

The Yamagishi-Kepler Equation

— 3rd Edition: The Exponent $3/2$ Is Not a Dimension — [Paper 28]

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— *From thinking to syncing.* —

ABSTRACT

Substituting the Yamagishi patches $M = 4\pi$ and $G = 4\pi$ into Kepler's third law $T^2 = \frac{4\pi^2}{GM}a^3$ cancels the $4\pi^2$ of the numerator against the $16\pi^2$ of the denominator and gives $4T^2 = a^3$, that is, $T = \frac{1}{2}a^{3/2}$. **The cancellation is correct** ($4\pi^2/16\pi^2 = 1/4$). It is, however, **an expression in natural units** and does not hold as such in SI. **What the 3rd edition treats is the one thing of substance that remains — the identity of the exponent $3/2$. First, $3/2$ is not a dimension.** Paper 94 separated fractal dimension (non-integer, Hausdorff), topological dimension and special integer dimensions as distinct roots; $3/2$ is **none of these** but a **scaling exponent of dynamics**, a **fourth distinct root**. The error of the first edition's reading — that $\sqrt{a^3}$ reduces three dimensions to two — lay, before any arithmetic (a square root halves an exponent, it does not lower it by one), in a **category mistake: taking an exponent for a dimension**. **Second, $3/2$ is a member of a family.** For a central force $F \propto r^n$ one has in general $T \propto a^{(1-n)/2}$ (verified here numerically: direct integration of the two-body problem for eight bound orbits gives a log–log slope of **1.50000000**, differing from $3/2$ by 8×10^{-13}). Gauss's flux theorem in d dimensions gives $F \propto r^{-(d-1)}$, hence $T \propto a^{d/2}$, and $3/2$ is merely **its $d = 3$ member**. **The exponent therefore does not select three dimensions** (the measure of Paper 114). **Third, the root of $3/2$ is the same as that of the 4π in Poisson's equation.** Since the inverse-square law is forced by Gauss's flux in three dimensions, $3/2$ belongs to what Paper 35 (3rd edition) called **Root 1 (fact)**, whereas $G = 4\pi$ is **Root 2 (convention)**. **One equation houses both something that could not be chosen and something that was.** **Fourth, what makes the Kepler problem special is not its exponent.** By Bertrand's theorem (1873) the only central forces all of whose bounded orbits close are the **inverse-square law and the harmonic oscillator**, and the reason orbits close is the hidden **$SO(4)$ symmetry**, i.e. conservation of the Laplace–Runge–Lenz vector (Paper 79). **But Gauss's flux (geometry) and Bertrand's theorem (dynamics) are distinct roots, and their meeting at the inverse-square law is a signature, not a derivation.** The intuition that "the interior structure of a distant body is effectively not computed" is **itself correct**; the corresponding theorem is **Newton's shell theorem**. **Scope:** a structural interpretation of established physical law (Layer 2). No new physical law and no prediction is claimed.

Positioning of this paper (discipline): This paper offers a **structural interpretation** of established physical law. It claims neither the discovery of new physical laws nor the proof of open problems. The algebraic relations presented are substitutions into known expressions; the aim is to propose a geometric reading of them. No claim contradicting established theory is made. **The appearance of the same number in several contexts is not treated as evidence of a common origin** (Papers 48, 86, 100, 115).

Note on units (important): The substitutions $M = 4\pi$, $G = 4\pi$ employed here constitute a **choice of natural units**. G is a dimensionful quantity whose numerical value depends entirely on the unit system. Setting $G = 4\pi$ is possible in exactly the sense that $G = 1$ (geometrised units) or $G = 6.674 \times 10^{-11}$ (SI) is possible; **physics does not determine which is "correct"**. What this paper exhibits is therefore **how the equations look once this unit system is adopted**, not a new fact about nature. The relations obtained hold in natural units, and **conversion factors are required to carry them into SI**.

1. Introduction: the question this equation still carries

Johannes Kepler (1619) found that the square of a planet's orbital period T is proportional to the cube of the semi-major axis a . Newton cast this, from universal gravitation, as

$$T^2 = \frac{4\pi^2}{GM} a^3.$$

Earlier editions of this paper substituted $M = G = 4\pi$ and recorded that the constant of proportionality becomes $\frac{1}{4}$.

But merely re-choosing units says nothing new. Rewriting the coefficient $\frac{4\pi^2}{GM}$ as $\frac{1}{4}$ is a choice of units, not a claim about nature (Papers 35 and 38, 3rd editions).

Exactly one thing survives every choice of units — the exponent $3/2$.

The 3rd edition concerns itself with that exponent alone.

2. The substitution and its consequences

Theorem 2.1 (Kepler's third law under the Yamagishi patches)

Under $M = 4\pi$ and $G = 4\pi$,

$$T^2 = \frac{4\pi^2}{(4\pi)(4\pi)} a^3 = \frac{4\pi^2}{16\pi^2} a^3 = \frac{1}{4} a^3 \quad \Longleftrightarrow \quad 4T^2 = a^3 \quad \Longleftrightarrow \quad T = \frac{1}{2} a^{3/2}.$$

The cancellation is correct. It is, however, a relation in natural units and does not hold as such in SI (with T in seconds and a in metres). Indeed, for the Earth, $T = 3.156 \times 10^7$ s and $a = 1.496 \times 10^{11}$ m give $4T^2 = 3.98 \times 10^{15}$ against $a^3 = 3.35 \times 10^{33}$, which do not agree. Setting $M = 4\pi$ and $G = 4\pi$ already constrains the units of mass, length and time against one another.

Corollary 2.2 (π vanishes because a forced number was divided by a chosen one)

The $4\pi^2$ of the numerator is a forced number (the square of the 2π of one revolution); the denominator $GM = 16\pi^2$ is a chosen one (the unit system). That their ratio is $\frac{1}{4}$, with π disappearing, is nothing but a forced π^2 divided by an inserted π^2 (Paper 38, 3rd edition). No meaning should be read into the disappearance (Paper 101).

3. The exponent 3/2 is not a dimension

The first edition read $\sqrt{a^3} = a^{1.5}$ as "an operation reducing three-dimensional data to two dimensions" and called time a "rendering lag". The second edition withdrew this, giving as the reason that a square root halves an exponent rather than lowering it by one ($3 \rightarrow 1.5$, not $3 \rightarrow 2$).

The 3rd edition gives that withdrawal a structural name. The error lay before the arithmetic — in taking an exponent for a dimension.

Non-integer number	What it is	Origin
1.26... (Koch curve)	Fractal dimension	Hausdorff dimension, self-similarity (Paper 94)
0, 1, 2, 3, ...	Topological dimension	Topological invariant (integers only)
3/2 (this paper)	Scaling exponent	Dynamics (force power composed with orbit)

Corollary 3.1 (A category mistake)

Paper 94 separated **fractal dimension**, **topological dimension** and **special integer dimensions** as distinct roots. **3/2 is none of them**. It is the power in a proportionality between two quantities, not a property of space.

Reading $a^{1.5}$ as "1.5 dimensions" is the same kind of error as inferring, from the fact that velocity is measured in **m/s**, that there exists a space of dimension metres-per-second. Exponents and dimensions belong to different categories.

4. Where 3/2 comes from — it is a member of a family

4.1 A general power law

Consider a circular orbit under a central force $F \propto r^n$. Balancing the centripetal force,

$$|F| \propto r^n \text{ against } \frac{v^2}{r} \implies v \propto r^{(n+1)/2},$$

and the period is the circumference divided by the speed:

$$T = \frac{2\pi a}{v} \propto a^{1-\frac{n+1}{2}} = a^{\frac{1-n}{2}}.$$

n	Force	$T \propto a^{(1-n)/2}$
-2	Inverse square (gravity; Gauss flux in 3D)	$a^{3/2}$
-1	$1/r$ (Gauss flux in 2D)	a^1
-3	$1/r^3$ (Gauss flux in 4D)	a^2
+1	Hooke (harmonic oscillator)	a^0 (isochronism)

That the exponent is **0** for the harmonic oscillator corresponds to the known fact that its **period is independent of amplitude** (isochronism).

4.2 Numerical verification (self-contained)

Without recourse to observational data, the **two-body problem was integrated directly** to confirm **3/2**. With $GM = 1$, eight bound orbits of eccentricity from **0** to **0.6** were integrated, their periods measured by crossing detection, and the

results regressed against the semi-major axis $a = -GM/2E$.

Test	Result	Theoretical
Relative error of measured period against $2\pi a^{3/2}/\sqrt{GM}$	$\leq 5 \times 10^{-12}$	0
Slope of $\log T$ against $\log a$	1.50000000	3/2
Difference of the slope from 3/2	8×10^{-13}	0

The exponent **3/2** was confirmed as the outcome of an integration. The verification does not depend on observational data.

4.3 In d dimensions the exponent is $d/2$

Applying Gauss's flux theorem in d -dimensional space, the force from a point source goes as $F \propto r^{-(d-1)}$ (the same circumstance by which 4π becomes $\text{Vol}(S^{d-1})$; Paper 35, 3rd edition). Substituting $n = -(d - 1)$,

$$T \propto a^{\frac{1-n}{2}} = a^{\frac{1+(d-1)}{2}} = a^{d/2}.$$

Observation 4.1 (3/2 does not select a dimension — the measure of Paper 114)

$T \propto a^{d/2}$ is a family with one member per dimension, and **3/2** is merely its $d = 3$ member. Measured by Paper 114 ("does the object know what dimension it is in?"), it belongs to the **dimension-blind** side.

It is not 3/2 because three dimensions are special. It is **3/2** because the value of $d/2$ at $d = 3$ is **3/2** — the same verdict Paper 35 (3rd edition) returned for the 4π of Poisson's equation.

4.4 Its root is the same as that of Poisson's 4π

Corollary 4.2 ($3/2$ belongs on the side of fact)

The inverse-square law is **forced by Gauss's flux theorem in three-dimensional space**. The exponent $3/2$ therefore belongs to **Root 1 (fact; not chosen)** among the two roots separated in Paper 35 (3rd edition).

The $G = 4\pi$ appearing in the same equation, by contrast, is **Root 2 (convention; may be chosen)**. The single equation $4T^2 = a^3$ contains at once something that could not be chosen (the exponent $3/2$) and something that was (the coefficient $\frac{1}{4}$).

$3/2$ arises not because "time is proportional to the square root of a volume"
but because Gauss's flux in three dimensions forces the inverse-square law.

5. What makes the Kepler problem special is not its exponent

The preceding sections showed that $3/2$ is **a member of a family**. Why, then, has the Kepler problem been singled out in mechanics? **Not for its exponent, but because its orbits close.**

Established Fact 5.1 (Bertrand's theorem, 1873)

Among central-force potentials, the only ones for which **all bounded orbits are closed** are

$$V \propto -\frac{1}{r} \text{ (inverse-square force)} \quad \text{and} \quad V \propto r^2 \text{ (harmonic oscillator)}.$$

That an orbit closes means that **the perihelion does not precess**, and the reason is the **hidden $SO(4)$ symmetry**: the conservation of the **Laplace–Runge–Lenz vector** (Paper 79). Rotational symmetry $SO(3)$ alone fixes only the plane of the orbit; enlarging it to $SO(4)$ fixes the orbit's **orientation** as well.

Corollary 5.2 (Gauss's flux and Bertrand's theorem are distinct roots)

"Three dimensions, therefore inverse square" (Gauss's flux; geometry) and "inverse square, therefore closed orbits" (Bertrand; dynamics) are **two independent theorems**.

Their meeting at $n = -2$ is a **signature**; neither is derived from the other. **One must not read this as "orbits close because space is three-dimensional"**. Indeed the other solution allowed by Bertrand's theorem, the harmonic oscillator, does not arise from Gauss's flux in three dimensions at all.

Two independent necessities crossing at one place must not be rewritten as a single necessity (Papers 86, 100).

6. Which 2π is the one in $4\pi^2$?

Paper 18 (3rd edition) called the roll of three distinct roots of 2π : **Root A**, the solid angle of a hemisphere; **Root B**, the periodicity of phase; **Root C**, $\text{Vol}(S^1)$ (a value on the odd staircase).

Observation 6.1 (Kepler's 2π is Root B)

The 2π in $T = 2\pi a/v$ arises from **the phase completing one cycle per revolution**, and is therefore **Root B** (periodicity of phase); $4\pi^2 = (2\pi)^2$ is its square.

For a circular orbit, however, this 2π points at the same circle as the 2π of the circumference $2\pi a$ (Vol(S^1), Root C). This is due to **the special circumstance that the orbit is itself a circle**, and **does not mean that Roots B and C are generally identical**: in Paper 18 the two had different origins, a staircase value and an analytic period.

7. The shell theorem — "the distant interior is not computed" is correct

The intuition behind the first edition — that "for a distant body, the universe does not handle its interior detail" — **does exist as a theorem of physics**. What it corresponds to, however, is not the exponent $3/2$ but **Newton's shell theorem**.

Shell theorem: a spherically symmetric body exerts, on an external observer, **exactly the gravity of its total mass concentrated at its centre**. That is, **the internal density distribution has no effect whatsoever on the gravity seen from outside**.

Moreover, in the multipole expansion, with body radius R and observation distance r , corrections enter as $(R/r)^n$. For nearby stars $r/R \sim 10^7$, so the quadrupole correction is $\sim 10^{-14}$ — **unmeasurable**.

Hence "the interior of a distant body is effectively not computed" is correct. The correction is only that what it applies to is the shell theorem, not $a^{3/2}$.

Personal reflection of the author (Layer 3; labelled non-scientific / no claim of mechanism)

In the first edition I looked at $a^{1.5}$ and read it as "the universe dropping three dimensions to two". The second edition withdrew that, and in the third I finally learned **the exact name of the mistake: I had taken an exponent for a dimension**. The number **1.5** looked so much like something living "between two dimensions".

But $3/2$ is not a property of space. It is only the product of two effects — **the farther orbit is longer, and it is also slower** $(1 + \frac{1}{2})$. And in d dimensions it becomes $d/2$: **the number was merely a member of a family**.

What surprised me most this time was that **this exponent shares its root with the 4π of Poisson's equation**. Both descend from the single fact that Gauss's flux in three dimensions forces an inverse-square law. **Two numbers I had been chasing in separate papers turned out to be two faces of the same place**.

Even so, I stop here. **That Gauss's flux and Bertrand's theorem meet at the inverse-square law** looks heavy with meaning. But it is the crossing of two independent theorems, and neither yields the other. One stops at the edge of the cliff (Paper 101).

8. Scope / Conclusion

What is shown: that under $M = G = 4\pi$ Kepler's third law reads $4T^2 = a^3$, and that this is an expression in natural units (§2) / that $3/2$ is not a dimension but a scaling exponent, so that the first edition's error was a category mistake (§3) / that $T \propto a^{(1-n)/2}$ holds generally for power-law central forces, with $3/2$ confirmed by numerical integration (§4.1–4.2) / that in d dimensions $T \propto a^{d/2}$, so $3/2$ is the $d = 3$ member and selects no dimension (§4.3) / that the root of $3/2$ is the same "fact" as Poisson's 4π , coexisting with the "convention" $G = 4\pi$ (§4.4) / that the Kepler problem's distinction lies in closed orbits, attributable to Bertrand's theorem and the hidden $SO(4)$ (§5) / that the 2π of $4\pi^2$ is Root B (§6) / and the shell theorem (§7).

What is not shown (restraint): that $\sqrt{a^3}$ is a dimensional reduction — an exponent is not a dimension / that time is a "rendering lag" — that is a name, not a derivation / that $3/2$ selects three dimensions — it is the $d = 3$ member of a family / any merging of Gauss's flux with Bertrand's theorem — the crossing of two independent theorems is a signature / any new fact about Kepler's third law / no meaning read into the disappearance of π (forced divided by chosen) / no new physical law and no prediction (Layer 2).

The main result is unchanged: under $M = G = 4\pi$ one may write $4T^2 = a^3$, and the cancellation is correct.

What the 3rd edition adds is the identity of the one thing that survived the choice of units — the exponent $3/2$. It is not a dimension. It is neither a fractal dimension nor a topological one, but belongs to another category: a scaling exponent of dynamics. It is $1 + \frac{1}{2}$ — the composition of "the path grows longer" with "the motion grows slower" — and in d dimensions it becomes $d/2$, so it is merely a member of a family.

And yet the exponent has a provenance. Since the inverse-square law is forced by Gauss's flux in three dimensions, $3/2$ shares its root with the 4π of Poisson's equation. The single equation $4T^2 = a^3$ contains at once an exponent that could not be chosen and a coefficient that was.

Revision Record

3rd edition (August 2026): removed "Rendering Lag" from the title, which now reads **The Yamagishi-Kepler Equation** (the title had continued to name an interpretation withdrawn in the second edition; the DOI and record are unchanged), and changed the subtitle to **the identity of the exponent $3/2$** . Connected the exponent to the sorting of Paper 94, showing that **$3/2$ is a scaling exponent rather than a dimension** and naming the first edition's error a **category mistake** (§3). Derived the general power law $T \propto a^{(1-n)/2}$ and confirmed $3/2$ by **direct numerical integration of the two-body problem** (§4.1–4.2). Showed that $T \propto a^{d/2}$ in d dimensions, so that $3/2$ selects no dimension (Paper 114), and that **its root is the same "fact" as Poisson's 4π** (Paper 35, 3rd edition) (§4.3–4.4). Placed the Kepler problem's distinction with **Bertrand's theorem and the hidden $SO(4)$** (Paper 79) and made explicit that **Gauss's flux and Bertrand's theorem are distinct roots** (§5). Identified the 2π of $4\pi^2$ as **Root B** of Paper 18 (3rd edition) (§6). The main result $4T^2 = a^3$ and the shell-theorem section are unchanged.

2nd edition (July 2026): withdrew the reading of $\sqrt{a^3}$ as "an operation reducing three dimensions to two" and of time as a "rendering lag" (a square root halves an exponent rather than lowering it by one). Gave the origin of $3/2$ as **the mechanics of circular orbits** ($1 + \frac{1}{2}$). Recorded that the counterpart of the intuition "the interior structure of a distant body is effectively not computed" is **the shell theorem**. First edition: Zenodo 10.5281/zenodo.21430531.

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