Flory’s formula returns a physically impossible value at $d = 5$, and writing the origin of the 4 as the count $2 + 2$.

1. Notation and Placement

Setting: place a chain of N units in d -dimensional space. When the end-to-end distance R goes as N^ν , the exponent ν is the exponent of the spread.

Chain	Constraint	ν (three dimensions)
Gaussian chain	none (it may overlap itself)	0.5
Self-avoiding chain	it avoids itself	0.58759

Question: adding one constraint changes the exponent. Where does that difference disappear?

The answer is four dimensions.And the 4 comes out of the count $2 + 2$.

2. Setting the Two Chains Side by Side

Check 2.1: compute $R \propto N^\nu$ for the two exponents, taking the constant of proportionality as 1.

N	Gaussian (0.5)	Self-avoiding (0.58759)	Ratio
10^2	10.0	15.0	1.4969
10^4	100.0	224.1	2.2406
10^6	1000.0	3353.8	3.3538
10^9	31622.8	194227.2	6.1420
10^{12}	10^6	1.1248×10^7	11.2481

Observation 2.2

the ratio keeps opening with N . The exponents differ by only 0.08759, and at $N = 10^{12}$ the ratio is **11**—a difference in a power tells once the orders of magnitude accumulate.

Observation 2.3

the ratio reaches 10 at $N = 10^{11.42}$. It follows from $1/(0.58759 - 0.5) = 11.42$ —the number of decades required is the reciprocal of the difference of exponents.

Observation 2.4

the separator—what has to be changed for the exponent to change. Whether the chain avoids itself, and nothing else. Neither the length of a unit, nor the flexibility of the chain, nor the temperature has been moved.

3. The Core: 2.11% in the Exponent Becomes 29.33% in the Length

Setting: Flory obtained $\nu = 3/(d + 2)$ from a balance of free energies, giving **0.6** in three dimensions. **The exact value, from numerical work and the renormalisation group, is 0.58759.**

Check 3.1: the error in the exponent is $(0.6 - 0.58759)/0.58759 = \mathbf{2.1120\%}$. See what it becomes in R .

N	Flory (0.6)	Exact (0.58759)	Excess
10^2	15.85	14.97	5.88%
10^4	251.19	224.06	12.11%
10^6	3981.07	3353.82	18.70%
10^9	251188.64	194227.18	29.33%

Observation 3.2

the error in the exponent is fixed at 2.11%, while the error in the length keeps growing,
 $5.88 \rightarrow 12.11 \rightarrow 18.70 \rightarrow 29.33$. A small error inside a power is amplified with the orders of magnitude.

Observation 3.3

Paper 256 counted the range needed to tell two exponents apart. This paper is its converse—widen the range and the error in the exponent becomes visible. The same relation seen from both ends.

Observation 3.4

that Flory’s formula is out by only 2.11% is rather remarkable. Such accuracy from the crude balancing of two terms in a free energy—whether that is fortuitous is not discussed here.

4. They Coincide in Four Dimensions, and There the Formula Ends

Check 4.1: compute Flory’s $3/(d + 2)$ by dimension.

d	Flory $3/(d+2)$	Gaussian	Difference
1	1.000000	0.5	+0.5000
2	0.750000	0.5	+0.2500
3	0.600000	0.5	+0.1000
4	0.500000	0.5	0.0000
5	0.428571	0.5	−0.0714

Observation 4.2

at $d = 4$ it becomes exactly 0.5 and the difference from the Gaussian chain is zero. Avoiding or not avoiding gives the same spread.

Observation 4.3

a point to state honestly—the **0.4286** at $d = 5$ is physically impossible. A chain that avoids itself cannot be more compact than one that does not— $\nu \geq 0.5$ must hold.

Observation 4.4

so Flory's formula is outside its range for $d \geq 4$. Correctly, $\nu = 0.5$ exactly for $d \geq 4$ (with logarithmic corrections at $d = 4$)—the formula returns the right answer exactly at $d = 4$ and breaks beyond it.

Observation 4.5

this is not a defect of the formula but a matter of range. Flory's argument was made for the case where avoiding tells, and owes nothing to the region where it has ceased to.

5. The 4 Comes Out of a Count

Observation 5.1

why 4?Count in dimensions whether a path meets itself.

Check 5.2: count the condition for two parts to intersect in d -dimensional space.

d	Sum of the two path dimensions	$2 + 2 - d$	Do they meet?
2	$2 + 2 = 4$	+2	they meet
3	$2 + 2 = 4$	+1	they meet
4	$2 + 2 = 4$	0	exactly the boundary
5	$2 + 2 = 4$	-1	they do not meet

Observation 5.3

the image of a random walk behaves, in a probabilistic sense, as an object of dimension 2.Place two of them in general position and the dimension of their intersection is $2 + 2 - d$ —negative for $d > 4$, so they do not meet.

Observation 5.4

so for $d > 4$, one may try to avoid, and there is nothing to run into.The constraint becomes vacuous and the exponent returns to the Gaussian—the constraint did not disappear; what it constrained did.

Observation 5.5

the separator—what has to be changed for the cost to disappear.The dimension of the space.Neither the chain nor the definition of the constraint has been moved.

6. The Same 4 as Paper 271

Observation 6.1

Paper 271 treated the upper critical dimension 4 of mean field (the Ising model). The 4 here is also an upper critical dimension—but in a different phenomenon.

Check 6.2: set the two side by side.

	Paper 271 (Ising)	This paper (self-avoiding chain)
What happens at $d \geq 4$	the exponents match mean field	ν matches the Gaussian chain
What disappears	the effect of fluctuations	self-intersection
Where the 4 comes from	power counting of the coupling	the count $2 + 2 - d$

Observation 6.3

the same 4 arrives by different roads. One is a dimensional count of a coupling, the other a geometric count of intersection—whether roots are shared is judged by the road, not the number.

Observation 6.4

a point to state honestly—the two are connected in the language of the renormalisation group (the self-avoiding walk corresponds to the $n \rightarrow 0$ limit of the ϕ^4 model). This paper does not enter that—it does not deny that the roots may be shared, but its tools cannot decide.

7. Roll Call and Closing

1. A factor of 3.3538 at $N = 10^6$ (Check 2.1): 1000.0 against 3353.8.
2. The ratio reaches 10 at $N = 10^{11.42}$ (Observation 2.3): the reciprocal of the difference.
3. The exponent error is 2.11% (Check 3.1): 0.6 against 0.58759.
4. It becomes 29.33% in length (Check 3.1): at $N = 10^9$.

5. **Zero difference at $d = 4$ (Check 4.1): exactly 0.500000.**
6. **The 0.4286 at $d = 5$ is impossible(Observation 4.3): the formula is out of range.**
7. **$2 + 2 - d$ is 0 at four(Check 5.2): beyond it they do not meet.**
8. **What it constrained disappeared(Observation 5.4): not the constraint.**
9. **The same 4 as Paper 271(Check 6.2): arriving by a different road.**

Closing: what changed the exponent of the chain was one constraint alone, that it avoid itself. In three dimensions 0.5 becomes 0.58759, and at $N = 10^{12}$ the lengths differ by 11.2481. And Flory's approximation, out by only 2.11% in the exponent, is out by 29.33% in the length at $N = 10^9$ —a small error inside a power is amplified with the orders of magnitude. But in four dimensions that difference disappears exactly. The reason is a count in geometry—the dimensions of two paths sum to $2 + 2 = 4$, so for $d > 4$ they do not meet in general position. The constraint did not disappear; what it constrained did. And there Flory's formula ends its office too—at $d = 5$ it returns 0.4286, the impossible claim that a chain avoiding itself is more compact than one that does not. One thing separates them—confirming by a count whether the constraint still tells. Confirm it, and the range in which the formula may be used becomes clear. Do not confirm it, and one reads 0.4286 as a property of a chain.

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Yuuki YAMAGISHI (山岸 勇輝)

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