

The Length That Divides Gravity from Surface Tension Is Nobody's Size

— Water Gives **2.7273** mm — Mercury Has **6.6621** Times the Surface Tension and Only **0.7009** Times the
Capillary Length — [Paper 309]

Yuuki YAMAGISHI (山岸 勇輝)

Yamagishi Complexity Torus Lab, Japan

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— *From thinking to syncing.* —

Abstract

A liquid carries one length, $\ell_c = \sqrt{\gamma/\rho g}$. This paper asks whose size that length is—the answer is nobody's. No new mathematical theorem and no new law is claimed. First, water gives **2.7273** mm. Mercury 1.9116 mm, ethanol 1.6977 mm, liquid helium 0.2625 mm (Section 2). Second, this is the core of the paper. Mercury has **6.6621** times the surface tension of water and a capillary length only **0.7009** times as long—its density is 13.5611 times greater, and only the square root of the ratio survives (Section 2). Third, the length is a boundary. The Bond number $\text{Bo} = (L/\ell_c)^2$ is exactly 1 there (Section 3). Fourth, capillarity is not what raises sap. In a $20\mu\text{m}$ vessel it lifts water only **0.74** m (Section 4). Fifth, in an alveolus it is γ that must move. Bare water gives 1456 Pa, surfactant 500 Pa (Section 5). Sixth, the separator is whether γ enters the formula. The break-up threshold $2\pi R$ does not contain it (Section 6).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed—the Laplace pressure, Jurin’s law, the Bond number and the Rayleigh–Plateau instability are all standard. We do not build fluid mechanics—all we use is one length and one square root. We do not treat the contact angle—we compute with $\cos \theta = 1$ (complete wetting). Real contact angles, hysteresis and roughness are not treated. We do not treat dynamics—we say whether a column breaks, never how fast. We do not build plant physiology—whether the cohesion–tension theory is right is out of scope. We give only the number showing capillarity does not suffice. We do not build lung physiology—neither surfactant composition nor real alveolar shape is treated. Only the Laplace estimate. We claim no accuracy for representative values— $\gamma = 0.0728 \text{ N/m}$ and $\rho = 998 \text{ kg/m}^3$ are values for showing orders and move with temperature. Relation to earlier papers: Paper 258 showed that 4π appears only when the source is a point, using capillaries as its example of a cylindrical source—this paper stays in the same capillary setting but asks about a length rather than a solid angle. 258 treats potential, this paper treats an interface, and they share not one quantity. Paper 291 showed one sea holding three lengths—the ℓ_c here is not a fourth length but a length that is the boundary between two forces. Paper 306 showed that the number of base units is a promise— ℓ_c is not a promise: it comes out of two material quantities. Paper 140 showed that symmetry fixes ratios and dynamics fixes the scale— ℓ_c is on the scale side. What is added is computing ℓ_c for five liquids, showing that surface tension and capillary length reverse their order for mercury, checking that the Bond number equals $(L/\ell_c)^2$, counting that capillarity reaches only 0.74 m against a tree’s height, and pointing out that the break-up threshold $2\pi R$ does not contain γ .

1. Notation and the Question

Setting: Build the length at which gravity balances surface tension.

$$\ell_c = \sqrt{\frac{\gamma}{\rho g}}$$

with γ the surface tension, ρ the density, $g = 9.80665 \text{ m/s}^2$.

Observation 1.1

Counting dimensions, $[\gamma] = \text{N/m} = \text{kg/s}^2$ and $[\rho g] = \text{kg}/(\text{m}^2\text{s}^2)$. The ratio is m^2 , and a square root makes it a length—there is only one way to combine them.

Question: Whose size is this length?

2. Five Liquids

Check 2.1:

Liquid	γ [N/m]	ρ [kg/m ³]	ℓ_c [mm]
Water (20 °C)	0.0728	998	2.7273
Mercury	0.4850	13534	1.9116
Glycerol	0.0634	1261	2.2643
Ethanol	0.0223	789	1.6977
Liquid helium 4	0.000098	145	0.2625

Observation 2.2

This is the core of the paper. Mercury's surface tension is **6.6621** times that of water—and yet its capillary length is **0.7009** times as long, i.e. shorter.

Observation 2.3

The reason is the density. Mercury is 13.5611 times denser than water, a larger rise than the surface tension's—and since ℓ_c takes only the square root of the ratio, $\sqrt{6.6621/13.5611} = 0.7009$.

Observation 2.4

The five values sit between 0.2625 and 2.7273 mm. γ spans a factor of 4950, from 0.000098 to 0.4850, while ℓ_c spans only 10.4—the square root acts twice and flattens the spread.

Observation 2.5

Paper 292 wrote that only the logarithm adds. What happens here is the other side of that—on a logarithmic scale, 3.69 decades of γ shrink to 1.02 decades of ℓ_c .

3. The Length Is a Boundary

Setting: The Bond number is $\text{Bo} = \rho g L^2 / \gamma$.

Check 3.1: Computed for water.

L [mm]	Bo	Which wins
0.1000	0.001344	surface tension
1.0000	0.134437	surface tension
2.7273	0.999967	balance
10.0000	13.443732	gravity
100.0000	1344.373173	gravity

Observation 3.2

$Bo = (L/\ell_c)^2$. The third row is 1 not by accident but because $L = \ell_c$ was substituted—the Bond number and the capillary length are two ways of writing one thing.

Observation 3.3

So ℓ_c is nobody’s size. It is fixed by the liquid alone and never looks at the object placed in it—what it looks at is whom that object’s size is compared with.

Observation 3.4

This is the scale side in the sense of Paper 140. Symmetry fixes the ratios and dynamics fixes the scale— ℓ_c is a scale built from two dynamical quantities, γ and ρg , and never from geometry.

4. Capillarity Is Not What Raises Sap

Setting: Jurin’s law: in a tube of radius r , $h = 2\gamma \cos \theta / (\rho g r)$. Take $\cos \theta = 1$.

Check 4.1:

r [μm]	h [m]
10	1.4877
20	0.7438
100	0.1488
1000	0.0149

Observation 4.2

Real xylem vessels have radii of 5–50 μm . There capillarity lifts **0.74 m**—tall trees reach 100 m.

Check 4.3: Inverting: $h = 100$ m demands $r = \mathbf{0.1488\mu\text{m}}$.

Observation 4.4

That is **134** times narrower than a real vessel. No such vessel exists—capillarity does not account for it.

Observation 4.5

A point to write honestly—this paper claims nothing about what does raise it. The cohesion--tension theory (transpiration at the leaves pulls the column) is the standard answer, but whether it is right is out of scope—all that is said is the one number: capillarity is not enough.

Observation 4.6

h grows as $1/r$, while ℓ_c contains no r at all. The same surface tension yields both a quantity that sees r and one that does not—the same shape as Paper 258's finding that the source's form fixes the solid angle.

5. In an Alveolus, What Must Move Is γ

Setting: The Laplace pressure of a sphere is $\Delta P = 2\gamma/R$. Take an alveolar radius $R = 100\mu\text{m}$.

Check 5.1:

State	γ [N/m]	ΔP [Pa]
Bare water	0.0728	1456.00
With surfactant	0.0250	500.00
At its lowest	0.0010	20.00

Check 5.2: Holding γ at bare water and moving only R .

R [μm]	ΔP [Pa]
50	2912.00
100	1456.00
200	728.00

Observation 5.3

At constant γ the smaller one is at higher pressure. If they are connected, the small alveolus empties into the large one and collapses——constant γ is unstable.

Observation 5.4

Surfactant lowers γ as the alveolus shrinks. As R falls so does γ , so $2\gamma/R$ does not run away——what removes the instability is not the value but the fact that γ is not a constant.

Observation 5.5

ℓ_c does not appear here. An alveolus at $100\mu\text{m}$ is 27 times smaller than $\ell_c = 2.7273 \text{ mm}$ ——it lies on the side where gravity does not act, so ρg drops out.

6. Separator: Whether γ Enters

Setting: Rayleigh–Plateau: a liquid column breaks when the wavelength exceeds $2\pi R$.

Check 6.1:

$R\text{ }[\mu\text{m}]$	Break-up wavelength $2\pi R\text{ }[\mu\text{m}]$
1000	6283.1853
100	628.3185
10	62.8319

Observation 6.2

Separator——whether γ enters the formula. The threshold $2\pi R$ does not contain γ ——surface tension does not decide whether it breaks, only how fast.

Check 6.3: Three quantities side by side.

Quantity	What enters	What it decides
$\ell_c = \sqrt{\gamma/\rho g}$	γ and ρg	which force wins
$h = 2\gamma/(\rho g r)$	$\gamma, \rho g, r$	how high it climbs
$\lambda > 2\pi R$	R alone	whether it breaks

Observation 6.4

Only the third row contains no material quantity. It is pure geometry——the same place as Paper 258’s solid angle, fixed by the shape of the source alone.

Observation 6.5

And ℓ_c is neither. It comes from material quantities, yet says nothing without something to compare——the number 2.7273 mm means something only once a 2.7273 mm object exists.

7. Roll Call and Closing

1. Water gives **2.7273 mm** (Check 2.1): $\ell_c = \sqrt{\gamma/\rho g}$.
2. Mercury has **6.6621** times the γ (Observation 2.2): and **0.7009** times the ℓ_c .
3. Density is **13.5611** times (Observation 2.3): **only the square root acts**.
4. **4950** in γ becomes **10.4** (Observation 2.4): **the spread flattens**.
5. $\text{Bo} = (L/\ell_c)^2$ (Check 3.1): **exactly 1 at $L = \ell_c$** .
6. **Nobody's size** (Observation 3.3): **it is the thing compared with**.
7. Capillarity gives **0.74 m** (Check 4.1): **trees reach 100 m**.
8. It would need **0.1488 μm** (Check 4.3): **134 times narrower**.
9. An alveolus at **500 Pa** (Check 5.1): **bare water gives 1456 Pa**.
10. **$2\pi R$ has no γ** (Check 6.1): **whether it breaks is geometry**.

Closing: A liquid carries one length, $\ell_c = \sqrt{\gamma/\rho g}$, and for water it is **2.7273 mm**—mercury 1.9116 mm, glycerol 2.2643 mm, ethanol 1.6977 mm, liquid helium 0.2625 mm. Mercury has **6.6621** times the surface tension of water and a capillary length only **0.7009** times as long—because its density is 13.5611 times greater, and ℓ_c takes only the square root of the ratio. A factor of 4950 in γ becomes a factor of 10.4 in ℓ_c . The length is a boundary—the Bond number $\text{Bo} = (L/\ell_c)^2$ is exactly 1 at $L = \ell_c$; below it surface tension wins, above it gravity does. So ℓ_c is nobody's size—it is whom an object's size is compared with. Capillarity is not what raises sap—in a 20 μm vessel it lifts **0.74 m**, and 100 m would need a 0.1488 μm tube, 134 times narrower than any real one. In an alveolus what acts is not R but γ —bare water gives 1456 Pa, surfactant **500 Pa**. What removes the instability is not the value but the fact that γ is not a constant. One thing separates them—whether γ enters the formula. The break-up threshold $2\pi R$ does not contain it and is pure geometry. Surface tension does not decide whether a column breaks, only how fast.

Revision Record

Second edition (2026-08-30): The subject of this paper has been replaced. The first edition, titled “Living Tissue Where 4π Appears, and Where It Does Not,” treated the solid angles of point, line and sheet sources and the diffusive reach around a capillary. That content duplicated Paper 258, “ 4π Appears Only When the Source Is a Point”—even the numbers (24.1144 μV , 111.0508 μV , 316.78 μm , 44.80 μm) agreed, and Paper 258 has priority. The second edition stays in the same capillary setting but moves to a quantity 258 did not treat: the interfacial length ℓ_c . All statements about solid angle have been removed from this paper; Paper 258 is the reference for them.

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Yuuki YAMAGISHI (山岸 勇輝)

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