

Grouping by Value Gives the Reverse of Grouping by Root

— The Golden Ratio Has Appeared a Third Time — Numerically (1) and (3) Agree to a Difference of 0 While (2) Differs, Yet the Verdict Puts (2) and (3) Together and (1) Apart — [Paper 301]

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— *From thinking to syncing.* —

Abstract

The golden ratio has appeared three times in this corpus—as the reason a crystal forbids five-fold symmetry (Paper 269), as the order of convergence of the secant method (Paper 288), and, added here, as the pole of the Fibonacci generating function. This paper asks whether the three share a root—the answer is two share one, and one does not. No new mathematical theorem and no new law is claimed. First, numerically (1) and (3) agree exactly. $2 \cos 72^\circ$ and the generating function's pole differ by 0; only (2) is a different number (Section 2). Second, this is the core of the paper. The verdict is the reverse—(2) and (3) share a root, and (1) does not (Sections 3--5). Third, Criterion A rejects (1). Moving n in $2 \cos(2\pi/n)$, the golden ratio appears at $n = 5$ alone (Section 3). Fourth, Criterion B joins (2) and (3). The exponents of e_0 and e_1 in the secant method's error are F_{n-1} and F_n themselves (Section 4). Fifth, Criterion C returns the same verdict. (1) comes from geometry; (2) and (3) from one and the same recursion (Section 5). Sixth, the three criteria agreed. All three rejected the grouping by value (Section 6).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed—the crystallographic restriction, the order of the secant method, the Fibonacci generating function, and Binet’s formula are all standard. We do not build number theory—all we use is one recursion and two quadratics. We claim no significance for the golden ratio—aesthetic properties and ubiquity in nature are not treated at all. Only the lineage of three appearances is adjudicated. We invent no criteria—the three used are Papers 31 and 101 (move the variable), Paper 56 (a correspondence preserving identity), and Paper 279 (the route), all already in the corpus. We do not use all five criteria—Paper 116 (a property of the quantity) and Paper 153 (the number of maps) are not in a form applicable here. Agreement of three does not license the claim that five would agree. We do not demote (1)—having a distinct root is not being inferior. Nothing in Paper 269’s conclusion is changed by this paper. We do not deny the algebraic relation— φ and $1/\varphi$ lie in the same field $\mathbb{Q}(\sqrt{5})$ and are roots of reciprocal polynomials. What this paper calls a distinct root is not that algebraic relation but the provenance within the corpus. Relation to earlier papers: Paper 300 gathered the five criteria of sameness of root the corpus had been using, and then warned itself that joinings went up while statements of criteria did not—this paper is the first to answer that warning, and applies the gathered criteria explicitly. Paper 288 noted that φ and $1/\varphi$ come from the same quadratic and withheld judgement, writing that it claimed no significance for the coincidence—this paper resolves that reservation using the criteria. Paper 269 showed that what a crystal forbids is periodicity—the (1) here is the number that follows from it. Paper 112 counted “six distinct roots sharing one rhyme”—this paper is the case of three rhymes and two roots. What is added is showing that grouping by value and grouping by verdict disagree completely, confirming that the secant method’s error exponents coincide with the Fibonacci numbers from $n = 2$ to 10, confirming that all three criteria return the same verdict, and making this the first response to Paper 300’s warning.

1. A Third Meeting

The corpus has met the golden ratio three times.

Provenance		Value
(1)	The crystallographic restriction, $2 \cos 72^\circ$ (Paper 269)	0.6180339887498949
(2)	The order of the secant method, $p^2 = p + 1$ (Paper 288)	1.6180339887498949
(3)	The pole of $G(x) = x/(1 - x - x^2)$ (this paper)	0.6180339887498949

Observation 1.1

Paper 288 noted this coincidence and withheld judgement: “this paper claims no significance for the coincidence.” At that time the criteria were still scattered across five papers—Paper 300 gathered them.

Observation 1.2

Paper 300 also issued itself a warning—“joinings went up; statements of criteria did not.” This paper is the first to answer that warning—it names the criteria and applies them.

2. The First Half of the Core: By Value, (1) Pairs with (3)

Check 2.1: We compare the three values.

Pair	Difference	
(1) and (3)	0.00×10^0	exactly equal
(1) and (2)	1.0000000000	different numbers
(2) and (3)	1.0000000000	different numbers

Observation 2.2

On the numbers alone the answer looks plain—(1) and (3) are the same and (2) is different. And the difference is not small but 0—they agree to the last bit of floating point.

Check 2.3: We also look at the quadratics they satisfy.

	Quadratic satisfied	Residual
$\varphi = 1.618\dots$	$x^2 - x - 1 = 0$	0.00×10^0
$1/\varphi = 0.618\dots$	$x^2 + x - 1 = 0$	1.11×10^{-16}

Observation 2.4

The two are not the same equation—they are reciprocal polynomials, lying in the same field $\mathbb{Q}(\sqrt{5})$ but distinct as polynomials.

Observation 2.5

So grouping by value gives $\{(1), (3)\}$ against (2) , and grouping by equation gives the same—everything visible supports one and the same grouping.

3. Criterion A (Move the Variable) Rejects (1)

Setting: The criterion of Papers 31 and 101 is not agreement at one point, but whether agreement persists when the variable is moved.

Check 3.1: We move n in the family $2 \cos(2\pi/n)$ to which (1) belongs.

n	$2 \cos(2\pi/n)$	
1	+2.0000000000	
2	−2.0000000000	
3	−1.0000000000	
4	+0.0000000000	
5	+0.6180339887	only here is it $1/\varphi$
6	+1.0000000000	
7	+1.2469796037	
8	+1.4142135624	

Observation 3.2

The golden ratio appears at $n = 5$ alone. Move n by one and it is 0; by one more and it is 1—move it and it vanishes.

Observation 3.3

(2) and (3) have no variable to move. Neither $p^2 = p + 1$ nor $1 - x - x^2 = 0$ carries an n —they are identically φ , not a value at a point.

Observation 3.4

We apply Paper 101's wording as it stands—"not agreement at one point, but whether it keeps agreeing as a function when the variable is moved." (1) is agreement at one point—Criterion A rejects (1).

Observation 3.5

We note the crystallographic side. $2 \cos(2\pi/n)$ is an integer for $n = 1, 2, 3, 4, 6$, and that it is not an integer at $n = 5$ is what forbids five-fold symmetry—so (1)'s golden ratio acts through the property "not an integer". That its value is φ plays no part in the prohibition.

4. Criterion B (A Correspondence Preserving Identity) Joins (2) and (3)

Setting: Definition 4.1 of Paper 56 requires that a correspondence preserving identity exist, and states that numerical agreement alone is not accepted.

Check 4.1: The secant method's error propagates as $e_{n+1} \sim C e_n e_{n-1}$. Expanding, we count the exponents of e_0 and e_1 .

n	Exponent of e_0	Exponent of e_1	F_{n-1}	F_n
2	1	1	1	1
3	1	2	1	2
4	2	3	2	3
5	3	5	3	5
6	5	8	5	8
7	8	13	8	13
8	13	21	13	21
9	21	34	21	34
10	34	55	34	55

Observation 4.2

From $n = 2$ to 10 they agree completely. The secant method’s error exponents are the Fibonacci numbers —not similar to them, but the same sequence.

Observation 4.3

This is not an agreement of values. What agrees is sequence with sequence, term corresponding to term—the “correspondence preserving identity” Paper 56 demanded is present as such.

Check 4.4: We turn to the generating function.

Quantity	Value
Poles of $G(x) = x/(1 - x - x^2)$	$+0.6180339887498949, -1.6180339887498949$
Radius of convergence (nearest pole)	$0.6180339887498949 = 1/\varphi$
Growth rate of F_n (1/radius)	$1.6180339887498947 = \varphi$

Observation 4.5

The denominator $1 - x - x^2$ is the Fibonacci recursion $F_n = F_{n-1} + F_{n-2}$ written out. And the secant exponents obey the same recursion——(2) and (3) are two faces of one recursion.

Check 4.6: We check with Binet's formula.

n	F_n	$(\varphi^n - \psi^n)/\sqrt{5}$	Difference
10	55	55.000000	1.42×10^{-14}
20	6765	6765.000000	4.55×10^{-12}
30	832040	832040.000000	8.15×10^{-10}
40	102334155	102334155.000000	1.34×10^{-7}

Observation 4.7

The differences are floating-point rounding only (relative 10^{-15}). φ sits inside the Fibonacci sequence by definition——it did not agree from outside.

Check 4.8: Convergence of the ratio F_{n+1}/F_n .

n	F_{n+1}/F_n	Difference from φ
5	1.6000000000000001	1.80×10^{-2}
10	1.6181818181818182	1.48×10^{-4}
20	1.6180339985218033	9.77×10^{-9}
30	1.6180339887505408	6.46×10^{-13}
40	1.6180339887498949	0.00×10^0

Observation 4.9

At $n = 40$ it reaches machine precision. The φ of (3) is generated by the sequence itself—and the φ of (2) is generated by the same sequence.

5. Criterion C (The Route) Returns the Same Verdict

Setting: Paper 279’s criterion is that sameness or distinctness of root is adjudicated by the route.

Check 5.1:

Where it came from		Kind
(1)	the pentagon’s diagonal ratio / the restriction on translation	geometry
(2)	the two-term error recursion $e_{n+1} \sim e_n e_{n-1}$	recursion
(3)	the denominator $1 - x - x^2$ of the generating function	recursion (the same)

Observation 5.2

(2) and (3) arrive at the same recursion. (1) does not go there—the pentagon’s diagonal ratio yields φ without passing through a recursion.

Observation 5.3

A point to write honestly—(1) does have relations to recursions. Nest pentagons and powers of φ line up, connecting to Fibonacci—but that is a different story, and not the route Paper 269 used. What this paper adjudicates is the route actually taken within the corpus.

6. The Three Criteria Agreed, and Rejected the Grouping by Value

Check 6.1: We collect the verdicts.

	Criterion A (move)	Criterion B (correspondence)	Criterion C (route)
(1) crystal	one point only	none	geometry
(2) secant	identical	agrees with F_n	recursion
(3) generating fn.	identical	is F_n	recursion

Observation 6.2

This is the core of the paper.

Way of grouping	Grouping obtained
By value	$\{(1), (3)\}$ against (2)
By verdict	$\{(2), (3)\}$ against (1)

Observation 6.3

The two disagree completely.(1) and (3), equal in value, are of distinct roots; (2) and (3), different in value, are of the same root—an agreement of difference 0 pointed at the wrong grouping.

Observation 6.4

Paper 56’s Definition 4.1 wrote that numerical agreement alone is not accepted.This paper shows something stronger—numerical agreement is not merely insufficient as evidence; it can point the opposite way.

Observation 6.5

We record too that the three criteria returned the same verdict independently.A rejects (1), B joins (2) and (3), and C confirms both—one criterion alone might be chance; three coincided.

Observation 6.6

A point to write honestly——Paper 300 gathered five criteria and this paper used only three. Paper 116 (a property of the quantity, i.e. mass dimension) and Paper 153 (the number of maps) are not in a form applicable here——agreement of three does not license the claim that five would agree.

Observation 6.7

And (1) is not demoted. Having a distinct root is not being inferior——nothing in Paper 269's conclusion changes. What changed is only the answer to the temptation to bind the three into one.

Observation 6.8

Separator——what must be moved for the verdict to swap? The object one looks at. Look at the number produced and you get $\{(1), (3)\}$; look at what produced the number and you get $\{(2), (3)\}$ ——the same three objects split two ways according to where one looks.

7. Roll Call and Closing

1. (1) and (3) differ by 0 (Check 2.1): **Equal to the last bit.**
2. φ and $1/\varphi$ are different polynomials (Check 2.3): **Reciprocal, not identical.**
3. $2 \cos(2\pi/n)$ only at $n = 5$ (Check 3.1): **Move it and it vanishes.**
4. (2)(3) have no variable to move (Observation 3.3): **Identical.**
5. Error exponents are F_{n-1}, F_n (Check 4.1): **Exact from $n = 2$ to 10.**
6. The denominator is the recursion (Observation 4.5): $1 - x - x^2$.
7. Binet differs only by rounding (Check 4.6): **Relative 10^{-15} .**
8. F_{41}/F_{40} differs by 0 (Check 4.8): **Machine precision at $n = 40$.**
9. Three criteria, one verdict (Check 6.1): **(2)(3) same, (1) distinct.**
10. Value and verdict disagree entirely (Observation 6.3): **A difference of 0 pointed the wrong way.**

Closing: The golden ratio has appeared three times in this corpus——as the reason a crystal forbids five-fold symmetry, as the order of the secant method, and as the pole of the Fibonacci generating function. On the numbers alone the answer looks plain——(1) and (3) agree to a difference of 0, to the last bit, and only (2) is a different

number. Yet the verdict is the reverse. Criterion A (move the variable) rejects (1)—in $2 \cos(2\pi/n)$ the golden ratio appears at $n = 5$ alone and vanishes when moved. Criterion B (a correspondence preserving identity) joins (2) and (3)—the exponents of e_0 and e_1 in the secant method's error are F_{n-1} and F_n themselves, agreeing exactly from $n = 2$ to 10. Criterion C (the route) returns the same verdict—(1) from geometry, (2) and (3) from one and the same recursion. One thing separates them—whether one looks at the number produced or at what produced it. By value the grouping is $\{(1), (3)\}$; by verdict it is $\{(2), (3)\}$ —an agreement of difference 0 pointed at the wrong grouping. Paper 56 wrote that numerical agreement alone is not accepted as sameness of root. What this paper adds is a sentence one degree stronger—numerical agreement is not merely insufficient as evidence; it can point the opposite way.

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