

Separation Improves Only as the Square Root of the Length

— Doubling the Resolution Takes a Column Four Times as Long and Four Times as Much Time — The Square
Roots of Three Fields Share One Root, and Raising Selectivity Is **3.64** Times Cheaper — [Paper 294]

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August 2026

— *From thinking to syncing.* —

Abstract

Chromatographic resolution goes as $R_s \propto \sqrt{N} \propto \sqrt{L}$. This paper asks where that square root comes from—the answer is the additivity of variance. No new mathematical theorem and no new law is claimed. First, the price is the square of the improvement sought. Doubling R_s takes **4** times the plates, **4** times the column length, and **4** times the time (Section 2). Second, this is the core of the paper. The same square root stands in three places—chromatography, diffusion, statistics—and the root is one and the same (Section 3). Third, we confirm it numerically. Summing n independent contributions multiplies the variance by n and the standard deviation by $\sqrt{n}$ —four times n halves the standard error exactly (Section 3). Fourth, selectivity is cheaper than length. Raising α from 1.05 to 1.10 cuts the plates required to one **3.64**th (Section 4). Fifth, there is an optimal flow rate. The van Deemter minimum sits at $u = \sqrt{B/C} = \mathbf{1.4142}$ mm/s with $H = \mathbf{20.97}$ μm (Section 5). Sixth, the separator is what is added independently. If variances are added, a square root; if expectations, first order (Section 6).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed—the resolution expression, the plate number, the van Deemter equation, the $\sqrt{2Dt}$ of diffusion, and the standard error $\sigma/\sqrt{n}$ are all standard. We do not build separation science—all we use is one square root and two divisions. We do not derive the van Deemter equation—we do not enter the physical content of A , B and C . We compute only where the minimum lies. We claim no accuracy for the representative values— $A = 4 \mu\text{m}$, $B = 12000 \mu\text{m}^2/\text{s}$, $C = 0.006 \text{ s}$ are values chosen to show the order for a typical HPLC, and they move greatly with packing, mobile phase and analyte. A peak shape is assumed—the resolution expression presumes Gaussian peaks of equal width. With tailing it does not hold. We do not treat pressure—lengthening a column raises the back pressure in proportion. In practice the pump sets the limit, but that is not treated here. We do not hide the independence behind $\sqrt{n}$ —the additivity of Section 3 assumes the contributions are independent. With correlation it is not $\sqrt{n}$. We do not say the three are “the same phenomenon”—what agrees is the structure by which variances add, not the phenomena. Relation to earlier papers: Paper 249 showed that the 4π that catches molecules by diffusion is the same root as the 4π by which a field spreads—this paper takes the square-root side of the same diffusion and confirms one root across three fields. Paper 112 counted “six distinct roots sharing one rhyme”—this paper is the case where both rhyme and root are the same, the counterpart to 112. Paper 204 showed that the power in an eigenvalue count is half the dimension—that $1/2$ comes from the dimension, whereas the $1/2$ here comes from the additivity of variance. The same $1/2$, different roots. Paper 188 treated cases exactly determined by fewer measurements than unknowns—this paper is the case where adding measurements improves things only as a square root. What is added is writing the price of resolution as “the square of the improvement sought”, identifying that the square roots of three fields come from the single root of variance additivity, computing that raising selectivity is 3.64 times cheaper, and giving the van Deemter optimum numerically.

1. Notation and the Question

Setting: How far two components have separated on a column is measured by the resolution

$$R_s = \frac{\sqrt{N}}{4} \cdot \frac{\alpha - 1}{\alpha} \cdot \frac{k}{1 + k}$$

where N is the plate number, α the selectivity, and k the retention factor.

Fact: Since $N = L/H$, we have $R_s \propto \sqrt{N} \propto \sqrt{L}$.

Question: Why a square root? And is the same square root standing in other fields the same root?

2. The Price Is the Square of the Improvement Sought

Check 2.1: With $\alpha = 1.10$ and $k = 3.0$, we quadruple the plate number.

Plate number N	$\sqrt{N}/4$	R_s	Column length	Time
2500	12.5000	0.852273	×1	×1
10000	25.0000	1.704545	×4	×4
40000	50.0000	3.409091	×16	×16
160000	100.0000	6.818182	×64	×64

Observation 2.2

Each doubling of R_s costs **4** times the plates, the length, and the time. To multiply R_s by 10 takes **100** times——the price is the square of the improvement sought.

Observation 2.3

Note that column length and time stand in the same column of factors. Leave the mobile-phase velocity alone and length and time are proportional——the currency in which separation is bought is time.

Check 2.4: We compute the plates needed for baseline separation, $R_s = 1.5$.

Quantity	Value
$N = 16(R_s/(\text{selectivity} \times \text{retention}))^2$	7744.0 plates

Observation 2.5

R_s enters squared. $R_s = 1.5$ needs 7744 plates; $R_s = 3.0$ needs 30976——“just a little more separation” is always expensive.

3. The Core: The Square Roots of Three Fields Share One Root

Setting: A square root of the same form stands in at least three fields.

Check 3.1:

Field	How it appears	What is under the root
Chromatography	band width $\sigma \propto \sqrt{L}$, $R_s \propto \sqrt{N}$	column length
Diffusion	displacement $\sqrt{\langle x^2 \rangle} = \sqrt{2Dt}$	time
Statistics	standard error $\sigma/\sqrt{n}$	sample size

Observation 3.2

This is the core of the paper. All three follow from the same single line: “summing n independent contributions multiplies the variance by n , so the standard deviation goes as $\sqrt{n}$ ”—not a rhyme, but the same root.

Check 3.3: We confirm numerically. We add n independent quantities of unit variance.

n	Variance	Standard deviation	Standard error of the mean
1	1.0000	1.000000	1.000000
4	4.0000	2.000000	0.500000
16	16.0000	4.000000	0.250000
100	100.0000	10.000000	0.100000
10000	10000.0000	100.000000	0.010000

Observation 3.4

Multiplying n by 4 doubles the standard deviation and exactly halves the standard error. This is precisely the “4 times for a doubling of R_s ” of Section 2—not a coincidence, but the same computation.

Check 3.5: We line up the three prices.

Quantity to improve	Price of doubling	Price of tenfold
Resolution R_s	$4\times$ column length	$100\times$
Diffusive reach	$4\times$ time	$100\times$
Measurement precision (1/standard error)	$4\times$ sample	$100\times$

Observation 3.6

The three rows carry exactly the same numbers. Different fields, different units, different mechanisms——because what is being added is the same.

Observation 3.7

Paper 112 counted “six distinct roots sharing one rhyme.” This paper is its counterpart——here the root is the same too, and that is why the numbers agree exactly. It is the same type of conclusion as Paper 249’s, that diffusion’s 4π and a field’s 4π share a root.

Observation 3.8

A point to write honestly——sharing a root does not make them the same phenomenon. What agrees is only the structure by which variances add——what happens inside a column and what happens when samples are collected are different events.

Observation 3.9

The $1/2$ of Paper 204 has a different root. There it comes from the dimension; here it comes from the additivity of variance——on meeting a $1/2$, one must check which root it has.

4. Selectivity Is Cheaper Than Length

Setting: Besides $\sqrt{N}$, the expression for R_s carries a selectivity term $(\alpha - 1)/\alpha$. That one acts at first order.

Check 4.1: Plates needed for $R_s = 1.5$, by α (with $k = 3.0$).

α	Plates required N	Relative to $\alpha = 1.05$
1.05	28224.0	1
1.10	7744.0	1/3.64
1.20	2304.0	1/12.25
1.50	576.0	1/49.00

Observation 4.2

Merely raising α from 1.05 to 1.10 cuts the plates needed to one **3.64th**. At 1.20, one **12.25th**—changing the mobile or stationary phase to raise selectivity is far cheaper than lengthening the column.

Observation 4.3

The reason is how they enter. N sits under the root; the α term is first order—moving a first-order variable first is always cheaper.

Observation 4.4

The first separator—which variable sits inside the square root. The same “improve the separation” costs a squared amount more when one moves what is inside rather than what is outside.

5. There Is an Optimal Flow Rate

Setting: The plate height H (the length of one plate) follows the van Deemter equation $H = A + B/u + Cu$. The smaller H , the more plates fit in the same length.

Check 5.1: With $A = 4 \mu\text{m}$, $B = 12000 \mu\text{m}^2/\text{s}$, $C = 0.006 \text{ s}$.

Quantity	Value
Optimal velocity $u = \sqrt{B/C}$	1414.21 $\mu\text{m/s}$ = 1.4142 mm/s
Minimum plate height $H = A + 2\sqrt{BC}$	20.9706 μm
Plates in a 15 cm column, $N = L/H$	7152.9

Observation 5.2

B/u is worse when slow (longitudinal diffusion acts); Cu is worse when fast (mass transfer cannot keep up). There is one point where they balance—going faster or slower both lose.

Observation 5.3

Note that B comes from the diffusion coefficient. The same diffusion that produced the square root of Section 3 here fixes the optimum—one mechanism both spoils the separation and sets its limit.

Observation 5.4

And a 15 cm column gives **7152.9** plates. That falls just short of the 7744 that Section 2 required for $R_s = 1.5$ —at $\alpha = 1.10$ this column does not reach baseline separation.

Observation 5.5

So there are two options—lengthen the column by **1.083**, or raise α slightly. As Section 4 showed, the latter is far cheaper.

6. Separator: What Is Added Independently

Observation 6.1

Separator——what must be moved for the square root to become first order?Whether what is added is a variance or an expectation.

What is added	Behaviour over n	Price of improvement
Expectation (signal)	$\propto n$	first order
Variance (noise)	$\propto n$; standard deviation $\propto \sqrt{n}$	squared

Observation 6.2

Signal is added in step and so goes as n ; noise points in scattered directions and grows only as $\sqrt{n}$.So the signal-to-noise ratio improves as $\sqrt{n}$ ——the square root is also good news.

Observation 6.3

Chromatography has the same figure.The retention time t_R is proportional to length, while the band width σ broadens only as the square root of length——which is why the ratio t_R/σ improves as $\sqrt{L}$.

Observation 6.4

So “lengthening improves it only as a square root” is half bad news and half good news.Had the band width also grown in proportion to length, no amount of lengthening would raise the resolution at all——it is precisely because variances add that separation is possible.

Observation 6.5

A point to write honestly——independence is the premise.If the contributions are correlated the variance does not go as n , and the $\sqrt{n}$ rule itself collapses——correlation is the enemy of separation and of measurement alike.

7. Roll Call and Closing

1. Doubling R_s takes $4\times$ plates (Check 2.1): And $4\times$ time.
2. Tenfold R_s takes $100\times$ (Observation 2.2): Price is the square.
3. $R_s = 1.5$ needs 7744 plates (Check 2.4): R_s enters squared.
4. One square root in three fields (Check 3.1): Chromatography, diffusion, statistics.
5. $4 \times n$ halves the standard error (Check 3.3): The same number as Section 2.
6. Three prices agree exactly (Check 3.5): Because the root is one.
7. $\alpha 1.05 \rightarrow 1.10$ gives one 3.64th (Check 4.1): First order.
8. Optimal velocity 1.4142 mm/s (Check 5.1): $H = 20.9706 \mu\text{m}$.
9. 7152.9 plates in 15 cm (Check 5.1): Just short of 7744.
10. Signal n , noise $\sqrt{n}$ (Observation 6.1): Which is why separation works.

Closing: Separation improves only as the square root of the length.Doubling R_s takes **4** times the plates, the column length and the time; a tenfold R_s takes **100** times——the price is the square of the improvement sought.That square root does not belong to chromatography alone——diffusion's $\sqrt{2Dt}$ and statistics' $\sigma/\sqrt{n}$ come from the same single root.The root is the additivity of variance——summing n independent contributions multiplies the variance by n , so the standard deviation grows only as $\sqrt{n}$.Which is why the three tables of prices carry exactly the same numbers.But there is a move——merely raising α from 1.05 to 1.10 cuts the plates required to one **3.64th**.One thing separates them——whether that variable sits inside the square root or outside it.Moving what is outside first is always cheaper.And the square root is only half bad news——it is precisely because the band width grows as a square root rather than in proportion to length that separation is possible at all.

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