

One Sea Holds Three Lengths, Spanning a Factor of 4.39×10^4

— All Contain the Coriolis Parameter, and Nothing Else in Them Overlaps — The Bathtub Vortex Has Rossby Number **3232**, and Seeing Rotation There Needs Flow Below **0.031** mm/s — [Paper 291]

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August 2026

— *From thinking to syncing.* —

Abstract

A rotating ocean has several characteristic lengths. Each is called “the length set by Coriolis.” This paper asks whether they are the same thing—the answer is that they are different things. No new mathematical theorem and no new law is claimed. First, we line up three lengths. At latitude 45° : Ekman layer **43.75** m, baroclinic deformation radius **48.48** km, barotropic deformation radius **1920.86** km (Section 2). Second, this is the core of the paper. Smallest to largest spans 4.3905×10^4 , and though all contain the Coriolis parameter f , nothing else in them overlaps (Section 2). Third, what separates baroclinic from barotropic is not depth. The ratio is **39.6182**, and what sets it is the stratification N (Section 3). Fourth, whether rotation acts is settled by one number. The Rossby number $Ro = U/(fL)$: Gulf Stream 0.0970, typhoon 0.5818 (Section 4). Fifth, the bathtub vortex has $Ro = 3232$. A force **3232** times stronger than Coriolis is turning it (Section 5). Sixth, the separator is Ro . To see Coriolis in a bathtub the flow must be below **0.031** mm/s (Section 5).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed—the Ekman layer, the Rossby deformation radius, the Rossby number, and geostrophic balance are all standard. We do not build ocean dynamics—all we use is three length formulas and one dimensionless number. We do not derive the Ekman spiral—we do not enter the boundary-layer equations. We do not take a roll call of forces—the classification of centrifugal, Coriolis, and pressure-gradient forces is not treated. This is a roll call of lengths. We claim no accuracy for the representative values—eddy viscosity $A_z = 0.01 \text{ m}^2/\text{s}$, stratification $N = 0.005/\text{s}$, and depths $H = 1000 / 4000 \text{ m}$ are values for showing orders and move by an order with place and season. A_z in particular ranges from 10^{-4} to 10^{-1} . We assert no values for the bathtub vortex— $U = 0.1 \text{ m/s}$ and $L = 0.3 \text{ m}$ are representative of a typical drain vortex. What this paper asserts is $Ro \gg 1$, not the value 3232. We do not explain the drain vortex’s sense of rotation—what actually decides it (residual initial circulation, vessel shape, how the plug is pulled) is not treated. We show only that it is not Coriolis. Relation to earlier papers: Paper 256 showed that the only scale at which dynamical similarity holds exactly is 1—the Ro here is likewise a dimensionless number that cannot be held when the scale changes, so a bathtub cannot be a model of an ocean. Paper 117 treated scale invariance fixing an exponent in four ways and wrote the condition that there be exactly one Π group—this paper is a case failing that condition, with three lengths standing independently. Paper 140 separated symmetry fixing ratios from dynamics fixing the scale—all three lengths here are on the dynamics side. Paper 112 counted “distinct roots sharing one rhyme”—the three lengths share the rhyme f and differ in root. What is added is lining up three lengths in one table and measuring the span 4.3905×10^4 , confirming that the ratio 39.6182 is set by stratification, computing the Rossby numbers of five phenomena, and deriving the flow speed 0.031 mm/s needed to see rotation in a bathtub.

1. Notation, and One Parameter

Setting: On the rotating Earth the Coriolis parameter

$$f = 2\Omega \sin \varphi$$

appears in every expression (Ω the rotation rate, φ the latitude).

Check 1.1: At latitude 45° .

Quantity	Value
Ω	$7.2921159 \times 10^{-5} \text{ rad/s}$
$f = 2\Omega \sin 45^\circ$	$1.0313 \times 10^{-4} / \text{s}$
Inertial period $2\pi/f$	16.9242 hours

Question: Are the lengths containing f all the same thing?

2. The Core: Three Lengths Spanning 4.39×10^4

Check 2.1: We take three lengths for the same sea at the same latitude.

Length	Formula	Value
Ekman layer depth	$\pi\sqrt{2A_z/f}, A_z = 0.01$	43.75 m
Baroclinic deformation radius	$NH/f, N = 0.005, H = 1000$	48.48 km
Barotropic deformation radius	$\sqrt{gH}/f, H = 4000$	1920.86 km

Observation 2.2

This is the core of the paper.Smallest to largest spans 4.3905×10^4 ——from 44 metres to 1900 kilometres, all “lengths set by Coriolis” in one and the same sea.

Check 2.3: We write out what enters besides f .

Length	Ingredients besides f	What layer
Ekman depth	eddy viscosity A_z	the friction layer
Baroclinic radius	stratification N , depth H	the density layer
Barotropic radius	gravity g , depth H	the free surface

Observation 2.4

The right two columns overlap in nothing.Viscosity, stratification, and gravity are mutually independent ——all they share is f .

Check 2.5: Even the manner of dependence on f differs.

Length	Dependence on f	Doubling f gives
Ekman depth	$\propto f^{-1/2}$	$\times 0.7071$
Baroclinic radius	$\propto f^{-1}$	$\times 0.5000$
Barotropic radius	$\propto f^{-1}$	$\times 0.5000$

Observation 2.6

Only the Ekman depth has a different exponent. Change the latitude and the very ratios among the three change——measuring one tells you nothing about the other two.

Observation 2.7

Paper 112 counted “six distinct roots sharing one rhyme.” The three lengths share the rhyme f and have entirely distinct roots——that the same parameter appears does not mean the same phenomenon.

3. What Separates Baroclinic from Barotropic Is Not Depth

Check 3.1: We take the ratio of the two deformation radii.

Quantity	Value
Barotropic / baroclinic	39.6182
As a formula, $\sqrt{gH_2}/(NH_1)$	f cancels

Observation 3.2

Inside the ratio, f cancels out. So changing the latitude does not change the ratio——it is not rotation that separates them.

Check 3.3: We move the stratification N .

N [s]	Baroclinic radius	Barotropic / baroclinic
0.005	48.48 km	39.6182
0.010	96.97 km	19.8091

Observation 3.4

Doubling N halves the ratio. What sets it is not depth but stratification—the degree to which the sea is layered makes this factor of 39.6.

Observation 3.5

Were the sea perfectly mixed ($N = 0$), the baroclinic radius would be 0. Only the barotropic one would remain—one of the three lengths exists only because there is stratification.

4. Whether Rotation Acts Is Settled by One Number

Setting: The Rossby number $Ro = U/(fL)$ is the ratio of inertial to Coriolis force. Rotation governs when $Ro \ll 1$ and is irrelevant when $Ro \gg 1$.

Check 4.1:

Phenomenon	U [m/s]	L [m]	Ro	Verdict
Gulf Stream	1.0	1.0×10^5	0.0970	rotation acts
Atmospheric anticyclone	10.0	1.0×10^6	0.0970	rotation acts
Typhoon	30.0	5.0×10^5	0.5818	rotation acts
Laboratory rotating tank	0.01	0.5	193.94	rotation irrelevant
Bathtub vortex	0.1	0.3	3232.29	rotation irrelevant

Observation 4.2

The Gulf Stream and the atmospheric anticyclone happen to share 0.0970. The speed is 10 times larger and the size is too, so the ratio is unchanged—— Ro sees only the ratio of U to L .

Observation 4.3

The typhoon at 0.5818 is close to 1. Rotation acts but does not govern——so near a typhoon's centre geostrophic balance fails and gives way to gradient-wind balance, a different balance.

5. The Core Continued: The Bathtub Vortex Has $Ro = 3232$

Observation 5.1

A bathtub drain vortex has $Ro = 3232.29$. A force 3232 times stronger than Coriolis is turning it——residual initial circulation, asymmetry of the vessel, and the act of pulling the plug.

Check 5.2: So what would make it visible? To get $Ro < 1$ one needs $U < fL$.

Vessel	Permitted flow speed $U < fL$
Bathtub (radius 0.3 m)	0.0309 mm/s
Tank of diameter 2 m	0.1031 mm/s
Pond of radius 100 m	10.3126 mm/s

Observation 5.3

To see Coriolis in a bathtub the flow must be below **0.031** mm/s. A real drain vortex runs at about 100 mm/s——**3232** times too fast.

Observation 5.4

That the same 3232 appears twice is no coincidence. It is the definition of Ro itself—but rewriting it as “3232 times too fast” shows what would be required.

Observation 5.5

In fact the experiment has succeeded. In a tank 1.8 m across, left still for over 18 hours and drained very slowly, the Coriolis sense appears—note that those 18 hours are close to the inertial period of 16.92 hours.

Observation 5.6

Separator—what must be moved for rotation to act? Making U/L smaller than f . Enlarge the vessel or slow the flow—changing the latitude moves f by at most $\sqrt{2}$, so that route cannot close the gap.

Observation 5.7

Paper 256 showed that the only scale at which dynamical similarity holds exactly is 1. Bathtub and ocean differ in Ro by 3.3×10^4 —a bathtub cannot be a model of an ocean.

6. Roll Call and Closing

1. $f = 1.0313 \times 10^{-4}$ /s (Check 1.1): Inertial period 16.9242 hours.
2. Ekman layer 43.75 m (Check 2.1): The friction layer.
3. Baroclinic radius 48.48 km (Check 2.1): The density layer.
4. Barotropic radius 1920.86 km (Check 2.1): The free surface.
5. Span 4.3905×10^4 (Observation 2.2): Same sea, same latitude.
6. Different exponents in f (Check 2.5): Only Ekman is $f^{-1/2}$.
7. Barotropic/baroclinic = 39.6182 (Check 3.1): f cancels.
8. Doubling N halves the ratio (Check 3.3): Stratification sets it.
9. Bathtub vortex $Ro = 3232.29$ (Check 4.1): Rotation irrelevant.

10. Flow needed: 0.031 mm/s (Check 5.2): One 3232nd of the real value.

Closing: One sea at one latitude holds three “lengths set by Coriolis.” Ekman layer **43.75** m, baroclinic deformation radius **48.48** km, barotropic deformation radius **1920.86** km——spanning **4.3905×10^4** . All contain the Coriolis parameter f , yet not one of their other ingredients overlaps——viscosity, stratification, gravity. Even the exponent of the dependence on f differs——only the Ekman depth goes as $f^{-1/2}$, so changing the latitude changes the ratios among the three. What separates baroclinic from barotropic is stratification, not depth—— f cancels in the ratio, and doubling N turns 39.6 into 19.8. And whether rotation acts is settled by one number, the Rossby number $Ro = U/(fL)$ ——Gulf Stream 0.0970, typhoon 0.5818, and the bathtub vortex **3232.29**. One thing separates them——whether U/L is smaller than f . Seeing Coriolis in a bathtub requires flow below **0.031** mm/s, and a real drain vortex is **3232** times faster. Changing the latitude moves f by at most $\sqrt{2}$, so that route cannot close the gap.

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