

The π That Carries No Dimension

— Ramanujan's Series, the AGM and BBP: the π of a Solid Angle and the π of a Period — [Paper 31]

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— *From thinking to syncing.* —

ABSTRACT

The corpus has followed 4π for a long time. This paper turns the same test on π itself: when the dimension d is moved, does that π move? **NO NEW MATHEMATICAL THEOREM IS CLAIMED.** First, this is one of the rare papers that confirms a shared root. The π of a solid angle and the π of a Gaussian have the same root: $\text{Vol}(S^{d-1}) = 2\pi^{d/2}/\Gamma(d/2)$, and the Gaussian integral over $\mathbb{R}^d$ is $\pi^{d/2}$. Integrating the ball volumes from a recursion that assumes no $\pi^{d/2}$ anywhere reproduces the closed form for $d = 2$ through 8 (differences of order 10^{-120}) (Section 2). Second, and this is the core. There is no d in the π of Ramanujan's series. No dimension appears anywhere in $1/\pi = (2\sqrt{2}/9801) \sum_k (4k)! (1103 + 26390k)/((k!)^4 396^{4k})$, so there is nothing to move. By the corpus's own test (dimension-independence, Paper 129) this π is not a solid angle — it is a period of a modular form (Section 3). Third, a census of digits. The Leibniz series gives 6 digits in 10^6 terms; Ramanujan gives about 8 digits per term and Chudnovsky about 14 (computed here). All three are linear, and only the constant differs (Section 4). Fourth, “fast” has three roots too. Gauss–Legendre (the AGM) reaches 3, 8, 19, 40, 84, 171, 345 digits, doubling each step — not a larger constant but a different growth law. BBP is fast in neither sense, yet it gives the n -th hexadecimal digit without the preceding $n - 1$ (positions 1, 2, 10, 50, 100, 200 all verified here) (Section 5). Fifth, the brake. $e^{\pi\sqrt{163}}$ is neither a coincidence nor an integer. It misses $640320^3 + 744$ by 7.49927×10^{-13} (computed here) (Section 6).

Scope of this paper: **NO NEW MATHEMATICAL THEOREM IS CLAIMED.** Ramanujan's series, Chudnovsky's series, the Gauss–Legendre algorithm, the BBP formula, and the imaginary quadratic fields of class number one with their j -invariants are all standard. What is added is turning the corpus's dimension test on π itself, confirming that the solid-angle π and the Gaussian π share a root, writing addresses for three senses of “fast”, and fencing both of the circulating claims about $e^{\pi\sqrt{163}}$.

1. Notation and position

Write $\text{Vol}(S^{d-1})$ for the area of the unit sphere in $\mathbb{R}^d$ and V_d for the volume of the unit ball. By “digits” is meant minus the common logarithm of the relative error against π .

Position: Paper 129 used dimension-independence to judge that a given 4π was not a solid angle, and Paper 130 conversely showed that the 4π of Biot–Savart moves with the dimension as $(d - 2)\text{Vol}(S^{d-1})$. This paper turns that test on π itself.

2. The solid-angle π and the Gaussian π share a root

Verification 2.1: Integrate the ball volumes from the recursion $V_d = \int_{-1}^1 V_{d-1}(1 - x^2)^{(d-1)/2} dx$, $V_1 = 2$, which assumes no $\pi^{d/2}$, and compare $\text{Vol}(S^{d-1}) = dV_d$ with $2\pi^{d/2}/\Gamma(d/2)$.

d	dV_d (integrated)	$2\pi^{d/2}/\Gamma(d/2)$	difference
2	6.2831853071796	6.2831853071796	0.0
3	12.566370614359	12.566370614359	0.0
4	19.739208802179	19.739208802179	0.0
5	26.318945069572	26.318945069572	-3.098×10^{-120}
6	31.0062766803	31.0062766803	0.0
7	33.07336179232	33.07336179232	-6.196×10^{-120}
8	32.469697011334	32.469697011334	0.0

Further, $\left(\int_{-\infty}^{\infty} e^{-t^2} dt\right)^d = \pi^{d/2}$ was checked at $d = 2, 3, 6$ (differences 0 or of order 10^{-120}). (QED)

Observation 2.2

The π of a solid angle and the π of a Gaussian have the same root. Both carry the exponent $d/2$ and move together when d moves — indeed the Gaussian integral is the very instrument by which one computes the area of a sphere.

Observation 2.3

It is rare for the corpus to confirm a shared root (this is the second time, after Paper 142). Confirming sameness is as much a part of a census as counting differences.

3. The core: there is no d in Ramanujan's π

Fact 3.1 (Ramanujan 1914):

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801} \sum_{k \geq 0} \frac{(4k)! (1103 + 26390k)}{(k!)^4 396^{4k}}.$$

Observation 3.2

Nowhere in this formula does a dimension d appear. Move d , and nothing whatever moves.

Observation 3.3

The corpus's test (Paper 129) reads dimension-independence as a verdict that the π in question is not a solid angle. Here the π appears as a period of a modular form, with no sphere involved. The same symbol is on the page for two different reasons.

Observation 3.4

This does not mean there are two π s. As a number it is one. What differs is the reason it has to be there — since Paper 95 the corpus has called that difference an address.

4. A census of digits

Verification 4.1

terms	Ramanujan (digits)	Chudnovsky (digits)
1	7	13
2	15	28
3	23	42
4	31	56
5	39	70
6	47	84

Leibniz terms	correct digits
10	1
10^3	3
10^5	5
10^6	6

(QED)

Observation 4.2

Leibniz needs about 10^k terms for k digits. Ramanujan gives about 8 digits per term and Chudnovsky about 14. Yet all three are of one type — a fixed number of digits per term — and only the constant differs.

5. “Fast” has three roots

Verification 5.1 (Gauss–Legendre, the AGM): the correct digits after each iteration were

$$3, \quad 8, \quad 19, \quad 40, \quad 84, \quad 171, \quad 345$$

(computed here at **500**-digit precision). (QED)

Observation 5.2

The digits double. This is not a bigger constant but a different law of growth — linear and quadratic are distinct roots.

Verification 5.3 (BBP): the n -th hexadecimal digit of π , computed without the preceding $n - 1$.

position	BBP	reference	match
1	2	2	yes
2	4	4	yes
10	5	5	yes
50	8	8	yes
100	c	c	yes
200	5	5	yes

(QED)

Observation 5.4

BBP is fast in neither earlier sense. It gives few digits per term and doubles nothing. What it gives is random access — and among the three “good algorithms” there is no ordering. Which is best depends on what one wants.

6. The brake: $e^{\pi\sqrt{163}}$ is neither a coincidence nor an integer

Verification 6.1

Observation 7.2

sense of “fast”	growth	example
many digits per term	linear	Ramanujan (8), Chudnovsky (14)
the digits double	quadratic	Gauss–Legendre (AGM)
the n -th alone	—	BBP

8. Conclusion

1. The solid-angle π and the Gaussian π share a root (Ver. 2.1): both carry the exponent $d/2$.
2. There is no d in Ramanujan's π (Obs. 3.2): so it is not a solid angle.
3. Digits grow linearly (Ver. 4.1): 8 and 14 differ only as constants.
4. The AGM doubles the digits (Ver. 5.1): 3, 8, 19, 40, 84, 171, 345.
5. What BBP gives is random access (Ver. 5.3): the three senses of fast are unordered.
6. $e^{\pi\sqrt{163}}$ is neither a coincidence nor an integer (Ver. 6.1): it misses by 7.49927×10^{-13} .

Closing: The habit of seeing a circle behind every π is a strong one. But move d , and some π s move while others do not. The π on Ramanujan's page did not come from a circle.

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