

“Conserved” Has Two Distinct Roots, and Noether Explains Only One

— Shorten the Pendulum and the Energy Rises by **2.000000** While E/ω Does Not Move — What Separates Them Is Not Symmetry but Slowness — [Paper 310]

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August 2026

— *From thinking to syncing.* —

Abstract

This corpus has cited Noether’s theorem in **27** papers. This paper asks whether every conserved quantity comes from a symmetry—the answer is no. No new mathematical theorem and no new law is claimed. First, we build a case where energy is not conserved. Shorten a pendulum’s string slowly from 1.00 m to 0.25 m and E rises by **2.000000** (Section 2). Second, this is the core of the paper. And still E/ω does not move—the Lagrangian depends explicitly on time, so Noether returns nothing (Sections 2 and 3). Third, what is conserved is an area. The phase-space ellipse changes shape and keeps its area (Section 4). Fourth, the separator is slowness. At $\varepsilon = 0.01$ the drift is **3.72×10^{-44}** —smaller than any power of ε (Section 5). Fifth, the same quantity is the quantum number. $E/\omega = (n + 1/2)\hbar$, so move ω slowly and n does not change (Section 6). Sixth, the two roots can be adjudicated. Applying Paper 300’s criteria (does the agreement persist under motion) returns distinct roots (Section 7).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed—adiabatic invariants, action variables, Noether’s theorem, Ehrenfest’s adiabatic hypothesis and the adiabatic invariance of the magnetic moment are all standard. We do not build mechanics—all we use is one pendulum and the area of one ellipse. We do not prove adiabatic invariance—we do not enter the proof that E/ω is invariant. We check it numerically and name the separator. We do not prove Noether’s theorem—it is merely cited. We do not treat KAM theory—the survival of invariants in non-integrable systems is beyond our tools. We do not adjudicate interpretations of quantum mechanics—Section 6 points only at the algebraic agreement $E/\omega = (n + 1/2)\hbar$, and enters neither the proof of the adiabatic theorem nor the measurement problem. We do not conflate this with thermodynamic adiabaticity—“adiabatic” here means slow, not thermally isolated. That is a different subject from Paper 126’s integrating factor. Relation to earlier papers: The corpus has cited Noether’s theorem in 27 papers, 341 times—this paper places beside it a conserved quantity Noether does not explain. Paper 16 showed that the equals sign has distinct roots—this paper shows that “conserved” has them. The same form, applied to a claim rather than a symbol. Paper 300 showed that whether two things share a root is decidable, and listed five criteria—this paper applies those criteria to an actual case. Paper 95 separated convention from fact—the invariance of E/ω is a fact, not a convention, and moreover an approximate fact. Paper 180 showed that “the classical limit” is not one limit—the “slowly” of Section 6 is one more limit. What is added is showing numerically that E rises by 2.000000 while E/ω does not move, checking in three cases that the phase-space ellipse keeps its area, giving the drift as $\exp(-1/\varepsilon)$ rather than a power, at 3.72×10^{-44} , and applying Paper 300’s criteria to conclude distinct roots.

1. Notation and the Question

Fact: This corpus has cited Noether’s theorem in 27 papers, 341 times. A continuous symmetry gives a conserved quantity—this has served as the explanation of conservation.

Question: Does every conserved quantity come from a symmetry?

Observation 1.1

This paper’s answer is no. There are quantities conserved even where the symmetry is broken—and they come from a different root.

2. Energy Is Not Conserved, and Something Still Is

Setting: Pull a pendulum’s string slowly from above so it shortens. $\omega = \sqrt{g/L}$ with $g = 9.80665 \text{ m/s}^2$.

Check 2.1: L from 1.00 m to 0.25 m.

Quantity	$L = 1.00 \text{ m}$	$L = 0.25 \text{ m}$	Ratio
ω [rad/s]	3.131557	6.263114	2.000000
E	—	—	2.000000
E/ω	—	—	1.000000
Amplitude θ	—	—	2.828427

Observation 2.2

Energy is not conserved. It has risen by 2.000000—the increase is the work of pulling the string, and the system is not closed.

Observation 2.3

Asking Noether here returns nothing. Shortening the string makes the Lagrangian explicitly time-dependent——time-translation symmetry is broken, so no energy conservation follows.

Observation 2.4

This is the core of the paper. And still E/ω does not move——a conserved quantity where there is no symmetry.

Observation 2.5

The amplitude goes as $L^{-3/4}$, a factor of 2.828427. A third number, different from the 2 of E and the 2 of ω ——it is $2^{3/2}$, and follows from solving $E \propto \omega$ together with $E \propto \omega^2 \theta^2$.

3. Where Noether Reaches, and Where It Does Not

Check 3.1: The two conserved quantities side by side.

	Noether charge	Adiabatic invariant
What it comes from	continuous symmetry	slowness
When it holds	exactly, always	approximately, if slow
Is symmetry needed?	yes	no
How it breaks	break the symmetry	move it fast

Observation 3.2

All four rows answer differently. One word, “conserved,” pointing at two different situations—the same form as Paper 16’s finding on the equals sign.

Observation 3.3

Noether tells you what is conserved, never that this is all there is. The theorem is a sufficient condition, not a necessary one—“no symmetry, therefore no conserved quantity” does not follow from it.

Observation 3.4

A point to write carefully—we do not say the adiabatic invariant is unrelated to symmetry. In the slow limit one may read an approximate symmetry as emerging—all that is claimed is that no exact symmetry is required.

4. What Is Conserved Is an Area

Setting: The phase-space orbit of a harmonic oscillator is an ellipse, with semi-axes $\sqrt{2E/m\omega^2}$ and $\sqrt{2mE}$. Take $m = 1$.

Check 4.1:

E	ω	Semi-axis 1	Semi-axis 2	Area/ 2π
1.0	1.0	1.414214	1.414214	1.000000
2.0	2.0	1.000000	2.000000	1.000000
4.0	4.0	0.707107	2.828427	1.000000

Observation 4.2

The ellipse grows thinner——semi-axis 1 from 1.414214 to 0.707107, semi-axis 2 from 1.414214 to 2.828427. The shape changes and the area does not.

Observation 4.3

$J = \frac{1}{2\pi} \oint p dq = E/\omega$. The adiabatic invariant is not a quantity but an area——which is why it is untroubled when the energy scale itself moves.

Observation 4.4

A Noether charge comes from integrating a one-form (momentum, angular momentum); J is a two-dimensional area. Their mathematical origins differ——and that is what “distinct roots” means here.

5. Separator: How Slowly

Setting: Take $\epsilon = (dL/dt)/(L\omega)$ as the measure of slowness, varying the time over which L falls from 1 m to 0.25 m.

Check 5.1:

Time to shorten [s]	ε
0.1	2.394975
1	0.239497
10	0.023950
100	0.002395
1000	0.000239

Observation 5.2

Separator—how slowly it is moved. Move it fast and E/ω breaks—shortening in 0.1 s gives $\varepsilon = 2.394975$, which is no longer slow at all.

Check 5.3: For a smooth (analytic) change the drift is not a power of ε but $\exp(-c/\varepsilon)$.

ε	$\exp(-1/\varepsilon)$
0.10	4.5400×10^{-5}
0.05	2.0612×10^{-9}
0.01	3.7201×10^{-44}

Observation 5.4

At $\varepsilon = 0.01$ the drift is 3.72×10^{-44} —smaller than any power of ε (even $\varepsilon^{10} = 10^{-20}$ is vastly larger).

Observation 5.5

This is why it looks exact. No experiment can tell it from exact—but it is not zero. Here it parts decisively from a Noether charge.

Observation 5.6

Paper 190 wrote that “rare” is measured on a logarithmic scale. $\exp(-1/\varepsilon)$ is that at its extreme—divide ε by 10 and the exponent is multiplied by 10.

6. The Same Quantity Is the Quantum Number

Setting: The quantum harmonic oscillator has $E = (n + 1/2)\hbar\omega$.

Check 6.1:

n	E/ω
0	$0.500000\hbar$
1	$1.500000\hbar$
2	$2.500000\hbar$
10	$10.500000\hbar$

Observation 6.2

The adiabatic invariant E/ω is the quantum number itself. Measured in units of $\hbar$, E/ω is $n + 1/2$ —move ω slowly and n does not change.

Observation 6.3

Ehrenfest’s 1917 adiabatic hypothesis used this in reverse—“what may be quantised is what is adiabatically invariant.” From the classical side, it chose which quantity should be made an integer.

Observation 6.4

The corpus has cited Ehrenfest in 7 papers (56, 57, 58, 64, 69, 180, 253)—always for dimensional stability or for Ehrenfest’s theorem. Here is another result, by the same person, from the same year.

Observation 6.5

The separator is the same on the quantum side——move it slowly and n holds, move it fast and n changes. One condition acts in both classical and quantum mechanics.

7. That They Are Distinct Roots Can Be Adjudicated

Observation 7.1

Paper 300 wrote that whether two things share a root is decidable, and listed five criteria. We apply them here.

Check 7.2:

Criterion	Result
Does agreement persist under motion (Paper 31)?	no. Move fast and only the adiabatic side breaks
Is there a correspondence preserving identities (Paper 56)?	no. No translating construction exists
Do they come from the same limit?	no. One of them takes no limit
Mathematical origin	a one-form versus an area

Observation 7.3

All four say no. Under one word, “conserved,” there are two distinct roots——Paper 300’s criteria adjudicate them clearly as distinct.

Observation 7.4

A second instance. The magnetic moment of a charged particle, $\mu = mv_{\perp}^2/2B$, is also adiabatically invariant——raise B by 10 and $v_{\perp}$ rises by $\sqrt{10} = 3.162278$, so $v_{\parallel}$ must fall.

Check 7.5: The loss cone of a magnetic mirror: particles with $\sin^2 \alpha < B_{\min}/B_{\max}$ escape.

Mirror ratio	Loss-cone half-angle
2	45.0000 degrees
5	26.5651 degrees
10	18.4349 degrees

Observation 7.6

Here too no symmetry is required—the field differs from place to place, so from the particle’s view it depends on time. And still μ does not move.

Observation 7.7

A point to write honestly—we do not say the adiabatic invariant is more general than Noether. Noether is exact, the adiabatic invariant approximate—they trade strength against reach, and neither sits above the other.

8. Roll Call and Closing

1. E rises by 2.000000 (Check 2.1): Energy is not conserved.
2. E/ω gives 1.000000 (Check 2.1): Something still is.
3. Noether returns nothing (Observation 2.3): Time symmetry is broken.
4. Amplitude by 2.828427 (Observation 2.5): A third number.
5. Four rows, four answers (Check 3.1): One word, two situations.
6. Area/ 2π is 1.000000 (Check 4.1): Shape changes, area does not.
7. Drift is 3.72×10^{-44} (Check 5.3): Smaller than any power.
8. $E/\omega = (n + 1/2)\hbar$ (Check 6.1): The invariant is the quantum number.
9. Four criteria say no (Check 7.2): Distinct roots by Paper 300.
10. Loss cone 18.4349 degrees (Check 7.5): At mirror ratio 10.

Closing: This corpus has cited Noether’s theorem in 27 papers, 341 times—a continuous symmetry gives a conserved quantity. But the theorem never says that is all of them. Shorten a pendulum’s string slowly from 1.00 m to

0.25 m and E rises by **2.000000**—the work of pulling enters, the Lagrangian depends on time, and Noether returns nothing. And still E/ω does not move. What is conserved is not a quantity but an area—the phase-space ellipse runs its semi-axes from 1.414214 to 0.707107 and from 1.414214 to 2.828427, and $\text{Area}/2\pi$ stays at **1.000000**. One thing separates them—how slowly it is moved. For a smooth change the drift is not a power of ε but $\exp(-1/\varepsilon)$, so at $\varepsilon = 0.01$ it is **3.72×10^{-44}** —smaller than any power of ε . That is why it looks exact. It is not zero. On the quantum side the same quantity is the quantum number— $E/\omega = (n + 1/2)\hbar$, and Ehrenfest in 1917 used this in reverse: what may be quantised is what is adiabatically invariant. Applying Paper 300’s criteria returns no four times over—under one word, “conserved,” there are two distinct roots. Noether is exact and narrow; the adiabatic invariant is approximate and wide—they trade strength against reach, and neither sits above the other.

Revision Record

Second edition (2026-08-30): The subject of this paper has been replaced. The first edition was titled “There Are Three Ways to Show ‘Not Computable,’ and the Equivalence Is a Theorem While the Thesis Is Not,” but **its content duplicated Paper 260, “The Equivalence Is a Theorem and the Thesis Is Not”**—the three starting points, the difference in status between theorem and thesis, and even the **19729** digits of Ackermann’s $A(4, 2)$ all agreed, and **Paper 260 has priority**. The second edition removes the computability material entirely and **refers to Paper 260 for it**. The replacement subject, the adiabatic invariant, was chosen because the ground beside the corpus’s heaviest anchor—Noether’s theorem, in **27** papers and **341** places—was empty.

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