

The Gap between the Buses You Board Is Twice the Average

— There Are Two “Averages,” and the One a Passenger Lands In Is the Longer — The Ratio Is $1 + CV^2$: One for Even Spacing, Exactly Two for Poisson — [Paper 308]

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— *From thinking to syncing.* —

Abstract

Suppose buses come at an average interval of 10 minutes. How long, on average, is the gap before the bus you board? This paper asks whether “the average interval” is one quantity—the answer is there are two. No new mathematical theorem and no new law is claimed. First, the bus-side average is 10 minutes. For a Poisson process the intervals are exponential with mean 10 minutes (Section 2). Second, this is the core of the paper. The average gap you land in is 20 minutes, exactly twice (Section 2). Third, the wait is 10 minutes too. So is the time since the previous bus, and $10 + 10 = 20$ (Section 3). Fourth, a discrete example can be checked by hand. With gaps 5, 5, 5, 5, 20: bus side 8.0 minutes, passenger side 12.5 (Section 4). Fifth, the general rule is $1 + CV^2$. Even spacing gives 1; Poisson gives 2 (Section 5). Sixth, the separator is what was sampled (Section 6).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed—the Poisson process, the memorylessness of the exponential distribution, the inspection paradox, and length bias are all standard. We do not build stochastic processes—all we use is one weighted mean and one coefficient of variation. We do not build queueing theory—congestion, service times, and multiple routes are not treated at all. We do not discuss real buses—real services run close to evenly spaced, not Poisson. The factor 2 is the value “if the intervals were entirely random”. We do not hide the Poisson assumption—Sections 2 and 3 rest on arrivals being a Poisson process. Real variability often has $CV < 1$, and the ratio is correspondingly below 2 (Section 5). Passenger arrival is assumed uniform too—it does not apply to passengers who consult a timetable. It is the case of arriving at a random time. Called a paradox, but no contradiction—the two averages are different quantities and are bound to differ. “Paradox” names the surprise, not a contradiction. Relation to earlier papers: Paper 220 counted four things called “independence,” saying nothing to one another—this paper is a case where “average” is two things, split by how one samples. Paper 194 showed the four means are one family—that concerns four means over one sample; this concerns two ways of sampling one quantity, splitting in a different place. Paper 292 showed that only the logarithm adds—this too is a case where the naive average misses. Paper 255 separated two things called “accuracy,” only one removed by calibration—the length bias here comes from the sampling and does not vanish with better precision. What is added is computing the passenger-side average as 20 minutes, twice the bus side, checking 8.0 against 12.5 by hand on a discrete example, writing the general rule as $1 + CV^2$, and placing the separator at what was sampled.

1. Notation and the Question

Setting: Buses arrive as a Poisson process with rate $\lambda = 1/10$ per minute, that is a mean interval of 10 minutes.

You go to the stop at a random time, without consulting a timetable.

Question: How long, on average, is the gap between the bus you board and the one before it?

Observation 1.1

Naively one wants to answer 10 minutes. Because the mean interval is 10 minutes—but that is wrong.

2. The Core: The Gap You Land In Averages 20 Minutes

Observation 2.1

The reason is length bias. The longer the gap, the likelier you are to land in it—a 20-minute gap occupies four times the time of a 5-minute gap, so you are four times likelier to fall into it.

Check 2.2: If the interval distribution is $f(x)$, the distribution of the gap you land in is $xf(x)/E[x]$ (weighted by length). For the exponential $f(x) = \lambda e^{-\lambda x}$ this is a gamma distribution of shape 2.

Quantity	Value
Mean interval between buses, $1/\lambda$	10.0000 min
Mean gap you land in, $2/\lambda$	20.0000 min
Ratio	2.0000

Observation 2.3

This is the core of the paper. The mean interval between buses is 10 minutes; the gap around the bus you board is 20—both correct, and differing by exactly 2.

Observation 2.4

No contradiction. The sampling differs—the bus side counts buses equally; the passenger side counts minutes equally. One sequence, viewed under two weights.

3. The Wait Is 10 Minutes Too, and $10 + 10 = 20$

Check 3.1:

Quantity	Value
Mean wait	10.0000 min
Mean time since the previous bus	10.0000 min
Total (the gap landed in)	20.0000 min

Observation 3.2

The wait is 10 minutes because of memorylessness. The exponential does not remember how long one has waited, so whenever one arrives, 10 minutes remain——after waiting five minutes, one still waits ten on average.

Observation 3.3

Note the symmetry front and back. Ten minutes before, ten after——hence one has landed in a gap of twenty.

Observation 3.4

Here is the source of the discomfort. “An average interval of 10 minutes” suggests “an average wait of 5,” but one waits 10——the wait too is doubled.

Observation 3.5

A point to write honestly——these values assume Poisson. Real services are run close to evenly spaced——and the ratio is correspondingly below 2 (Section 5).

4. A Discrete Example Can Be Checked by Hand

Setting: Take a day with gaps 5, 5, 5, 5, 20 minutes (forty minutes in all).

Check 4.1:

	Computation	Value
Bus-side average	$40/5$	8.0000 min
Passenger-side average	$\sum g^2 / \sum g = 500/40$	12.5000 min
Ratio		1.5625

Observation 4.2

The 20-minute gap occupies **50%** of the forty minutes. Yet by count it is one bus in five, only **20%**—the difference between counting by time and counting by bus.

Observation 4.3

So half the passengers land in the 20-minute gap. From the bus side, “most services run five minutes apart”—from the passenger side, “half the time it is the twenty-minute one”.

Observation 4.4

This example can be verified entirely by hand. $500 = 5^2 \times 4 + 20^2 = 100 + 400$ and $500/40 = 12.5$ —the same phenomenon appears without any probability theory.

5. The General Rule Is $1 + CV^2$

Setting: With mean μ , standard deviation σ , and coefficient of variation $CV = \sigma/\mu$, the ratio is $1 + CV^2$.

Check 5.1:

Service	CV	Ratio = $1 + CV^2$
Even spacing (10, 10, 10, 10, 10)	0.0000	1.0000
The example of Section 4 (5, 5, 5, 5, 20)	0.7500	1.5625
Poisson (exponential)	1.0000	2.0000

Observation 5.2

With even spacing there is no bias (ratio 1). If all gaps are equal it does not matter where one lands—the bias is born of the variability.

Observation 5.3

And Poisson gives exactly 2. Because the exponential has CV exactly 1— $1 + 1^2 = 2$.

Observation 5.4

If CV exceeds 1 the ratio exceeds 2. When buses bunch ($CV > 1$), waits get worse still—real bunching arises because a late bus picks up more passengers, which is a separate mechanism.

Observation 5.5

Paper 255 separated two things called “accuracy,” noting that calibration removes only one. The length bias here comes from the sampling—a better clock and a larger sample do not remove it.

6. Separator: What Was Sampled

Observation 6.1

Separator—what must be moved for 10 and 20 to exchange? What is counted equally.

	What is counted equally	Answer
Bus-side average	one bus at a time	10 min
Passenger-side average	one minute at a time	20 min

Observation 6.2

Both rows are correct averages. Only the sample differs—the question “what is the average interval” covers two questions.

Observation 6.3

The same structure appears elsewhere.

Setting	Object-side average	Sample-side average
Buses	one bus at a time	weighted by time
Class size	one class at a time	weighted by pupils (more sit in big classes)
Number of friends	one person at a time	weighted by edges (the popular are picked more)

Observation 6.4

All three are the same length bias. “Your friends have, on average, more friends than you” comes from here—one mechanism covering three phenomena.

Observation 6.5

Paper 220 counted four things called “independence,” saying nothing to one another. The two averages here likewise say nothing to one another—knowing one does not give the other without knowing *CV*.

Observation 6.6

A point to write honestly—this is not a contradiction. “Paradox” names the surprise, not a contradiction—the two averages are different quantities and are bound to differ.

7. Roll Call and Closing

1. **Bus-side average 10.0000 min** (Check 2.2): **A Poisson process.**
2. **Passenger-side average 20.0000 min** (Check 2.2): **Exactly twice.**
3. **The wait is 10.0000 min too** (Check 3.1): **By memorylessness.**
4. **Symmetric front and back** (Observation 3.3): **$10 + 10 = 20$.**
5. **Discrete: bus side 8.0000 min** (Check 4.1): **Passenger side 12.5000.**
6. **The long gap is 50% of the time** (Observation 4.2): **Only 20% of buses.**
7. **Even spacing gives ratio 1.0000** (Check 5.1): **Bias comes from variability.**
8. **Poisson has $CV = 1$** (Observation 5.3): **$1 + 1^2 = 2$.**
9. **Class size and friend counts too** (Observation 6.3): **One mechanism.**
10. **Not a contradiction** (Observation 6.6): **Different quantities.**

Closing: When buses come at an average interval of 10 minutes, the gap around the bus you board averages **20**. Exactly twice, and both are correct—the bus side counts buses equally, the passenger side counts minutes equally. The wait follows the same reasoning and is 10 minutes—by memorylessness, ten before and ten after, so one has landed in a gap of twenty. A discrete example can be checked by hand—with gaps 5, 5, 5, 5, 20, the bus side is **8.0** minutes and the passenger side **12.5**. The 20-minute gap is only 20% of the buses but **50%** of the time. The general rule is **$1 + CV^2$** —even spacing gives **1** (no bias), and Poisson, with $CV = 1$, gives exactly **2**. The bias is born of the variability. One thing separates them—what was counted equally. The same mechanism acts on class sizes and on counts of friends. “Your friends have, on average, more friends than you” comes from here.

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