

The Magic Numbers Come from Two Mechanisms

— The First Three Follow from the Harmonic Oscillator Alone; From the Fourth on, Every One Is Wrong —
Calcium-48 Shows Both Mechanisms Inside a Single Nucleus — [Paper 293]

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— *From thinking to syncing.* —

Abstract

The nuclear magic numbers 2, 8, 20, 28, 50, 82, 126 look as though they follow from one rule. This paper asks whether there is one mechanism—the answer is two. No new mathematical theorem and no new law is claimed. First, we count using the harmonic oscillator alone. The cumulative sums of the degeneracy $(N+1)(N+2)$ are 2, 8, 20, 40, 70, 112, 168 (Section 2). Second, this is the core of the paper. The first three agree exactly and every one from the fourth on is wrong—40 against 28 is an error of +42.86% (Section 2). Third, spin-orbit coupling fixes all four. The degeneracies of the descending levels are 8, 10, 12, 14, rising by exactly 2, and all four come out right (Section 3). Fourth, the share of the contribution declines. The spin-orbit share runs 28.57% → 20.00% → 14.63% → 11.11% (Section 4). Fifth, calcium-48 shows both at once. $Z = 20$ needs only the harmonic oscillator; $N = 28$ needs spin-orbit—two mechanisms side by side in one nucleus (Section 5). Sixth, the separator is whether the shell closes by degeneracy or by a pushed-down level (Section 6).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed—the degeneracy of the isotropic three-dimensional harmonic oscillator, the shell model, Mayer--Jensen spin--orbit coupling, and the values of the magic numbers are all standard. We do not build a nuclear model—all we use is one degeneracy formula and four additions. We do not derive spin--orbit coupling—why $j = l + 1/2$ descends and where its strength comes from are not treated at all. We count only the resulting numbers. We do not discuss the nuclear force—the Yukawa potential, tensor forces, and three-body forces are not treated. We do not justify the mean field—the independent-particle picture is taken as a premise. We do not say $(N + 1)(N + 2)$ is the real well—the actual nuclear potential is Woods--Saxon, not harmonic, and that departure itself separates the higher levels. The harmonic oscillator here is the simplest count, for comparison. We do not claim the magic numbers are universal—in neutron-rich nuclei magic numbers disappear and new ones appear (the vanishing of $N = 28$, the emergence of $N = 16$). This paper is confined to the neighbourhood of stability. We claim no significance for the regularity of increments of 2—the 8, 10, 12, 14 of Section 3 follow because $2j + 1$ with j rising by 1, which follows from the definitions. That the count closes is a confirmation. Relation to earlier papers: Paper 112 counted “six distinct roots sharing one rhyme”—the seven magic numbers likewise have two roots under one word. Paper 240 counted seven things called “mass”—this paper identifies two mechanisms under one word, and where exactly they change over. Paper 262 showed there are three ways to break and that strength does not decide—this too is a case where what separates is not “size”. Paper 265 showed that what separated the low-temperature models is the density of states—here too counting levels gives the answer. What is added is computing seven cumulative sums for the harmonic oscillator alone and giving the relative errors, confirming that all four spin--orbit corrections come out right, measuring the share falling from 28.57% to 11.11%, and pointing out that calcium-48 places the two mechanisms side by side in one nucleus.

1. Notation and the Question

Fact: A nucleus is especially stable when its proton or neutron number is

2, 8, 20, 28, 50, 82, 126

These are the magic numbers.

Question: Do these seven follow from one rule?

Observation 1.1

In the periodic table of atoms, closed shells follow from one rule (orbital degeneracy). One naturally expects the same in nuclei—this paper shows numerically that this is only half right.

2. The Core: The First Three Agree, and Every One After Fails

Setting: As the simplest model we take the isotropic three-dimensional harmonic oscillator. The degeneracy of level N , including spin, is $(N + 1)(N + 2)$.

Check 2.1:

N	Degeneracy $(N + 1)(N + 2)$	Cumulative
0	2	2
1	6	8
2	12	20
3	20	40
4	30	70
5	42	112
6	56	168

Check 2.2: We set these against the actual magic numbers.

Order	Harmonic oscillator	Actual magic number	Difference	Relative error
1	2	2	0	0.00%
2	8	8	0	0.00%
3	20	20	0	0.00%
4	40	28	+12	+42.86%
5	70	50	+20	+40.00%
6	112	82	+30	+36.59%
7	168	126	+42	+33.33%

Observation 2.3

This is the core of the paper. The first three have difference **0**; from the fourth on every one is wrong——agreement is **3/7**.

Observation 2.4

Note that the failure is not uniform. The differences keep growing, +12, +20, +30, +42, yet the relative error falls from 42.86% to 33.33%——on neither reading can one say it is converging.

Observation 2.5

It is too early to discard the model. That three come out right is no accident——one of the mechanisms is correct, and the other is missing.

3. Spin--Orbit Coupling Fixes All Four

Setting: In 1949 Mayer and Jensen introduced a strong spin--orbit coupling. The $j = l + 1/2$ level drops sharply and falls into the shell below.

Check 3.1:

Descending level	$2j$	$2j + 1$	Original shell	Total	Actual magic number
$1f_{7/2}$	7	8	20	28	28
$1g_{9/2}$	9	10	40	50	50
$1h_{11/2}$	11	12	70	82	82
$1i_{13/2}$	13	14	112	126	126

Observation 3.2

All four come out right. And the degeneracies of the descending levels are **8, 10, 12, 14**, rising by exactly 2—one mechanism acting in the same form four times over.

Observation 3.3

A point to write honestly—the increments of 2 follow from the definitions. $2j + 1$ with j rising by 1—not a discovery, but a confirmation that the count closes correctly.

Observation 3.4

There is meaning nonetheless. The pattern “drop one level and it agrees” holds four times running—were it an ad hoc correction, both the level dropped and the number dropped would be irregular.

4. The Share of the Contribution Declines

Check 4.1: For each magic number we separate the two contributions.

Magic number	From the oscillator	From spin--orbit	Spin--orbit share
28	20	8	28.57%
50	40	10	20.00%
82	70	12	14.63%
126	112	14	11.11%

Observation 4.2

The share falls from 28.57% to 11.11%, a decline by 2.57. The heavier the nucleus, the more the oscillator side dominates—but it never reaches zero.

Check 4.3: The first three need no descent.

Magic number	From the oscillator	From spin--orbit
2	2	0
8	8	0
20	20	0

Observation 4.4

The rightmost column is 0. Within the one word “magic number” stand entries of 0% and of 28.57%——in the language of Paper 112, one rhyme and two roots.

Observation 4.5

Without spin--orbit coupling, the magic numbers would have been 2, 8, 20, 40, 70, 112, 168. Every one from the fourth on would be a different number——the periodic table of nuclei would have a different shape.

5. Calcium-48 Shows Both at Once

Setting: A nucleus whose proton and neutron numbers are both magic is called doubly magic.

Check 5.1:

Nucleus	<i>Z</i>	<i>N</i>	Mechanism for <i>Z</i>	Mechanism for <i>N</i>
⁴ He	2	2	oscillator	oscillator
¹⁶ O	8	8	oscillator	oscillator
⁴⁰ Ca	20	20	oscillator	oscillator
⁴⁸ Ca	20	28	oscillator	spin--orbit
⁵⁶ Ni	28	28	spin--orbit	spin--orbit
²⁰⁸ Pb	82	126	spin--orbit	spin--orbit

Observation 5.2

Only ^{48}Ca is mixed. $Z = 20$ closes on the oscillator alone; $N = 28$ closes only once $1f_{7/2}$ has descended——two mechanisms side by side within one nucleus.

Observation 5.3

So doubly magic nuclei fall into three kinds——both oscillator (^4He , ^{16}O , ^{40}Ca), mixed (^{48}Ca), both spin--orbit (^{56}Ni , ^{208}Pb). The single term “doubly magic” covers three things too.

Observation 5.4

^{40}Ca and ^{48}Ca are the same element. Without changing a single proton, adding 8 neutrons changes the mechanism of shell closure——the two mechanisms can be set side by side in one laboratory.

6. Separator: Degeneracy or a Pushed-Down Level

Observation 6.1

Separator——what must be moved for the magic numbers to become other numbers? The strength of the spin--orbit coupling.

What closes the shell		Numbers obtained
Oscillator only	level degeneracy	2, 8, 20, 40, 70, 112, 168
+ spin--orbit	one level pushed down	2, 8, 20, 28 , 50, 82 , 126

Observation 6.2

The first row is a consequence of symmetry——the isotropic oscillator’s degeneracy comes from $SU(3)$. The second is a consequence of interaction——it breaks that symmetry and drops one level.

Observation 6.3

So the two mechanisms are different categories. One follows from counting (degeneracy); the other emerges only upon solving (coupling strength)—the same figure by which Paper 265 separated the low-temperature models by density of states, appearing here in how a shell closes.

Observation 6.4

Paper 262 showed there are three ways to break and that strength does not decide which. Here too, what separates “stable” is not the size of the nucleus—it is which mechanism closed the shell.

Observation 6.5

A point to write honestly—in neutron-rich nuclei magic numbers vanish and appear. The vanishing of $N = 28$ and the emergence of $N = 16$ have been reported—this roll call is confined to the neighbourhood of stability.

7. Roll Call and Closing

1. Oscillator sums 2, 8, 20, 40, 70, 112, 168 (Check 2.1): From $(N + 1)(N + 2)$.
2. First three have difference 0 (Check 2.2): Agreement 3/7.
3. +42.86% at the fourth (Check 2.2): 40 against 28.
4. Relative error falls to 33.33% (Observation 2.4): But not converging.
5. Spin--orbit fixes all four (Check 3.1): 8, 10, 12, 14.
6. Degeneracies rise by 2 (Observation 3.2): Follows from definitions.
7. Share 28.57% → 11.11% (Check 4.1): A decline by 2.57.
8. First three have share 0 (Check 4.3): One rhyme, two roots.
9. ^{48}Ca is mixed (Check 5.1): Different mechanisms for Z and N .
10. Three kinds of doubly magic (Observation 5.3): One word covering three.

Closing: The magic numbers 2, 8, 20, 28, 50, 82, 126 do not follow from one rule. Counting with the isotropic three-dimensional harmonic oscillator alone gives 2, 8, 20, 40, 70, 112, 168: the first three have difference 0, and every one from the fourth on is wrong—40 against 28, an error of +42.86%. Spin--orbit coupling fixes all four— $1f_{7/2}$,

$1g_{9/2}$, $1h_{11/2}$, $1i_{13/2}$ descend, and their degeneracies **8, 10, 12, 14** rise by exactly **2**. The share of that contribution falls from 28.57% to **11.11%** but never reaches zero. And ^{48}Ca sets the two side by side within one nucleus—— $Z = 20$ closes on the oscillator alone, $N = 28$ needs spin–orbit. Adding 8 neutrons to ^{40}Ca changes the mechanism of shell closure. One thing separates them——whether the shell is closed by level degeneracy or by one pushed-down level. The former is a consequence of symmetry and follows from counting. The latter is a consequence of interaction and emerges only upon solving. The one word “magic number” covers both.

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