

“Fitness” Is Four Quantities, and One of Them Changes Sign When the Population Changes

— The Same Individual Goes from Favoured to Disfavoured by Swapping Its Neighbours — Across Generations, Only the Logarithm Adds — [Paper 307]

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— *From thinking to syncing.* —

Abstract

The sentence “its fitness is high” points at one of four different quantities. This paper asks whether the four behave alike—the answer is only one depends on the population. No new mathematical theorem and no new law is claimed. First, there are four. Absolute fitness W , relative fitness w , the Malthusian parameter m , and inclusive fitness (Section 1). Second, this is the core of the paper. The same individual keeps $W = 2.0$ and $m = +0.693147$, yet w alone falls from 1.3333 to 0.8000 (Section 2). Third, the sign changes too. Favoured or not is decided at $w = 1$, so the same individual goes from favoured to disfavoured (Section 2). Fourth, across generations only the logarithm adds. The arithmetic mean of W is 1.25, yet the true factor after four generations is **1.000000** (Section 3). Fifth, inclusive fitness counts a different set. Under Hamilton’s rule, two brothers and eight cousins both come to 1.0 (Section 4). Sixth, the separator is what one divided by (Section 5).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed——absolute and relative fitness, the Malthusian parameter, inclusive fitness, and Hamilton’s rule are all standard.We do not build evolutionary biology——all we use is one division and one logarithm.We do not measure fitness——how W is estimated in real organisms is not treated at all. Only the relations among the definitions are.We do not discuss the theory of selection——we do not enter the Price equation or the dynamics of population genetics.The numbers are invented examples——the $W = 2.0, 1.0, 3.0$ of Section 2 are numbers placed to display the structure, not measurements.We assert nothing about applying Hamilton’s rule——both the definition of relatedness r and the measurement of inclusive fitness are debated. This paper uses only the form of the inequality $rB > C$.We do not apply it to humans——it is not used to explain behaviour. It is a roll call of definitions.Relation to earlier papers: Paper 292 showed that only the logarithm adds——the Malthusian parameter here is that logarithm, the same structure standing in another field.Paper 240 counted seven things called “mass,” one changing with the direction of the push——“fitness” here is four things, one changing with its neighbours.Paper 112 counted “six distinct roots sharing one rhyme”——this too is several under one word.Paper 194 showed the four means are one family——Section 3 here is a case where which member of that family is used changes the answer.What is added is separating the four by whether they depend on the population, computing that w falls from 1.3333 to 0.8000 and changes sign for one unchanged individual, showing the arithmetic mean 1.25 against the true factor 1.000000, and confirming that two brothers and eight cousins both give 1.0.

1. Four “Fitnesses”

Check 1.1:

Name	Definition	Unit	Depends on the population?
Absolute fitness W	offspring per individual	individuals	no
Relative fitness w	$w = W/\bar{W}$	dimensionless	yes
Malthusian parameter m	$m = \ln W$	1/time	no
Inclusive fitness	self + kin $\times$ relatedness	individuals	counts a different set

Observation 1.2

In the rightmost column only the second row says “yes.”And the fourth row asks a different question altogether——one sentence, “its fitness is high,” covering all four.

Question: How large is the difference among the four, in fact?

2. The Core: Swap the Neighbours and the Same Individual Turns Disfavoured

Setting: Let an individual, “A,” have absolute fitness $W = 2.0$. We change only its neighbours.

Check 2.1:

Population	Neighbour’s W	$\bar{W}$	A’s $w = W/\bar{W}$	A’s $m = \ln W$
I	1.0	1.5000	1.3333	+0.693147
II	3.0	2.5000	0.8000	+0.693147

Observation 2.2

This is the core of the paper. A’s W stays 2.0 and its m stays +0.693147—yet w alone falls to **0.6000** of its former value.

Observation 2.3

A has changed in nothing. The same number of offspring, the same lifespan—only the neighbours changed.

Check 2.4: We look at the sign. Relative fitness decides favoured or not at $w = 1$.

Population	A’s w	Verdict
I	1.3333	favoured ($w > 1$)
II	0.8000	disfavoured ($w < 1$)

Observation 2.5

The same individual went from favoured to disfavoured. The sentence “A has high fitness” has no truth value until the population is named—settled if said of absolute fitness, unsettled if said of relative.

Observation 2.6

This is not a defect. What natural selection actually sees is the relative one——leaving more than the neighbours is what moves frequencies. Which is precisely why the two senses must be kept apart.

3. Across Generations, Only the Logarithm Adds

Setting: A lineage passes through $W = 2.0, 0.5, 2.0, 0.5$ over four generations.

Check 3.1:

Method	Result
Arithmetic mean of W	1.2500 (looks like growth)
True factor (product)	1.000000
Sum of $m = \ln W$	0.000000
Exponentiated	1.000000 (agrees)

Observation 3.2

The arithmetic mean of W is $1.25 > 1$, so it looks like growth. In fact it is **1.000000**, that is no growth——exactly the error Paper 292 showed for rates of return.

Observation 3.3

The Malthusian parameter $m = \ln W$ does add. The sum is 0.000000, exponentiating to 1.000000——generations compose by multiplication, so one can add only after moving to logarithms.

Observation 3.4

This also explains why m carries a unit of inverse time. W is a factor (dimensionless); $m = \ln W$ is a growth rate per unit time——what adds is a rate.

Observation 3.5

Paper 194 showed that the four means are one family. Here the choice within that family gives 1.25 or 1.000—the geometric mean is right, and the arithmetic mean is answering a different question.

4. Inclusive Fitness Counts a Different Set

Setting: Hamilton's rule says that helping evolves when $rB > C$, with relatedness r , benefit B , cost C .

Check 4.1: With $B = 3.0$ and $C = 1.0$.

Kin	Relatedness r	rB	$rB > C?$
Brother	0.500	1.500	yes
Nephew or niece	0.250	0.750	no
Cousin	0.125	0.375	no
Stranger	0.000	0.000	no

Check 4.2: We check Haldane's remark—"I would lay down my life for two brothers or eight cousins."

Number $\times$ relatedness		Total
Brothers	2×0.5	1.0
Cousins	8×0.125	1.0

Observation 4.3

Both come to exactly 1.0. Equal to one of oneself—Haldane's remark was not a figure of speech but a calculation.

Observation 4.4

Inclusive fitness compares against a different object than W , w and m .The three count only one’s own offspring; inclusive fitness counts kin’s offspring too, weighted by relatedness—the unit is the same (individuals), the set counted is not.

Observation 4.5

A point to write honestly—both the definition of relatedness and the measurement of inclusive fitness are debated.This paper uses only the form of the inequality $rB > C$ —which definition is right is not treated.

5. Separator: What One Divided By

Observation 5.1

Separator—what divides the four?What one divided by, and what one counted.

	Divided by	Counted
Absolute fitness W	nothing	own offspring
Relative fitness w	the population mean $\bar{W}$	own offspring
Malthusian parameter m	nothing; took a logarithm	own offspring
Inclusive fitness	nothing	own + kin’s offspring

Observation 5.2

Only the one divided by the population mean depends on the population.Obvious as it looks, that it is obvious is the point—the dependence enters through the division, not through any property of the organism.

Observation 5.3

Paper 240 counted seven things called “mass,” one changing with the direction of the push. “Fitness” here is four, one changing with the neighbours——both are cases where the quantity alone does not determine its value.

Observation 5.4

And which is right depends on the question.

Question	Quantity to use
How many will this individual leave?	absolute fitness W
Will this trait’s frequency rise?	relative fitness w
By what factor after n generations?	Malthusian m (it adds)
Will helping evolve?	inclusive fitness

Observation 5.5

All four rows are correct. They answer different questions——say only “its fitness is high” and one cannot tell which of the four.

6. Roll Call and Closing

1. There are four (Check 1.1): Only one depends on the population.
2. A’s W stays 2.0 (Check 2.1): And m stays +0.693147.
3. w falls 1.3333 $\rightarrow$ 0.8000 (Check 2.1): A factor of 0.6000.
4. Favoured to disfavoured (Check 2.4): The same individual.
5. Arithmetic mean of W is 1.25 (Check 3.1): The truth is 1.000000.
6. Sum of m is 0.000000 (Check 3.1): Exponentiating agrees.
7. Brother gives $rB = 1.500$ (Check 4.1): Cousin gives 0.375.

8. Two brothers and eight cousins give 1.0 (Check 4.2): Haldane was calculating.
9. Dependence enters through division (Observation 5.2): Not through biology.
10. All four are correct (Observation 5.5): They answer different questions.

Closing: “Its fitness is high” points at one of four different quantities——absolute fitness W , relative fitness w , the Malthusian parameter m , and inclusive fitness. And only one depends on the population——an individual keeps $W = 2.0$ and $m = +0.693147$ unchanged, yet swapping its neighbours drops w from **1.3333** to **0.8000**. Since $w = 1$ is the boundary, the same individual goes from favoured to disfavoured——having changed in nothing. Across generations there is a second trap——the arithmetic mean of W is 1.25 and looks like growth, while the true factor after four generations is **1.000000**. Only the logarithm (the Malthusian parameter) adds, the same structure Paper 292 showed for rates of return. Inclusive fitness counts a different set again——under Hamilton’s rule, two brothers and eight cousins both come to exactly **1.0**. One thing separates them——what one divided by, and what one counted. Only the quantity divided by the population mean depends on the population. The dependence enters through the division, not through any property of the organism.

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