

“It Must Appear” Does Not Tell You When

— Checking $R(5, 5)$ by Exhaustion Would Take $10^{271.8}$ Colourings, **191.8** Orders More Than There Are Atoms
in the Universe — Existence Can Be Proved; Size Is Not Given — [Paper 302]

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— *From thinking to syncing.* —

Abstract

Ramsey’s theorem says that if it is large enough, order must appear. This paper asks how large “large enough” is —the answer is the theorem does not say. No new mathematical theorem and no new law is claimed. First, only two values are settled. $R(3, 3) = 6$ and $R(4, 4) = 18$; $R(5, 5)$ is pinned only to the range $[43, 48]$ (Section 2). Second, this is the core of the paper. Checking $R(5, 5) = 43$ by exhaustion needs $2^{903} = 10^{271.8}$ colourings, **191.8** orders more than the atoms in the universe (Section 3). Third, each step up jumps. $R(3, 3)$ is 32768 colourings, within reach by hand; $R(4, 4)$ is 1.14×10^{46} , which computers barely reached (Section 3). Fourth, the lower-bound proof does not give the value either. Erdos’s probabilistic method gives $R(k, k) > 2^{k/2}$, which at $k = 10$ is one **24.94**th of the truth (Section 4). Fifth, the upper bound is further off still. $W(3, 3) = 27$, yet the bound from the classical proof was of Ackermann type (Section 5). Sixth, the separator is existence against quantity. A different cut from Paper 163’s existence against construction (Section 6).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed——Ramsey’s theorem, the known values and ranges of Ramsey numbers, van der Waerden numbers, and Erdos’s probabilistic lower bound are all standard. We do not build combinatorics——all we use is one binomial coefficient and two logarithms. We do not prove Ramsey’s theorem——we merely quote it. We do not compute Ramsey numbers——no attempt is made to find $R(5, 5)$. We count only the effort it would take. We assert no values for the ranges—— $R(5, 5) \in [43, 48]$ and $R(6, 6) \in [102, 160]$ are ranges known at one epoch and may be improved. The claim is that the range is not empty, not the endpoints. We do not say exhaustion is the only route——actual searches cut enormously by symmetry and pruning. 2^{903} is a naive upper bound, not the necessary work. The point is the order of magnitude by which it remains out of reach. We do not use “Ackermann type” strictly——it indicates that the pre-Gowers bound was of a tower-of-growing-height kind. This paper asserts no classification of the bound, only its divergence from the true value. Relation to earlier papers: Paper 163 separated “a good code exists” from “here is a good code”——that is existence against construction; this is existence against quantity. It treats the case where one can construct yet cannot reach, so the cut differs. Paper 179 showed that “cannot be constructed” has distinct roots——this paper treats what can be constructed yet is out of reach. Paper 190 measured “rare” on a logarithmic scale——the 271.8 orders here are read logarithmically too. Paper 259 showed there are two roads to independence——this paper likewise asks what kinds of showing there are. What is added is computing the exhaustion for $R(5, 5)$ as $10^{271.8}$ and measuring the gap to the atoms of the universe as 191.8 orders, lining up the jumps at $R(3, 3)$, $R(4, 4)$, $R(5, 5)$, computing that the probabilistic lower bound is one 24.94th at $k = 10$, and placing the separator at existence against quantity.

1. Notation and the Question

Theorem (Ramsey, 1930): If n is large enough, then colouring the edges of the complete graph on n vertices with two colours, a monochromatic K_k must appear.

Definition: The least such n is written $R(k, k)$.

Question: The theorem says it must appear. What, then, is $R(k, k)$?

Observation 1.1

The proof guarantees existence. It does not give the value——this paper measures that gap numerically.

Observation 1.2

Erdos put it famously—if aliens demanded $R(5, 5)$ humanity should commit all its computing resources, but if they demanded $R(6, 6)$ we should rather fight. This paper checks that remark against numbers.

2. Only Two Values Are Settled

Check 2.1:

	Lower	Upper	Settled?
$R(3, 3)$	6	6	yes
$R(4, 4)$	18	18	yes
$R(5, 5)$	43	48	no
$R(6, 6)$	102	160	no
$R(7, 7)$	205	492	no

Observation 2.2

Only the first two are settled. $R(4, 4) = 18$ was settled in 1995—sixty-five years after the theorem was proved.

Observation 2.3

The range for $R(6, 6)$ is $[102, 160]$, a width of 58. For $R(7, 7)$ it is $[205, 492]$, a width of 287—each step up increases the ignorance.

Observation 2.4

Yet “it exists” has been proved from the start. For every k , $R(k, k)$ is finite—known to be finite and unknown in value.

3. The Core: $10^{271.8}$ Colourings to Exhaust

Setting: Suppose we check $R(5, 5) = 43$ by exhaustion. We two-colour every edge of the complete graph on 43 vertices.

Check 3.1:

Quantity	Value
Vertices	43
Edges $\binom{43}{2}$	903
Colourings 2^{903}	$10^{271.8}$
Atoms in the observable universe	10^{80}
Orders short	191.8

Observation 3.2

This is the core of the paper. Turn every atom into a computer and one is still **191.8** orders short——what is lacking is not the speed of computers but the quantity of matter.

Check 3.3: We step down to see where reach begins.

	Vertices	Edges	Colourings
$R(3, 3)$	6	15	32768 (countable by hand)
$R(4, 4)$	18	153	1.14×10^{46} (by computer, 1995)
$R(5, 5)$	43	903	$10^{271.8}$ (out of reach)

Observation 3.4

From $R(3, 3)$ to $R(4, 4)$ is 10^{41} ; from $R(4, 4)$ to $R(5, 5)$ is 10^{226} ——each step up jumps further than the last.

Observation 3.5

A point to write honestly—— 2^{903} is a naive upper bound.Real searches cut it enormously by symmetry and pruning——only the fact that $R(5, 5)$ has still not been reached grounds this paper.

4. The Lower-Bound Proof Does Not Give the Value Either

Setting: Erdos (1947) showed $R(k, k) > 2^{k/2}$ by the probabilistic method.Colour at random; a configuration with no monochromatic K_k exists with positive probability.

Check 4.1:

k	$2^{k/2}$	Known lower bound	Ratio
3	2.83	6	2.12
4	4.00	18	4.50
5	5.66	43	7.60
6	8.00	102	12.75
7	11.31	205	18.12
10	32.00	798	24.94

Observation 4.2

The probabilistic bound is one **24.94**th of the truth at $k = 10$.And the ratio widens with k ——it does not converge.

Observation 4.3

There is a second gap here.The method shows that a colouring with no monochromatic K_k exists, and exhibits not one of them——the existence against construction of Paper 163.

Observation 4.4

So the gap is twofold.

What is known	What is not
$R(k, k)$ is finite	its value (this paper)
A colouring meeting the bound exists	which colouring (Paper 163)

Observation 4.5

The two rows are different gaps. The upper is quantity, the lower construction—one theorem carrying two kinds of “does not say” at once.

5. The Upper Bound Is Further Off Still

Setting: Van der Waerden’s theorem has the same shape——“if it is long enough, an arithmetic progression must appear.”

Check 5.1: Known values.

	Value
$W(2, 3)$	9
$W(3, 3)$	27
$W(4, 3)$	76
$W(2, 4)$	35
$W(2, 5)$	178

Observation 5.2

$W(3, 3) = 27$. Yet the bound from the classical proof was of a tower-of-growing-height kind (Ackermann type)—the true value is 27, and the bound could not even be written down.

Observation 5.3

Gowers (2001) removed the tower and brought the bound down to an elementary form. The gap to the true value remains—that a change of technique moves the bound by orders means the bound was not measuring the object.

Observation 5.4

This is the paper's second core. A theorem's upper bound measures the power of the proof's tools, not the size of the object—change the tools and the bound moves; the object does not.

Observation 5.5

A point to write honestly—"Ackermann type" is used as an indication. What this paper asserts is not a classification of the bound but only that the divergence from the true value moved by orders.

6. Separator: Existence against Quantity

Observation 6.1

Separator—what must be moved to turn "it must appear" into "when it appears"? Nothing moves. They are different questions, and an answer to one does not contain the other.

Question	Type of answer	Here
Does it appear?	yes/no	yes (settled 1930)
At what size?	a number	open for $k \geq 5$
Which configuration?	a construction	not exhibited

Observation 6.2

The three rows are answered independently. The first may be “yes” while the second and third stay open——the single word “proved” points at only one of the three.

Observation 6.3

Paper 163 separated “a good code exists” from “here is a good code.” This paper sets a third sentence beside them——“a good code lies at or below n .” Existence, quantity and construction are mutually independent.

Observation 6.4

Paper 190 measured “rare” on a logarithmic scale. The **271.8** orders here are the same——a size meaningless as a ratio, yet expressible as “191.8 orders short”.

Observation 6.5

Finally we reread Erdos’s remark numerically. $R(5, 5)$ is $10^{271.8}$ colourings; $R(6, 6)$ has $\binom{102}{2} = 5151$ edges, hence $2^{5151} = 10^{1550.6}$ ——a gap of 1278.8 orders. “Rather fight” was not hyperbole but a statement of orders.

7. Roll Call and Closing

1. Settled: $R(3, 3) = 6$, $R(4, 4) = 18$ (Check 2.1): $R(5, 5)$ only a range.
2. $R(4, 4)$ settled in 1995 (Observation 2.2): 65 years after the theorem.
3. $R(5, 5)$ exhaustion is $10^{271.8}$ (Check 3.1): 903 edges.

4. **191.8 orders past the atoms** (Check 3.1): **Matter, not speed, is lacking.**
5. **One step is 10^{226}** (Observation 3.4): **The jumps grow.**
6. **Lower bound is one 24.94th at $k = 10$** (Check 4.1): **The ratio widens.**
7. **The gap is twofold** (Observation 4.4): **Quantity and construction.**
8. **$W(3, 3) = 27$** (Check 5.1): **The bound was a tower.**
9. **The bound measures the tools** (Observation 5.4): **The object does not move.**
10. **$R(6, 6)$ is $10^{1550.6}$** (Observation 6.5): **1278.8 orders past $R(5, 5)$.**

Closing: Ramsey’s theorem says that if it is large enough, order must appear. But it does not say how large. Only $R(3, 3) = 6$ and $R(4, 4) = 18$ are settled, and $R(5, 5)$ is pinned only to the range $[43, 48]$ —even settling $R(4, 4)$ took 65 years from the theorem. Counting the work of exhaustion shows why— $R(5, 5) = 43$ has **903** edges and $2^{903} = 10^{271.8}$ colourings, **191.8 orders** more than the atoms in the observable universe. What is lacking is not the speed of computers but the quantity of matter. The lower bound is no better—the probabilistic method gives $R(k, k) > 2^{k/2}$, one **24.94th** of the truth at $k = 10$, and the ratio widens with k . The upper bound is further off still— $W(3, 3)$ is truly **27**, yet the classical bound could not be written down. A theorem’s upper bound measures the power of the proof’s tools, not the size of the object. One thing separates them—whether it exists, how large it is, and which one it is, are three different questions. The first may be “yes” while the other two stay open. The single word “proved” points at only one of the three.

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