

A Strategy That Wins 99.9999% of the Time Has Expectation Exactly 0

— Push the Win Rate Towards 1 and the Loss Grows at the Same Speed — The Fence Is Built of Capital, Not Probability, and Doubling Merely Multiplies the Standard Deviation by 1024 — [Paper 298]

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— *From thinking to syncing.* —

Abstract

The doubling strategy—stake twice as much after every loss—can push the win rate arbitrarily close to 1. This paper asks whether the expectation moves with it—the answer is it stays exactly 0. No new mathematical theorem and no new law is claimed. First, the win rate can be raised without limit. A bankroll of $2^k - 1$ survives k consecutive losses, and at $k = 20$ the win rate is **99.9999046%** (Section 2). Second, this is the core of the paper. The expectation is nevertheless exactly 0 for every k — $1 - 2^{-k} - 1 + 2^{-k} = 0$ (Section 2). Third, the product of loss and its probability exactly balances the gain. $2^{-k} \times (2^k - 1) = 1 - 2^{-k}$, which tends to 1 (Section 3). Fourth, what moved is the variance. At $k = 20$ the standard deviation becomes **1024** times larger while the expectation stays 0 (Section 3). Fifth, with a house edge it gets worse. To win the same 1 unit, the expected loss becomes **26.07** times larger (Section 4). Sixth, the separator is capital, not probability. Make the capital unbounded and all three premises of the optional stopping theorem fail (Section 5).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed—the doubling strategy, martingales, the optional stopping theorem and its three sufficient conditions are all standard. We do not build probability theory—all we use is one geometric series and one expectation. We do not prove the optional stopping theorem—the sufficient conditions are merely quoted, and we do not enter the proof. We do not recommend gambling—this paper only counts how the expectation behaves. As Section 4 shows, in a game with a house edge the expectation is negative. We do not discuss optimal strategy—we do not enter the Kelly criterion, utility functions, or minimising the probability of ruin. We do not discuss real casinos—table limits and the practicalities of capital constraints are not treated. The bankroll $2^k - 1$ is a mathematical setting. We do not call variance evil—the factor of 1024 in Section 3 is a statement of fact, and we do not claim that large variance is always bad. We do not treat the tails—Paper 190 treated heavy tails and large deviations. This paper does not enter the shape of the distribution and counts only expectation and variance. Relation to earlier papers: Paper 143 showed that minimum entropy production is not a principle but a theorem with a fence—the optional stopping theorem here is likewise a theorem with a fence, of the same shape: remove the fence and the conclusion collapses. Paper 194 showed that the four means are one family with only one fence of equality—here too there is only one fence. Paper 220 counted four things called “independence”—the successive bets here are independent in one of those senses. Paper 190 measured “rare” on a logarithmic scale—the 2^{-20} here is likewise read logarithmically. What is added is computing that the expectation is exactly 0 for every k , showing that the product of loss and probability is $1 - 2^{-k}$ and balances the gain exactly, measuring the standard deviation as 1024 times larger, computing that a house edge makes the expected loss 26.07 times larger, and naming the fence as capital rather than probability.

1. Notation and the Question

Setting: Consider a fair game (win probability $1/2$, a win returns the stake). The strategy is: double the stake after a loss, stop after a win.

Observation 1.1

If the first win comes on round k , the losses so far are $1 + 2 + \dots + 2^{k-1} = 2^k - 1$ and that round returns 2^k . The net is $+1$ —whenever the win comes, one is 1 unit ahead.

Question: Does this strategy make money?

Observation 1.2

It does not. This paper shows that numerically and separates why——what stops it is not probability but capital.

2. The Core: Raise the Win Rate and the Expectation Stays Exactly 0

Setting: Take the bankroll to be $2^k - 1$. It survives k consecutive losses and is ruined when it cannot pay the $(k + 1)$ th stake.

Check 2.1:

k	Bankroll $2^k - 1$	Win rate $1 - 2^{-k}$	Loss when it happens	Expectation
1	1	50.0000000%	1	0
5	31	96.8750000%	31	0
10	1023	99.9023438%	1023	0
20	1048575	99.9999046%	1048575	0
30	1073741823	99.9999999%	1073741823	0

Observation 2.2

This is the core of the paper. The win rate tends to 1 with k , yet the expectation is strictly 0 for every k .

Check 2.3: We confirm algebraically.

$$EV = (1 - 2^{-k}) \cdot (+1) + 2^{-k} \cdot (-(2^k - 1)) = 1 - 2^{-k} - 1 + 2^{-k} = 0$$

Observation 2.4

The terms cancel exactly. -2^{-k} against $+2^{-k}$, and $+1$ against -1 ——not approximately but identically 0.

Observation 2.5

Read the $k = 20$ row. With probability **99.9999%** one gains 1 unit; with probability **0.0001%** one loses **1,048,575**——“almost surely a little, very rarely everything.”

3. What Balances Is the Product; What Moved Is the Variance

Check 3.1: We take the product of the losing probability and the loss.

k	Losing probability 2^{-k}	Loss $2^k - 1$	Product
5	3.125000×10^{-2}	31	0.9687500000
10	9.765625×10^{-4}	1023	0.9990234375
20	9.536743×10^{-7}	1048575	0.9999990463
30	9.313226×10^{-10}	1073741823	0.9999999991
40	9.094947×10^{-13}	1099511627775	1.0000000000

Observation 3.2

The product is $1 - 2^{-k}$ and tends to 1. The gain side is $(1 - 2^{-k}) \times 1$, the same value——the loss has grown by exactly as much as the probability has shrunk.

Observation 3.3

This is the content of “the expectation does not move.” Lowering the losing probability and raising the loss are one and the same operation——one cannot move only one of them.

Check 3.4: Then what did move? We compute the variance at $k = 20$.

	A single bet	Doubling ($k = 20$)
Expectation	0	0
Variance	1	1.0486×10^6
Standard deviation	1	1024.0

Observation 3.5

The doubling strategy merely multiplied the standard deviation by **1024**. The expectation stayed 0——what the strategy moved is not the first moment but the second.

Check 3.6: And how many bets?

k	Expected number of bets
10	1.998047
20	1.999998

Observation 3.7

On average it ends in 2 bets. Most rounds win on the first or second bet——ruin looks rare because it is rare. And being rare, it is large.

4. With a House Edge, It Is 26 Times Worse

Setting: Bet on red at roulette. With the zero present, the win probability is $18/37 = 0.486486$.

Check 4.1:

Quantity	Value
Expectation of a single bet (per unit staked)	−2.7027%

Check 4.2: We run the same doubling on this unfavourable game.

k	Win rate	Loss	Expectation
5	96.429250%	31	−0.142640
10	99.872497%	1023	−0.305626
20	99.999837%	1048575	−0.704659

Observation 4.3

What was 0 in the fair game now opens negative. And it gets worse as k rises——doubling does not erase a disadvantage.

Check 4.4: We compare two ways of pursuing the same aim, “win 1 unit.”

Method	Expected loss
A single bet	0.027027
Doubling ($k = 20$)	0.704659
Ratio	26.07

Observation 4.5

To chase the same 1 unit, doubling makes the expected loss **26.07** times larger. The price of raising the win rate is loaded straight onto the expectation——in an unfavourable game, adding variance also worsens the expectation.

Observation 4.6

In a fair game the expectation does not move; in an unfavourable one it worsens. In neither case does it improve——these are the two faces of “strategy cannot move the expectation.”

5. Separator: The Fence Is Built of Capital

Observation 5.1

Here something strange happens. With unbounded capital there is no ceiling on k , and the win rate becomes exactly 1—a certain gain of +1 unit out of a fair game.

Observation 5.2

That contradicts the optional stopping theorem. The theorem says that stopping a fair game does not change the expectation—but it carries premises.

Check 5.3: Any one of three sufficient conditions is needed.

Premise	In the doubling strategy
(a) The stopping time is bounded	fails (one does not know when the win comes)
(b) The increments are bounded	fails (the stake grows as 2^k)
(c) Uniform integrability	fails (the loss at ruin diverges)

Observation 5.4

All three fail. So the theorem does not apply, and no contradiction arises—the theorem is not wrong; one is outside its premises.

Observation 5.5

Separator—what must be moved for this strategy to work? The capital. No change of probability is needed—keep the game fair and make the capital unbounded, and it becomes a sure-win method.

Observation 5.6

Paper 143 showed that minimum entropy production is not a principle but a theorem with a fence. The optional stopping theorem is the same—it holds only inside its fence (boundedness, uniform integrability). And the fence here stands on the side of capital, not probability.

Observation 5.7

Hence the figure “a win rate of 99.9999%” guarantees nothing. The same figure means expectation 0 under finite capital and expectation +1 under unbounded capital—the probability alone does not tell you which.

6. Roll Call and Closing

1. 99.9999046% at $k = 20$ (Check 2.1): Bankroll 1048575.
2. Expectation 0 for every k (Check 2.3): Identically cancelling.
3. The product is $1 - 2^{-k}$ (Check 3.1): Exactly balancing the gain.
4. At $k = 40$ the product is 1.0000000000 (Check 3.1): Converging to 1.
5. Standard deviation $\times 1024$ (Check 3.4): Expectation still 0.
6. Expected bets 1.999998 (Check 3.6): It ends in about two.
7. -0.704659 at roulette (Check 4.2): Worse as k rises.
8. Expected loss $\times 26.07$ (Check 4.4): Chasing the same 1 unit.
9. All three premises fail (Check 5.3): No contradiction arises.
10. The fence is on the capital side (Observation 5.5): Not probability.

Closing: A strategy that wins 99.9999% of the time has expectation exactly 0. A bankroll of $2^k - 1$ survives k consecutive losses, and at $k = 20$ the win rate is 99.9999046%—yet $(1 - 2^{-k}) - 2^{-k}(2^k - 1) = 0$ holds identically. What balances is the product—lowering the losing probability to 2^{-k} and raising the loss to $2^k - 1$ are one and the same operation, and one cannot move only one of them. What moved was the variance alone—doubling multiplied the standard deviation by 1024 and left the expectation at 0. With a house edge it is worse still—chasing the same 1 unit makes the expected loss 26.07 times larger. One thing separates them—not probability, but capital. With unbounded capital all three premises of the optional stopping theorem fail, and a fair game yields a certain profit. The fence stands on the side of capital, not of probability. So the figure “a win rate of 99.9999%” guarantees nothing on its own.

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