

The Yamagishi Double-Torus Stellar Fortress Equation — 2nd Edition: on the $256\pi^4$ obtained from $\bar{\rho} = 4\pi n$ in the self-gravitational energy — [Paper 30]

Yuuki YAMAGISHI (山岸 勇輝)
Yamagishi Complexity Torus Lab, Japan
July 2026

— *From thinking to syncing.* —

ABSTRACT

We rewrite the self-gravitational potential energy of a uniform sphere, $U = -\frac{3GM^2}{5R}$, using $M = \frac{4}{3}\pi R^3 \bar{\rho}$ and the Yamagishi mass patch $\bar{\rho} = 4\pi n$, and examine the origin of the resulting constant $256\pi^4 = (16\pi^2)^2$.

Principal withdrawal in the 2nd edition: The first edition claimed to have explained hydrostatic equilibrium. This is **withdrawn**. The paper merely rewrites the self-gravitational energy U and **never writes down a pressure equation at all**. Hydrostatic equilibrium is $dP/dr = -Gm(r)\bar{\rho}/r^2$, and stability follows from the virial theorem as the condition $\gamma > 4/3$. **From the expression for U alone, nothing can be said about stability** (Section 5).

The reading of $256\pi^4$ as a "double toroidal barrier" is likewise withdrawn: $256\pi^4$ is a **consequence of choosing the mass unit $m = 4\pi$** , and setting $m = 7$ would produce a different constant with equal justification (Section 4).

Scope of this paper: This paper offers a **structural interpretation** of existing physical law. It does not claim the discovery of new physical laws or the proof of open problems. **Note on units:** Substitutions such as $m = 4\pi$ are a **choice of natural units**. G and $\hbar$ are dimensionful; their numerical values depend entirely on the unit system. Setting $G = 4\pi$ is legitimate in exactly the same sense as setting $G = 1$ (geometric units) or $G = 6.674 \times 10^{-11}$ (SI); **physics does not single out one of these as "correct."** What this paper exhibits is therefore **how the equations look in this unit system**, not a new fact about nature.

Introduction

Self-gravitational potential energy of a uniform sphere

For a uniform sphere of radius R and mass M , the gravitational self-energy is

$$U = -\frac{3GM^2}{5R}$$

Density expansion

Writing $M = \frac{4}{3}\pi R^3 \bar{\rho}$ gives

$$U = -\frac{16\pi^2 G \bar{\rho}^2 R^5}{15}$$

Substituting $\bar{\rho} = 4\pi n$

Proposition

Substituting $\bar{\rho} = 4\pi n$ into the self-energy:

$$U = -\frac{256\pi^4 G n^2 R^5}{15} \quad (256\pi^4 = (16\pi^2)^2)$$

Proof. Since $\bar{\rho}^2 = (4\pi n)^2 = 16\pi^2 n^2$, substitution gives $U = -\frac{16\pi^2 G \cdot (16\pi^2 n^2) R^5}{15} = -\frac{256\pi^4 G n^2 R^5}{15}$.

[2nd edition note: the algebra is correct]. Numerical verification confirms that substituting $M = \frac{4}{3}\pi R^3 \bar{\rho}$ into $U = -\frac{3}{5} \frac{GM^2}{R}$ gives the coefficient $\frac{3}{5} \cdot \left(\frac{4}{3}\right)^2 = \frac{16}{15}$, and that substituting $\bar{\rho} = 4\pi n$ gives $\frac{16}{15} \cdot 16 = \frac{256}{15}$. There is no computational error. What requires correction is the interpretation below.

[2nd Edition] Corrections to the reading of $256\pi^4$

Correction 1: $256\pi^4$ is a consequence of the chosen mass unit (error pattern A)

$\bar{\rho} = mn$ (mass density = particle mass $\times$ number density) is a **definition**; substituting $m = 4\pi$ gives $\bar{\rho} = 4\pi n$. Hence $U \propto \bar{\rho}^2 = m^2 n^2$, so the constant is fixed by m^2 :

Mass unit	Coefficient of U ($\times G n^2 R^5$)
$m = 4\pi$ (this paper)	$-\frac{256\pi^4}{15} = -1662.4485$
$m = 1$	$-\frac{16\pi^2}{15} = -10.5276$
$m = 7$	$-\frac{16\pi^2 \cdot 49}{15} = -515.8513$

$256\pi^4$ appears **because m was set to 4π** , and means nothing beyond that. The "doubling" arises because $U \propto M^2$ (hence $\propto \bar{\rho}^2$, hence $\propto m^2$), **not because tori are nested**. The description of a "double toroidal barrier being deployed" is withdrawn.

Correction 2: the first $16\pi^2$ comes from the ball-volume formula (error pattern C)

The $16\pi^2$ appearing in Section 1.2 originates from **squaring the ball volume** $\frac{4}{3}\pi R^3$: $(\frac{4}{3}\pi)^2 = \frac{16\pi^2}{9}$. The "16" is simply 4^2 .

This is of different origin from the other occurrences of $16\pi^2$ in this corpus:

Origin	Context	Source
square of the ball-volume factor $(4\pi/3)^2$	self-gravitational energy	this paper
$\text{Vol}(S^2 \times S^2)$	chiral anomaly normalisation	Paper 29
radius-2 S^3 : $2\pi^2 \cdot 2^3$	Hopf decomposition	Papers 08, 45
inverse of $\Omega_3/(2\pi)^4 \cdot \frac{1}{2}$	4D momentum measure	Papers 46, 47
$n = 2$ term of $(4\pi)^2 n!$	Gauss–Bonnet normalisation	Paper 48
$(2\pi) \times (8\pi)$	tokamak torus	Paper 22

Following the discipline established in Paper 48, **these must not be merged**.

[2nd Edition] Withdrawal of the claim about hydrostatic equilibrium

The first edition stated in its title and abstract that it **explains hydrostatic equilibrium**, describing it as "the maintenance process of a double-torus packaging fortress". **This claim is withdrawn in full.**

Reason 1: no pressure equation is written

Hydrostatic equilibrium is the balance

$$\frac{dP}{dr} = - \frac{G m(r) \bar{\rho}(r)}{r^2}$$

between the pressure gradient and gravity. This paper only rewrites the self-gravitational energy U ; **the symbol P never appears**. The subject it claims to explain is never engaged.

Reason 2: stability cannot be read off from U

Whether a star collapses under its own weight is determined not by the value of U but by the **equation of state**. By the virial theorem, the system is stable when the adiabatic index satisfies

$$\gamma > \frac{4}{3},$$

and for $\gamma < 4/3$ it collapses *whatever* the value of U . **This paper makes no reference to that condition.** The statement that "a $256\pi^4$ double barrier prevents collapse" has no dynamics behind it (error pattern A: presenting as a mechanism something that is not one).

Reason 3: a uniform sphere is a crude stellar model

The expression $U = -\frac{3}{5} \frac{GM^2}{R}$ assumes **uniform density**. Real stars are centrally condensed: for an $n = 3$ polytrope, appropriate to the Sun, $U \simeq -1.5 \frac{GM^2}{R}$, **about 2.5 times the uniform-sphere value**. This paper does not address stellar interior structure.

Explicit Scope

- **What this paper establishes:** that substituting $\bar{\rho} = 4\pi n$ into the self-gravitational energy of a uniform sphere gives $U = -\frac{256\pi^4 G n^2 R^5}{15}$. **The algebra is correct** (verified).
- **What it does not establish:** (i) hydrostatic equilibrium (no pressure equation is written); (ii) stellar stability (no reference to $\gamma > 4/3$); (iii) any topological meaning for $256\pi^4$ (it is a consequence of the mass unit); (iv) stellar interior structure (uniform density assumed).
- **Character of this paper:** a rewriting consequent on a choice of units, containing no independent physical result. The title and abstract have been adjusted accordingly.
- **Status of $\bar{\rho} = 4\pi n$:** this is the definition $\bar{\rho} = mn$ with $m = 4\pi$ substituted — **a choice of units, not a derivation**. The first edition did not make this explicit.

Conclusion

Substituting $M = \frac{4}{3} \pi R^3 \bar{\rho}$ and $\bar{\rho} = 4\pi n$ into the self-gravitational energy $U = -\frac{3GM^2}{5R}$ of a uniform sphere gives $U = -\frac{256\pi^4 G n^2 R^5}{15}$. **The algebra is correct.**

However, the first edition's reading — that $256\pi^4$ is a double toroidal barrier which thereby produces hydrostatic equilibrium — is withdrawn. $256\pi^4$ follows from choosing the mass unit $m = 4\pi$, and since this paper writes no pressure equation, it does not address hydrostatic equilibrium at all.

References

1. Chandrasekhar, S. (1939). *An Introduction to the Study of Stellar Structure*. University of Chicago Press.

2. Yamagishi, Y. (2026a). *The Uniqueness Theorem of $\mathfrak{n} = \mathbf{2}$ for Spherically Symmetric Density Distributions* [Paper 01]. Zenodo. 10.5281/zenodo.21403186

3. Yamagishi, Y. (2026ab). *The Yamagishi-Adler-Bardeen Note* [Paper 29]. Zenodo. 10.5281/zenodo.21601581

— *From thinking to syncing.* —

Yuuki YAMAGISHI (山岸 勇輝)

© 2026 Yamagishi Complexity Torus Lab, Japan · Licensed under Creative Commons Attribution 4.0 International (CC BY 4.0)