

What Saturates Is Not the Earthquake but the Scale

— Measured by m_b , the 2011 Tohoku Earthquake's Energy Is Estimated at One **5623**rd — A Scale Saturates
When the Source Duration Exceeds the Period It Observes — [Paper 282]

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— *From thinking to syncing.* —

Abstract

“Magnitude” is not one thing. There are four— M_L , m_b , M_S , M_w —and all but M_w hit a ceiling for great earthquakes. This paper asks whether what hits the ceiling is the earthquake or the scale—the answer is the scale. No new mathematical theorem and no new law is claimed. First, the four scales observe different periods. M_L at 0.1 s, m_b at 1 s, M_S at 20 s, and M_w choosing no period at all (Section 2). Second, this is the core of the paper. m_b saturates at $M \simeq 6.5$, so measuring the $M_w = 9.0$ Tohoku earthquake with it estimates the energy at one **5623**rd (Section 3). Third, even M_S falls short by a factor of 7.94. The difference of 0.6 between $M_S = 8.4$ and $M_w = 9.0$ is a factor of **7.9433** in energy (Section 3). Fourth, it does not happen for small earthquakes. For 1995 Southern Hyogo the difference is 0.10, a factor of only 1.41 in energy (Section 4). Fifth, the cause is the duration of rupture. The Tohoku rupture lasted **150 s**, **7.5** times the 20 s that M_S observes (Section 5). Sixth, the separator is whether the source duration exceeds the observed period. M_w alone does not saturate because it chooses no period and measures M_0 directly (Section 6).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed——

$M_w = \frac{2}{3}\log_{10} M_0 - 6.07$, $\log_{10} E = 1.5M + 4.8$, the saturation values of each scale, and the rupture duration of the Tohoku earthquake are all standard. We do not build seismology——all we use is two linear expressions and one division. We do not predict earthquakes——time, place, and size are not treated at all. We do not discuss source processes——neither rupture propagation nor slip distribution is treated. The duration is merely placed as a representative value. We claim no accuracy for the saturation values—— $M_L \simeq 6.8$, $m_b \simeq 6.5$, $M_S \simeq 8.5$ are approximate, moving by about ± 0.3 with network and procedure. We do not assert the ω^{-2} model——the 1.7501 of Section 5 is an upper bound the model gives, not a value that accounts for the observed 0.60. This paper does not explain the gap between model and observation. We do not say M_w is perfect—— M_w carries its own error in estimating M_0 ; it merely does not saturate. We assert no individual earthquake’s values——the M_w and M_S of the tables are representative values widely used in the literature, differing by about ± 0.1 between agencies. Relation to earlier papers: Paper 117 treated the Gutenberg–Richter power law——that concerns the relation between the number and the size of earthquakes, while this paper concerns the construction of the scale itself. The material is the same earthquakes; the question differs. Paper 190 measured “rare” on a logarithmic scale——this paper likewise treats how a difference of 0.6 on a logarithmic scale becomes a factor of 7.94. Paper 255 separated two things called “accuracy,” only one of which calibration removes——the saturation here is on the side that calibration does not remove, arising from the construction of the scale. Paper 140 separated symmetry fixing ratios from dynamics fixing the scale——this paper treats the case where the scale side breaks. What is added is arranging the four scales by observed period, computing that m_b estimates the Tohoku energy at one 5623rd, confirming that the same procedure is off by only a factor of 1.41 for small earthquakes, and writing the ratio 7.5 of rupture duration to observed period as the separator.

1. Notation and Standing

Only two expressions are used.

$$M_w = \frac{2}{3}\log_{10} M_0 - 6.07, \qquad \log_{10} E = 1.5M + 4.8$$

Here M_0 is the seismic moment (Nm) and E the energy (J).

Check 1.1: We substitute $M_w = 9.0$.

Quantity	Value
$M_0 = 10^{(9.0+6.07)\times 1.5}$	4.0272×10^{22} Nm
$E = 10^{1.5\times 9.0+4.8}$	1.9953×10^{18} J

Observation 1.2

An increase of 1 in M multiplies the energy by **31.6228**. An increase of 2 multiplies it by exactly **1000**—— even 0.1 is a factor of 1.41, so the first decimal place matters.

Question: M_L , m_b and M_S hit a ceiling for great earthquakes. Is it the earthquake or the scale that hits the ceiling?

2. The Four Scales Observe Different Periods

Check 2.1: We line up what each of them looks at.

Scale	Period observed	Saturation value
M_L (Richter, local)	0.1 s	$M \simeq 6.8$
m_b (body wave)	1 s	$M \simeq 6.5$
M_S (surface wave)	20 s	$M \simeq 8.5$
M_w (moment)	chooses no period	none

Observation 2.2

The order of saturation values matches the order of observed periods. 0.1 s $\rightarrow$ 6.8, 1 s $\rightarrow$ 6.5, 20 s $\rightarrow$ 8.5—— the longer the period a scale observes, the later it hits its ceiling.

Observation 2.3

M_w alone stands outside the column. M_w looks at the amplitude of no period's wave——it looks at $M_0 = \mu A \bar{d}$, that is at rigidity, rupture area, and mean slip.

Observation 2.4

That M_L saturates later (6.8) than m_b (6.5) conflicts with the order of periods. Written honestly, saturation values are not set by period alone but also by network and procedure——this paper treats period as the principal factor, but does not claim it is the only one.

3. The Core: m_b Estimates Tohoku at One 5623rd

Setting: The 2011 Tohoku earthquake is $M_w = 9.0$, $E = 1.9953 \times 10^{18}$ J. What do saturated scales give?

Check 3.1:

Scale	Value given	Estimate of E	Fraction of true value
m_b	6.5	3.5481×10^{14} J	1/5623.4
M_L	6.8	1.0000×10^{15} J	1/1995.3
M_S	8.4	2.5119×10^{17} J	1/7.9433
M_w	9.0	1.9953×10^{18} J	1

Observation 3.2

Measured by m_b , the energy is estimated at one **5623**rd. The number $m_b = 6.5$ means “a moderate earthquake”——of the same event, m_b says moderate and M_w says fourth largest ever recorded.

Observation 3.3

Even M_S falls short by **7.9433**. The difference between $M_S = 8.4$ and $M_w = 9.0$ is only 0.6, but it is 0.6 on a logarithmic scale——as Paper 190 wrote, it must be read as a ratio, not a difference.

Check 3.4: We see whether the same happens for other great earthquakes.

Earthquake	M_w	M_S	Difference	Underestimate of E
1960 Chile	9.5	8.5	+1.00	31.6228 ×
1964 Alaska	9.2	8.4	+0.80	15.8489 ×
2004 Sumatra	9.1	8.8	+0.30	2.8184 ×
2011 Tohoku	9.0	8.4	+0.60	7.9433 ×

Observation 3.5

The larger M_w , the larger the difference—but not monotonically. 2004 Sumatra has $M_w = 9.1$ yet a difference of only 0.30— M_w alone does not settle it; the rupture duration acts (Section 5).

4. It Does Not Happen for Small Earthquakes

Check 4.1: We put earthquakes below the saturation values into the same table.

Earthquake	M_w	M_S	Difference	Underestimate of E
1995 Southern Hyogo	6.9	6.8	+0.10	1.4125 ×
1906 San Francisco	7.9	7.8	+0.10	1.4125 ×
2011 Tohoku	9.0	8.4	+0.60	7.9433 ×

Observation 4.2

Below the M_S saturation value of 8.5, the difference stays around 0.1. M_S is not broken; it is being used outside its range—the scale is not at fault.

Observation 4.3

Its standing differs from the distinction of Paper 255. The systematic error of Paper 255 is removed by calibration, whereas this saturation is not—the construction of the scale itself makes it unable to see large earthquakes.

Observation 4.4

Hence one cannot “correct M_g into M_w .” The quantity to be corrected has not been observed—a 20 s wave carries only part of a 150 s rupture.

5. The Cause Is the Duration of Rupture

Setting: An earthquake is not instantaneous. A fault takes time to tear from end to end.

Check 5.1: For the Tohoku earthquake we compare two times.

Quantity	Value
Duration of rupture	about 150 s
Period of the wave M_g measures	20 s
Ratio	7.5

Observation 5.2

A 20 s wave sees only part of an event that took 150 s. By the time the far end of the fault is tearing, the near end is already finished—they are not summed together.

Check 5.3: We rewrite in frequency.

Quantity	Value
Corner frequency $f_c = 1/150$	0.006667 Hz
M_S observing frequency $1/20$	0.050000 Hz
f/f_c	7.50
Amplitude ratio in the ω^{-2} model, $(f/f_c)^{-2}$	1/56.25
In magnitude units	1.7501

Observation 5.4

The model gives a shortfall of 1.7501; the observation is 0.60. The model gives only an upper bound——because the M_S procedure uses a band of periods, not a single frequency.

Observation 5.5

A point to write honestly——this paper does not explain the gap between 1.7501 and 0.60. What can be shown is the direction and the order of magnitude, not the value——only that the model predicts a shortfall in the right direction and of sufficient size.

6. The Separator, and the Span One Word Covers

Observation 6.1

Separator——what must be moved for saturation to occur or not? Whether the source duration exceeds the scale’s observed period.

Observation 6.2

This shows why M_w alone does not saturate. M_w observes no wave of any period——it observes M_0 itself, and so has no period to compare against.

Check 6.3: We measure the span the single word “magnitude” covers.

	M	E
Laboratory microfracture	−2.0	63.096 J
1960 Chile	9.5	1.1220×10^{19} J
Ratio	11.5	$1.7783 \times 10^{17} \times$

Observation 6.4

One word covers a factor of 1.78×10^{17} . No single scale can measure such a span, so having four is natural —the problem is not that they are separate but that one fails to say which was used.

Observation 6.5

Paper 117 treated the power law between the number and the size of earthquakes. This paper treats the instrument that measures the size—the material is the same earthquakes, and the questions are orthogonal.

7. Roll Call and Closing

1. 1 in M is $31.6228 \times$ (Observation 1.2): 2 is exactly $1000 \times$.
2. Observed periods of the four scales (Check 2.1): 0.1 / 1 / 20 s / none.
3. m_b gives one 5623rd (Check 3.1): It calls Tohoku moderate.
4. M_S still $7.9433 \times$ short (Check 3.1): The content of a 0.6 difference.
5. Same for four great earthquakes (Check 3.4): But not monotonically.
6. Small earthquakes: $1.4125 \times$ (Check 4.1): Inside the range nothing breaks.
7. Calibration does not remove it (Observation 4.3): It arises from construction.
8. Rupture 150 s vs period 20 s (Check 5.1): Ratio 7.5.
9. Model 1.7501, observation 0.60 (Check 5.3): An upper bound only.
10. One word covers 1.78×10^{17} (Check 6.3): From $M = -2$ to 9.5.

Closing: When magnitude hits a ceiling for great earthquakes, it is not the earthquake that hits the ceiling. Measuring the 2011 Tohoku earthquake with m_b gives 6.5, that is an energy estimated at one **5623rd**—of the same event, m_b says moderate and M_w says fourth largest ever recorded. Even M_S falls short by **7.9433**—the difference is only 0.6, but it is 0.6 on a logarithmic scale. And it does not happen for small earthquakes—for 1995 Southern Hyogo it stays at 1.4125. The scale is not broken; it is being used outside its range. The cause is the duration of rupture—the Tohoku rupture lasted **150 s**, **7.5** times the 20 s that M_S observes. A 20 s wave carries only part of a 150 s event. One thing separates them—whether the source duration exceeds that scale’s observed period. If it does, no amount of calibration removes the saturation. If it does not, the four scales agree well. M_w alone escapes saturation not because it is superior but because it has no period to compare against.

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