

“Forbidden” Means “Slower by a Factor of 7.62×10^7 ”

— Hydrogen’s $2s \rightarrow 1s$ Does Not Fail to Happen; It Takes **0.1216** Seconds — The **21** cm Line Takes **11** Million Years, Yet 10^{60} Atoms Emit 2.9×10^{45} Photons a Second — [Paper 285]

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— *From thinking to syncing.* —

Abstract

A selection rule is written as “this transition is forbidden.” Yet forbidden transitions are actually observed. This paper asks what “forbidden” means as a number—the answer is a matter of rate. No new mathematical theorem and no new law is claimed. First, we compare two lifetimes in the hydrogen atom. $2p \rightarrow 1s$ (E1 allowed) is 1.596×10^{-9} s; $2s \rightarrow 1s$ (E1 forbidden) is **0.1216** s (Section 2). Second, this is the core of the paper. The ratio is **7.6190×10^7** —it is not forbidden. It is merely slow (Section 2). Third, the estimate can be written with the fine-structure constant. The E1-to-M1 ratio is $\alpha^2 = \mathbf{5.325} \times 10^{-5}$, and two-photon is $\alpha^4 = 2.836 \times 10^{-9}$ (Section 3). Fourth, the 21 cm line is slower still. Lifetime 3.47×10^{14} s = **11** million years, **2.17×10^{23}** times that of $2p$ (Section 4). Fifth, it is nevertheless visible because of numbers. With 10^{60} hydrogen atoms, **2.88×10^{45}** emit each second (Section 5). Sixth, the separator is “exactly zero, or merely small”. What a selection rule sets to zero is a matrix element within one approximation, not the total rate (Section 6).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed—the E1 selection rules, two-photon emission, hyperfine structure, the lifetime of the 21 cm line, and the α^2 estimate are all standard. We do not build quantum electrodynamics—all we use is three lifetimes and two divisions. We do not derive the selection rules—we do not enter the Wigner–Eckart theorem. That is the side of Paper 140. We do not compute transition probabilities— 1.596×10^{-9} , 0.1216, and 3.47×10^{14} are merely quoted from the literature, and this paper only takes ratios. We do not say α^2 is an exact ratio—the actual E1-to-M1 ratio moves by orders with the matrix elements. $\alpha^2 = 5.325 \times 10^{-5}$ is an estimate, not a prediction. We do not say selection rules are meaningless—a factor of 7.6×10^7 is indistinguishable from prohibition in practice in many settings. What this paper says is only that “forbidden” and “slow” have different standings. We do not discuss interstellar physics—the number 10^{60} is a representative value for showing orders, and cloud mass, temperature, and excitation mechanism are not treated at all. Relation to earlier papers: Paper 140 treated selection rules from the group-theoretic side, showing that symmetry fixes ratios—this paper is on the rate side, writing the same selection rules as lifetimes. It treats the place where a theorem’s zero set is not in fact zero. Paper 190 measured “rare” on a logarithmic scale—the 23.34 decades here must likewise be read logarithmically. Paper 272 showed that one and the same “doubling” opens by a factor of 6.70 on the stimulus side—this paper is the case where one and the same electromagnetic transition opens by 2.17×10^{23} on the rate side. Paper 269 showed that what a crystal forbids is not the symmetry but the periodicity—this paper likewise asks what the word “forbid” contains. There the prohibition is exact; here it is a matter of rate, and the two stand in contrast. What is added is lining up three transition lifetimes in one table and measuring a spread of 2.17×10^{23} , writing the 7.6190×10^7 of $2s \rightarrow 1s$ as the numerical content of “prohibition”, filling in why the 21 cm line is visible as “ 10^{60} atoms give 2.88×10^{45} per second”, and placing as the separator that what goes to zero is a matrix element, not a rate.

1. Notation and the Question

The electric-dipole (E1) selection rules are commonly written:

$$\Delta\ell = \pm 1, \quad \Delta m = 0, \pm 1$$

A transition failing these is called “E1 forbidden.”

Question: Does “forbidden” mean it does not happen, or it is slow?

Observation 1.1

Hydrogen’s $2s$ level has $\ell = 0$, and so does $1s$. Since $\Delta\ell = 0$, it is E1 forbidden—yet the $2s$ level does not live forever.

Observation 1.2

Paper 140 treated selection rules from the group-theoretic side——by the Wigner–Eckart theorem, a certain matrix element is exactly zero by symmetry. This paper treats what lies beyond——what remains outside the matrix element that went to zero.

2. The Core: A Factor of 7.6190×10^7

Check 2.1: We line up two lifetimes of the hydrogen atom.

Transition	Type	Lifetime τ
$2p \rightarrow 1s$	E1 allowed	$1.596 \times 10^{-9} \text{ s}$
$2s \rightarrow 1s$	E1 forbidden (two-photon)	0.1216 s
Ratio		7.6190×10^7

Observation 2.2

This is the core of the paper. $2s \rightarrow 1s$ does not fail to happen; it takes 0.1216 s——it is not forbidden. It is 7.62×10^7 times slower.

Observation 2.3

0.1216 s is a length a human can feel. Shorter than a blink——that is how slow the word “forbidden” is pointing at.

Observation 2.4

In logarithms it is **7.88** decades. As Paper 190 wrote, such quantities are read in orders, not differences——7.88 decades is large, but not infinite.

Observation 2.5

Why does it stay finite? Even when the one-photon E1 goes to zero, the road of emitting two photons remains —a selection rule closes one road, not every road.

3. The Estimate Can Be Written with the Fine-Structure Constant

Setting: There is a rough estimate of how much slower. Each step up in the order of radiation brings a power of α .

Check 3.1:

Quantity	Value	Meaning
α	0.007297353	fine-structure constant
α^2	5.325135×10^{-5}	estimate for M1 against E1
$1/\alpha^2$	18778.8650	how many times slower
α^4	2.835707×10^{-9}	estimate for two-photon (E1E1)

Observation 3.2

$1/\alpha^4 = 3.53 \times 10^8$. The measured 7.62×10^7 is one 4.6th of that—the order agrees; the value does not.

Observation 3.3

A point to write honestly—this paper does not explain the factor of 4.6. The powers of α are an estimate that only counts the order, while matrix elements and phase-space factors act separately— α^4 says which decade, not which value.

Observation 3.4

Why it works as an estimate at all—because α is dimensionless. The same number for any atom and any system of units—only ratios are transferable (Paper 140).

4. The 21 cm Line Is 10^{23} Times Slower Still

Setting: The hydrogen ground state has two levels according to the relative orientation of electron and proton spins (hyperfine structure). The transition between them is the 21 cm line.

Check 4.1: We line up three transitions in one table.

Transition	Type	Lifetime τ	Times that of $2p$
$2p \rightarrow 1s$	E1 allowed	$1.596 \times 10^{-9} \text{ s}$	1
$2s \rightarrow 1s$	E1 forbidden, two-photon	0.1216 s	7.619×10^7
21 cm	hyperfine, M1	$3.47 \times 10^{14} \text{ s}$	2.1742×10^{23}

Observation 4.2

$3.47 \times 10^{14} \text{ s}$ is **11 million years**. One hydrogen atom emits this light on average once in 11 million years—the probability per year is 9.09×10^{-8} .

Observation 4.3

The total spread is 2.17×10^{23} , that is **23.34 decades**. This much, within the same hydrogen atom and the same electromagnetic transition—two words, allowed and forbidden, cover that entire span.

Observation 4.4

The 21 cm line is M1. The α^2 factor makes it 1.88×10^4 times slower, but most of the **23 decades** is not that—the transition energy is extremely small at $5.9 \times 10^{-6} \text{ eV}$, and the rate goes as E^3 .

5. It Is Visible Because of Numbers

Question: Why is something that emits once in 11 million years visible to a radio telescope?

Check 5.1:

Quantity	Value
Rate at which one atom emits, $1/\tau$	2.8818×10^{-15} per second
Atoms needed for one emission per second	3.47×10^{14}
Hydrogen atoms in an interstellar cloud (representative)	$\sim 10^{60}$
Emissions per second from that	2.8818×10^{45}

Observation 5.2

Only 3.47×10^{14} atoms are needed for one emission per second. In mass that is 5.8×10^{-13} kg—not remotely a microgram.

Observation 5.3

10^{60} hydrogen atoms weigh 1.67×10^{33} kg, that is about 841 solar masses. From that, 2.88×10^{45} photons emerge each second—once in 11 million years for each atom, and 10^{45} per second for the whole.

Observation 5.4

So there is no mystery in “a forbidden transition being visible.” Even with a rate of 10^{-15} , a count of 10^{60} gives a product of 10^{45} —what decides visibility is not the rate but the product.

Observation 5.5

This is Paper 190 turned over. There, “rare” was measured logarithmically—here, a rare event is multiplied by a large number and brought back to everyday orders.

6. Separator: What Goes to Zero Is the Matrix Element, Not the Rate

Observation 6.1

Separator——what is exactly zero, and what is merely small?

Exactly zero	the matrix element of the E1 operator (by symmetry)
Merely small	the total rate of the transition (other roads remain)

Observation 6.2

A selection rule speaks about the first row. But an experiment measures the second——the word “forbidden” is what makes the two be taken for one another.

Observation 6.3

Paper 269 showed that what a crystal forbids is not the symmetry but the periodicity. That prohibition is exact——a five-fold-symmetric periodic structure does not exist. The prohibition here is a matter of rate——the same “forbid” with a different standing.

Observation 6.4

Hence when reading a selection rule one must check inside which approximation it speaks. Within the E1 approximation it is exactly zero; step outside that approximation and it is finite——the zero is a property of the inside of a theory, not a property of nature.

7. Roll Call and Closing

1. $2p \rightarrow 1s$ is 1.596×10^{-9} s (Check 2.1): E1 allowed.

2. $2s \rightarrow 1s$ is 0.1216 s (Check 2.1): Shorter than a blink.

3. The ratio is 7.6190×10^7 (Check 2.1): 7.88 decades.

4. $\alpha^2 = 5.325 \times 10^{-5}$ (Check 3.1): The M1 estimate.

5. $1/\alpha^4 = 3.53 \times 10^8$ (Observation 3.2): **Measurement is one 4.6th.**
6. The **21 cm line is 11 million years** (Check 4.1): **2.17×10^{23} times $2p$.**
7. **23.34 decades in total** (Observation 4.3): **Same atom, same electromagnetic transition.**
8. **One per second needs 3.47×10^{14} atoms** (Check 5.1): **5.8×10^{-13} kg.**
9. **10^{60} gives 2.88×10^{45} per second** (Check 5.1): **841 solar masses.**
10. **The zero is the matrix element** (Observation 6.1): **Not the rate.**

Closing: “Forbidden,” as a number, means a matter of rate. Hydrogen’s $2s \rightarrow 1s$ is E1 forbidden, yet it does not fail to happen; it takes **0.1216 s**— **7.6190×10^7 times**, that is 7.88 decades, slower than $2p \rightarrow 1s$. The 21 cm line is slower still, once in 11 million years, **2.17×10^{23} times** that of $2p$ —and it is nevertheless visible not because the rate rose but because the count is large: 10^{60} atoms emit **2.88×10^{45} photons a second**. One thing separates them—that what is exactly zero is not the rate but a matrix element within one approximation. The selection rule speaks about that matrix element; the experiment measures the total rate. The zero is a property of the inside of the E1 approximation, not a property of nature. So when a “forbidden” line is seen in the sky, no contradiction has occurred.

References

1. Breit, G. and Teller, E. (1940). Metastability of Hydrogen and Helium Levels. *The Astrophysical Journal*, 91.
 2. Wigner, E. P. (1959). *Group Theory and Its Application to the Quantum Mechanics of Atomic Spectra*. New York: Academic Press.
 3. Bethe, H. A. and Salpeter, E. E. (1957). *Quantum Mechanics of One- and Two-Electron Atoms*. Berlin: Springer.
 4. Wild, J. P. (1952). The Radio-Frequency Line Spectrum of Atomic Hydrogen and Its Applications in Astronomy. *The Astrophysical Journal*, 115.
 5. Ewen, H. I. and Purcell, E. M. (1951). Radiation from Galactic Hydrogen at 1420 Mc/sec. *Nature*, 168(4270).
 6. Sobelman, I. I. (1979). *Atomic Spectra and Radiative Transitions*. Berlin: Springer.
 7. Yamagishi, Y. (2026). *Symmetry Fixes the Ratios, Dynamics Fixes the Scale* [Paper 140]. Zenodo. 10.5281/zenodo.22081997
 8. Yamagishi, Y. (2026). “Rare” Is Measured on a Logarithmic Scale [Paper 190]. Zenodo. 10.5281/zenodo.22106182
 9. Yamagishi, Y. (2026). *What a Crystal Forbids Is Not the Symmetry but the Periodicity* [Paper 269]. Zenodo. 10.5281/zenodo.22169855
 10. Yamagishi, Y. (2026). *One and the Same “Twice as Strong” Opens by a Factor of 6.70 on the Stimulus Side* [Paper 272]. Zenodo. 10.5281/zenodo.22169965
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