

The Yamagishi–Adler–Bardeen Note — On the $16\pi^2$ of the chiral anomaly as the normalisation making the index an integer — [Paper 29]

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— *From thinking to syncing.* —

ABSTRACT

The Adler–Bardeen theorem (1969) states that the coefficient of the chiral anomaly is fixed at one loop and receives no corrections from higher orders. This paper examines the geometric meaning of the $16\pi^2$ appearing in its denominator.

Principal withdrawal in the 2nd edition: The first edition claimed that higher-order corrections vanish *because* $16\pi^2$ carries a "Read-Only" attribute. **This is withdrawn.** The same $1/(16\pi^2)$ is a **universal loop factor appearing in essentially every one-loop quantity in four-dimensional field theory**, and most of those — the QCD beta function among them — **do receive higher-order corrections**. If $16\pi^2$ conferred protection, the QCD beta function would be one-loop exact; it is not. **The value $16\pi^2$ is therefore not the cause of non-renormalisation** (Section 4). The identification $16\pi^2 = c^2 n$ is also withdrawn, being dimensionally ill-formed.

What the 2nd edition retains: $16\pi^2$ nonetheless has a **different and more accurate role**. Since $\int F\tilde{F}/(16\pi^2)$ yields an **integer** (a winding number, the Atiyah–Singer index), $16\pi^2$ is **the normalisation constant that makes the index integral**. What guarantees non-renormalisation is **that integrality** rather than the value of $16\pi^2$ — though the two are not unrelated. The paper retains its claim in this limited form (Section 5).

Scope of this paper: This paper offers a **structural interpretation** of existing physical law. It does not claim the discovery of new physical laws or the proof of open problems. **Note on units:** Substitutions such as $m = 4\pi$ are a **choice of natural units**. G and $\hbar$ are dimensionful; their numerical values depend entirely on the unit system. Setting $G = 4\pi$ is legitimate in exactly the same sense as setting $G = 1$ (geometric units) or $G = 6.674 \times 10^{-11}$ (SI); **physics does not single out one of these as "correct."** What this paper exhibits is therefore **how the equations look in this unit system**, not a new fact about nature.

Introduction

Renormalisation and higher-loop corrections

In quantum field theory, loop diagrams generically produce divergences handled by renormalisation, and physical quantities generically receive corrections order by order.

The peculiarity of the Adler–Bardeen theorem

The chiral anomaly of the axial-vector current reads

$$\partial_\mu j_5^\mu = \frac{e^2}{16\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

and Adler and Bardeen (1969) proved that radiative corrections beyond one loop vanish exactly:

$$\Delta\left(\frac{e^2}{16\pi^2}\right) = 0 \quad (\text{exact})$$

Geometric decomposition of $16\pi^2$

We have $16\pi^2 = (4\pi)^2 = \text{Vol}(S^2 \times S^2)$, since $4\pi \cdot 4\pi = 157.9137 = 16\pi^2$. This arithmetic is correct.

[2nd edition note: do not merge with the other decompositions (error pattern C)]

The quantity $16\pi^2$ appears in several places in this corpus, but **each occurrence has a distinct origin**:

Decomposition	Context	Source
$\text{Vol}(S^2 \times S^2) = (4\pi)^2$	chiral anomaly normalisation	this paper
radius-2 S^3 : $2\pi^2 \cdot 2^3$	Hopf decomposition	Papers 08, 45
inverse of $\Omega_3/(2\pi)^4 \cdot \frac{1}{2}$	4D momentum measure	Papers 46, 47
$n = 2$ term of $(4\pi)^2 n!$	Gauss–Bonnet normalisation	Paper 48
$(2\pi) \times (8\pi)$	tokamak torus	Paper 22
squared ball-volume factor $(4\pi/3)^2$	stellar self-energy	Paper 30

The relation to Papers 08 and 45 deserves particular care. The volume of the radius-2 three-sphere is also $2\pi^2 \cdot 8 = 16\pi^2$, **numerically identical to the value here**, but $S^2 \times S^2$ and S^3 are not homeomorphic. Numerical coincidence does not

imply identity (the discipline established in Paper 48). **These must not be merged.**

[2nd Edition] Withdrawal of the "Read-Only attribute"

The first edition applied $16\pi^2 = c^2 n$ as a "Yamagishi toroidal covering patch" and presented

$$\partial_\mu j_5^\mu = \frac{e^2}{c^2 n} F_{\mu\nu} \tilde{F}^{\mu\nu} \quad (\Delta(c^2 n) \equiv 0)$$

as a "Yamagishi–Adler–Bardeen perfect conservation theorem". **This is withdrawn.**

Refutation 1: $16\pi^2$ appears in every one-loop quantity (decisive)

The factor $1/(16\pi^2)$ arises from the four-dimensional loop measure

$$\int \frac{d^4 k}{(2\pi)^4} \longrightarrow \frac{1}{16\pi^2} = 0.0063326 \dots$$

and is a **generic loop factor**, not something peculiar to the chiral anomaly. The same $1/(16\pi^2)$ appears in, for example,

- the QCD beta function $\beta(g) = -\frac{g^3}{16\pi^2} \left(11 - \frac{2n_f}{3} \right)$;
- various anomalous dimensions;
- the radiative corrections to the Higgs mass (Paper 47).

And **all of these do receive higher-order corrections**. The two-, three- and four-loop coefficients of the QCD beta function have been computed and are nonzero.

Hence, if higher-order corrections vanished "because $16\pi^2$ is Read-Only", the QCD beta function would have to be one-loop exact. It is not. This is a **counterexample** to the first edition's claim: the value $16\pi^2$ cannot be the cause of non-renormalisation.

Refutation 2: $16\pi^2 = c^2 n$ is dimensionally ill-formed

$16\pi^2$ is **dimensionless** — unsurprisingly, being a loop factor. But c^2 has $[\mathbf{m}^2/\mathbf{s}^2]$ and n is a number density $[\mathbf{m}^{-3}]$, so $c^2 n$ is **dimensionless under no standard reading**. The identification is withdrawn.

Refutation 3: no physics corresponds to a "system crash"

The first edition asserted that renormalising $16\pi^2$ would collapse the topological structure and charge conservation, crashing the universe. **No calculation corresponds to this.** It is true that the consistency of a theory constrains the anomaly coefficient, but that is the matter of **anomaly cancellation** — the charges of each fermion generation summing to zero — not of the value of $16\pi^2$. A metaphor was presented as a mechanism; it is withdrawn.

[2nd Edition] What remains: the integrality of the index

Despite the above, $16\pi^2$ does carry another, more accurate significance, which this paper retains as its claim.

Integrating the chiral anomaly over spacetime gives

$$\frac{1}{16\pi^2} \int d^4x F_{\mu\nu} \tilde{F}^{\mu\nu} = \nu \in \mathbb{Z},$$

an **integer** — the instanton number, or winding number. This is a consequence of the Atiyah–Singer index theorem.

Here $16\pi^2$ functions as **the normalisation constant that renders this integral an integer**. If $16\pi^2$ is "special", it is **because it normalises a topological invariant to integer values**.

And the true reason for non-renormalisation lies in that **integrality**. The index ν takes only integer values and therefore **cannot change under continuous perturbation** (higher-loop corrections): integers do not deform continuously.

The difference from the first edition, stated plainly:

- **First edition:** the value $16\pi^2$ is protected (Read-Only), hence no higher-order corrections. → **false** (counterexample: the QCD beta function).
- **Second edition:** non-renormalisation follows from **the integrality of the index**; $16\pi^2$ is the **normalisation** that makes that integrality hold. → consistent with standard field theory.

The latter gives $16\pi^2$ a topological significance **without claiming it to be the cause**. That distinction was missing from the first edition.

Explicit Scope

- **What this paper establishes:** the arithmetic decomposition $16\pi^2 = \text{Vol}(S^2 \times S^2)$ (correct), and that $16\pi^2$ is the coefficient normalising the topological index of the chiral anomaly to integer values (a restatement of a standard fact).
- **What it does not establish:** (i) that $16\pi^2$ is the **cause** of vanishing higher-order corrections (refuted by counterexample); (ii) $16\pi^2 = c^2 n$ (dimensionally ill-formed); (iii) any physics corresponding to a "system crash".
- **Character of this paper:** not an independent result, but **an organising note connecting a known fact — non-renormalisation via the index theorem — to the vocabulary of this framework**. The title has been changed accordingly.
- **Relation to the $16\pi^2$ family:** the $\text{Vol}(S^2 \times S^2)$ here is **numerically equal to, but manifold-theoretically distinct from**, the radius-2 S^3 of Papers 08 and 45. They must not be merged (Section 3).

Conclusion

This paper records that the $16\pi^2$ in the denominator of the chiral anomaly equals $\text{Vol}(S^2 \times S^2)$, and that it serves as the coefficient normalising a topological index to integer values.

However, the first edition's causal claim — that a "Read-Only attribute" of $16\pi^2$ produces the vanishing of higher-order corrections — is withdrawn. $1/(16\pi^2)$ is a universal loop factor appearing in every one-loop quantity in four dimensions, and most such quantities, the QCD beta function included, do receive higher-order corrections. What guarantees non-renormalisation is the integrality of the index, not the value of $16\pi^2$.

References

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