

That the Earth's Field Begins at the Dipole Is Not a Property of the Earth

— The $l = 0$ Term Vanishes Identically Because There Are No Monopoles — There Are Three “Norths,” and
Two of Them Are **1289** km Apart — [Paper 290]

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— *From thinking to syncing.* —

Abstract

Expanded in spherical harmonics, the Earth's magnetic field begins at the dipole ($l = 1$). One is tempted to read the absence of a monopole term as “a property of the Earth.” This paper asks whether that is a fact about the Earth or about electromagnetism—the answer is about electromagnetism. No new mathematical theorem and no new law is claimed. First, the $l = 0$ term vanishes identically. $\nabla \cdot \mathbf{B} = 0$ gives $\oint \mathbf{B} \cdot d\mathbf{A} = 0$, so the monopole term vanishes for any body whatsoever (Section 2). Second, we measure the dipole from the actual coefficients. From IGRF-13 (2020), $B_1 = \mathbf{29805.9244}$ nT and $m = \mathbf{7.7084} \times 10^{22}$ A·m² (Section 3). Third, this is the paper's other core. Two 4π 's stand in the moment formula and cancel outright— $4\pi a^3/\mu_0 = a^3 \times 10^7$ (Section 3). Fourth, the dipole axis is tilted by $\mathbf{9.4105^\circ}$ (Section 3). Fifth, there are three “norths.” The geomagnetic north pole and the magnetic north pole are **1289.1** km apart, different places (Section 4). Sixth, the separator is what does the erasing. What erases the monopole term is not the Earth but the fact that no magnetic monopole has been found (Section 5).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed——spherical harmonic expansion, the Gauss coefficients, the IGRF model, the dipole moment formula, and the definitions of the three “norths” are all standard. We do not build geophysics——all we use is one square root, one arccosine, and one great-circle distance. We do not discuss the dynamo——the generation of the field, reversals, and outer-core convection are not treated at all. We do not treat the inverse problem——Gauss (1839) separated internal from external sources using surface data alone, but this paper stays with the forward expansion. We do not deny the existence of monopoles——the $l = 0$ term vanishes from the condition that there is no net magnetic charge inside the Earth. This is not a proof that monopoles do not exist. We assert no values for the IGRF——the Gauss coefficients are observed values updated every five years, and these are the 2020 values. The pole positions move year by year. We do not predict the magnetic pole’s motion——55 km per year is a recent speed, and we do not extrapolate. We claim no significance for the 4π cancellation——the cancellation in Section 3 arises from the SI definition of μ_0 , and has a different standing from the geometric 4π of Paper 02. Relation to earlier papers: Papers 146, 147 and 151 treated Dirac monopole quantisation——this paper is on the consequence side and uses only the one point that the absence of monopoles fixes the form of the expansion. Paper 02 showed that a pure solid angle 4π appears in inverse-square fields——the 4π here stands twice and cancels, so by the conditions of Paper 258 (point source, three dimensions, isotropy) it is not a geometric 4π . Paper 99 treated the inverse problem of reading the interior from the boundary——this paper is its forward direction. Paper 112 counted “distinct roots sharing one rhyme”——the three “norths” here are likewise three distinct definitions under one word. What is added is placing the reason the $l = 0$ term vanishes outside the Earth, confirming that the two 4π ’s cancel, computing the dipole tilt as 9.4105° , and measuring the mutual distances of the three “norths” as 1289.1 / 1034.1 / 389.2 km.

1. Notation and the Question

Setting: Outside the Earth’s surface the magnetic field has a potential V , expanded in spherical harmonics:

$$V = a \sum_{l=1}^{\infty} \sum_{m=0}^l \left(\frac{a}{r} \right)^{l+1} (g_l^m \cos m\phi + h_l^m \sin m\phi) P_l^m(\cos \theta)$$

Note that the sum begins at $l = 1$.

Question: Where did the $l = 0$ term go? Is it something the Earth does not have, or something that cannot exist in principle?

Observation 1.1

Saying “the Earth’s field is a dipole” sounds like a statement about the Earth. This paper regards that as half right——that the dipole is the lowest order is not a property of the Earth.

2. The $l = 0$ Term Vanishes Identically

Observation 2.1

The $l = 0$ term is a potential going as $1/r$, that is, a magnetic monopole.

Observation 2.2

One of Maxwell's equations is $\nabla \cdot \mathbf{B} = 0$. By the divergence theorem, over any closed surface

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$

——the net flux is necessarily zero.

Observation 2.3

With an $l = 0$ term, the normal component of $\mathbf{B}$ would be uniformly of one sign over a sphere. The integral would not vanish——so the $l = 0$ term vanishes identically.

Observation 2.4

This is the paper's first core. One knows this term is absent without looking inside the Earth——the same holds for Jupiter, for the Sun, and for a laboratory electromagnet. It is not the Earth that erases it.

Observation 2.5

Papers 146, 147 and 151 treated Dirac monopole quantisation. There the question was what would happen if monopoles existed——this paper is on the consequence side. Positing that they do not exist fixes the first term of the expansion.

Observation 2.6

A point to write honestly——this is not a proof that monopoles do not exist.All that was used is the condition that there is no net magnetic charge inside the sphere considered——so long as that holds, the first term is the dipole.

3. Measuring the Dipole: Two 4π’s That Cancel

Check 3.1: From the first-degree Gauss coefficients of IGRF-13 (epoch 2020), we take the dipole magnitude.

Coefficient	Value [nT]
g_1^0	−29404.8
g_1^1	−1450.9
h_1^1	+4652.5
$B_1 = \sqrt{(g_1^0)^2 + (g_1^1)^2 + (h_1^1)^2}$	29805.9244

Check 3.2: We compute the dipole moment $m = (4\pi a^3/\mu_0)B_1$ with $a = 6371.2$ km.

Quantity	Value
$\mu_0 = 4\pi \times 10^{-7}$	1.2566371×10^{-6}
$4\pi a^3/\mu_0$	2.5862×10^{27}
m	$7.7084 \times 10^{22} \text{ A}\cdot\text{m}^2$

Observation 3.3

This is the paper’s other core.Two 4π’s stand in the formula——the $4\pi a^3$ in the numerator and the $\mu_0 = 4\pi \times 10^{-7}$ in the denominator.And

$$\frac{4\pi a^3}{\mu_0} = \frac{4\pi a^3}{4\pi \times 10^{-7}} = a^3 \times 10^7$$

so they cancel outright.

Observation 3.4

Hence no 4π remains in the dipole moment formula. It is $m = a^3 \times 10^7 \times B_1$ —seeing two 4π ’s is an artefact of the SI way of writing.

Observation 3.5

Paper 258 wrote that 4π appears only when the source is a point. Here the source is not a point and there is no inverse square—so its standing differs from the geometric 4π of Paper 02. That it cancels at all is the evidence that it comes from convention.

Check 3.6: The tilt of the dipole axis.

Quantity	Value
$ g_1^0 /B_1$	0.986542
Tilt $\arccos(g_1^0 /B_1)$	9.4105 degrees

Check 3.7: How dominant the dipole is, compared with the second degree.

Quantity	Value [nT]
First degree (dipole) B_1	29805.9
Second degree (quadrupole) B_2	5238.5
Ratio B_1/B_2	5.6898

Observation 3.8

The dipole is only **5.69** times the quadrupole. The approximation “the Earth’s field is a dipole” carries an error of some 15%—being the lowest order and being dominant are different things.

4. Three “Norths,” Two of Them 1289 km Apart

Check 4.1: The three “norths” in 2020.

Name	Definition	Position
Geographic north pole	rotation axis	90.0°N
Geomagnetic north pole	intersection of the dipole axis	80.7°N, 72.7°W
Magnetic north pole	point of 90° inclination	86.5°N, 164.0°E

Check 4.2: We measure great-circle distances.

Pair	Distance
Geomagnetic — magnetic north pole	1289.1 km
Geographic — geomagnetic north pole	1034.1 km
Geographic — magnetic north pole	389.2 km

Observation 4.3

The geomagnetic and magnetic north poles are **1289** km apart. Farther than Tokyo is from Sapporo——saying “magnetic north” gives different answers depending on which is meant.

Observation 4.4

Unexpectedly, the magnetic north pole is the closer to the geographic pole (389 km against 1034 km). The dipole axis is the more displaced——because the higher-order terms pull the point of 90° inclination back towards the pole.

Observation 4.5

The magnetic north pole moves. About 2300 km over the 120 years from 1900 to 2020, an average of 19.17 km per year—the present rate of 55 km per year is **2.87** times that average.

Observation 4.6

Separator—without saying which “north,” even the sign of the declination is undetermined. Referred to the geomagnetic pole or to the magnetic pole, the declination at one place differs by several degrees—a “north” without a definition is not a quantity.

5. Separator: What Does the Erasing

Observation 5.1

Separator—what must be moved for an $l = 0$ term to appear? The existence of monopoles.

	First term	Reason
Gravitational field	$l = 0$ (monopole)	mass has no negative
Electrostatic field (net charge)	$l = 0$ (monopole)	charge can be separated
Magnetic field	$l = 1$ (dipole)	no net magnetic charge

Observation 5.2

Laid out in three rows it becomes plain. What fixes the lowest order is whether a net quantity can be left on the source—not a property of the body.

Observation 5.3

Paper 02 showed that a pure solid angle 4π appears in inverse-square fields. The $l = 0$ term of gravity is exactly that 4π 's address—the magnetic field has no such term, so its 4π has a different standing (it cancelled in Section 3).

Observation 5.4

Paper 99 treated the inverse problem of reading the interior from boundary data. In 1839 Gauss separated internal from external sources using surface field data alone—this paper stays forward, but the constraint that $l = 0$ is absent removes one unknown from that inverse problem.

6. Roll Call and Closing

1. $l = 0$ vanishes identically (Observation 2.3): From $\oint \mathbf{B} \cdot d\mathbf{A} = 0$.
2. Known without looking inside (Observation 2.4): Same for Jupiter or an electromagnet.
3. $B_1 = 29805.9244$ nT (Check 3.1): IGRF-13, 2020.
4. Two 4π 's cancel (Observation 3.3): $4\pi a^3/\mu_0 = a^3 \times 10^7$.
5. $m = 7.7084 \times 10^{22}$ A·m² (Check 3.2): No 4π remains.
6. Tilt 9.4105° (Check 3.6): The dipole axis.
7. $B_1/B_2 = 5.6898$ (Check 3.7): Lowest is not dominant.
8. Geomagnetic – magnetic = 1289.1 km (Check 4.2): Different places.
9. The magnetic pole is closer (389.2 km) (Observation 4.4): Higher terms pull it back.
10. 55 km/yr, 2.87 times the average (Observation 4.5): It is fast now.

Closing: That the Earth's field begins at the dipole is not a property of the Earth. Since $\nabla \cdot \mathbf{B} = 0$ forces $\oint \mathbf{B} \cdot d\mathbf{A} = 0$, the $l = 0$ term vanishes identically for any body—for Jupiter, for the Sun, for a laboratory electromagnet alike. What erases it is a fact on the side of electromagnetism: that no magnetic monopole has been found. We also measured the actual values—from IGRF-13 (2020), $B_1 = 29805.9244$ nT, $m = 7.7084 \times 10^{22}$ A·m², and a dipole tilt of 9.4105° . Two 4π 's stand in that moment formula, and $4\pi a^3/\mu_0 = a^3 \times 10^7$ cancels them outright—that they cancel is itself the evidence that this 4π comes from convention. And there are three “norths”—the geomagnetic and magnetic north poles are 1289.1 km apart, different places. One thing separates

them——whether a net quantity can be left on the source of the field. Gravity can, and begins at $l = 0$; the magnetic field cannot, and begins at $l = 1$. No amount of studying the body will produce that difference.

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