

This Corpus Has Used “Synchrony” in Four Senses, and Only the Fifth Has a Threshold

— Double the Coupling Past Threshold and the Order Parameter Reaches Only **0.707107** — $r = 0.99$ Costs **50.25** Times — [Paper 313]

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— *From thinking to syncing.* —

Abstract

This corpus has used the word “synchrony” in 20 papers. This paper asks whether they mean the same thing—the answer is they mean four different things. No new mathematical theorem and no new law is claimed. First, we call the four existing senses. A clock convention (Paper 24), an equality between units (Paper 16), a phase that closes on itself (Paper 57), and an explicitly declared metaphor (Paper 76) (Section 2). Second, the four are distinct roots by Paper 300’s criteria (Section 3). Third, this is the core of the paper. Only a fifth sense, entrainment, has a threshold—for a Lorentzian it is exactly $K_c = 2\gamma$ (Section 4). Fourth, it rises as a square root. Double the coupling past threshold and the order parameter reaches only **0.707107** (Section 4). Fifth, the last stretch is expensive. $r = 0.99$ costs **50.25** times, $r = 0.999$ costs **500.25** (Section 5). Sixth, the separator is whether a state of not being in step can exist (Section 6).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed—the Kuramoto model, its critical coupling $K_c = 2\gamma$, the order parameter $r = \sqrt{1 - K_c/K}$ and Huygens’ observation of two pendulum clocks are all standard. We do not build a theory of synchronisation—all we use is one threshold and one square root. We do not solve the Kuramoto model—we do not enter the derivation of the self-consistency equation. We quote the known answer under its premises: $N \rightarrow \infty$, a Lorentzian, all-to-all coupling. We do not treat finite N —neither finite-size fluctuations nor finite-time transients are treated. We do not generalise the coupling—synchronisation on lattices and complex networks is not treated. All-to-all only. We do not revisit earlier claims—we touch neither the history of Paper 16’s retraction of its “grand unification” nor Paper 76’s decision to mark “synchrony” as a metaphor. Each conclusion is quoted as it stands. Relation to earlier papers: Paper 24 showed the one-way speed of light cannot be measured, and treated the clock-synchrony convention ε —this paper counts it as the first synchrony. Paper 16 showed the equals signs of the “Yamagishi all-synchrony equation” to have distinct roots—the second. 16’s conclusion (three kinds of equals) is quoted, not reopened. Paper 57 saw in Kaluza–Klein single-valuedness the picture of “a phase coming round and matching again”—the third. Paper 76 stated explicitly that “Sync” was a lens, a metaphor and not a testable claim—the fourth, and this paper adopts that self-discipline unchanged. Paper 300 showed that whether two things share a root is decidable—this paper applies those criteria to five. What is added is separating the corpus’s “synchrony” into four, placing entrainment as a fifth and quoting its threshold $K_c = 2\gamma$, computing r at six points and confirming the exponent is $1/2$, giving the coupling needed for $r = 0.99$ and 0.999 as 50.25 and 500.25 , and putting the separator at whether a state of not being in step can exist.

1. Notation and the Question

Fact: This corpus has used “synchrony” in 20 papers, 78 times. The lab’s motto is From thinking to syncing.

Question: Do they mean the same thing?

Observation 1.1

This paper’s answer is no. They mean at least four different things, all covered by the one word “in step”.

2. Calling the Four Existing Senses

Check 2.1:

	What comes into step	Source	Status
First	the readings of distant clocks	Paper 24	convention (not fixed by measurement)
Second	an equality of four physical quantities	Paper 16	an equals sign (of three kinds)
Third	the phase of a wave gone round once	Paper 57	a requirement (single-valuedness)
Fourth	position matched with velocity	Paper 76	a metaphor (declared as a lens)

Observation 2.2

All four have different status. Convention, equals sign, requirement, metaphor——not one is the same kind of claim.

Observation 2.3

In the first (Paper 24) the way of matching is chosen. Whatever ε is taken, the round-trip speed is c and no observable moves——what measurement does not fix is a convention (Paper 95).

Observation 2.4

In the third (Paper 57) matching is required. The wave function must return to itself after one turn, and from that the momentum is quantised——the option of not matching does not exist.

Observation 2.5

The fourth (Paper 76) calls itself a metaphor. It states that “cosmic OS” and “Sync” are lenses for interpretation and not testable claims——this paper quotes that self-discipline unchanged.

3. The Four Are Distinct Roots

Check 3.1: Applying Paper 300’s criteria.

Criterion	Result	Why
Is there a correspondence preserving identities?	no	not one translating construction exists
Does agreement persist under motion?	no	the first moves with ϵ , the third cannot move
Do the same quantities appear?	no	time, units, phase, coordinates
Is it refutable?	it varies	a convention is not, a requirement is internal, a metaphor is not a claim

Observation 3.2

All four say no or vary—the four synchronies are distinct roots.

Observation 3.3

This is the same shape as what Paper 16 showed for the equals sign.16 found six roots under “=” and this paper finds four under “synchrony”—in both, one symbol or one word covered claims of differing status.

Observation 3.4

And none of the four has a threshold.A convention is chosen, an equality either holds or not, a requirement is imposed, a metaphor is not a claim—the state of “gradually coming into step” exists in none of them.

4. The Core: Only the Fifth Has a Threshold

Setting: Couple oscillators of differing natural frequency weakly and all-to-all (the Kuramoto model).For a Lorentzian frequency spread of half-width γ , as $N \rightarrow \infty$

$$K_c = 2\gamma, \quad r = \sqrt{1 - \frac{K_c}{K}} \quad (K > K_c)$$

Check 4.1:

γ [rad/s]	$K_c = 2\gamma$
0.01	0.02
0.05	0.10
0.10	0.20

Observation 4.2

This is the core of the paper. There is a threshold—below K_c no oscillator comes into step at all ($r = 0$). A state absent from all four of Section 2 exists here.

Check 4.3: The rise, with $K_c = 0.10$.

K	K/K_c	r
0.10	1.0	0.000000
0.11	1.1	0.301511
0.15	1.5	0.577350
0.20	2.0	0.707107
0.40	4.0	0.866025
1.00	10.0	0.948683

Observation 4.4

Doubling the coupling past threshold brings r only to **0.707107**. Ten times gives 0.948683—not a switch: it rises as a square root.

Observation 4.5

The exponent is exactly $1/2$. Checking $r = 0.301511$ at $K/K_c = 1.1$: $\sqrt{1 - 1/1.1} = \sqrt{0.090909} = 0.301511$ ——agreeing to the sixth decimal.

Observation 4.6

Coupling is not the only thing that moves the threshold. Since $K_c = 2\gamma$, halving the spread and doubling the coupling do the same work——to bring things into step, pull harder or make them more alike.

Observation 4.7

In 1665 Huygens observed two pendulum clocks hung from one beam coming into antiphase. All 46 occurrences of Huygens in this corpus are “Huygens’ principle” (sharp wave propagation in odd dimensions), and this observation of the clocks appears nowhere——the same name on different work (Paper 266 raised the same caution about Nyquist).

5. The Last Stretch Is Expensive

Check 5.1: Solving back for the coupling a target r demands: $K/K_c = 1/(1 - r^2)$.

Target r	Required K/K_c
0.5	1.3333
0.9	5.2632
0.99	50.2513
0.999	500.2501

Observation 5.2

Raising r from 0.99 to 0.999 alone costs **9.95** times the coupling. Perfect step is unreachable—— $r = 1$ requires infinite coupling.

Observation 5.3

This is the same shape as Paper 304’s “capacity is a supremum, not a maximum.” $r = 1$ is attained at no finite K —an unattainable bound.

Observation 5.4

Set beside Paper 311. The Carnot efficiency can be attained, at zero output— $r = 1$ cannot be attained at all. One word, “unreachable,” with three shapes.

Why it is out of reach	
Capacity (Paper 304)	unattainable in principle at finite length
Carnot (Paper 311)	attainable, and then useless
$r = 1$ (this paper)	it would take infinite coupling

6. Separator: Whether Not Being in Step Can Exist

Observation 6.1

Separator—whether a state of not being in step can exist.

Synchrony	Can it fail to hold?	Therefore
Clock convention (24)	no (any choice is consistent)	a convention
Equality of units (16)	no (it holds or it does not)	an equals sign
Phase round a circle (57)	no (single-valuedness forbids it)	a requirement
The lens (76)	not askable	a metaphor
Entrainment (this paper)	yes ($K < K_c$)	a phenomenon

Observation 6.2

Only the fifth row says yes. Because failing to come into step can happen, coming into step is a claim—in the first four, the state of not being in step is not even defined.

Observation 6.3

The corpus has written that zero-tension needs a null hypothesis that can be violated. The same discipline lands here—only when it can be written in a form that could fail is it a phenomenon.

Observation 6.4

A point to write honestly—that the four are not phenomena is not a defect. A convention works correctly as a convention, a requirement as a requirement—all that is said is that calling all five by one word hides their status.

Observation 6.5

And a word about the lab's motto. The syncing of From thinking to syncing. is the fourth sense, the lens—exactly as Paper 76 declared, and this paper does not promote it to a phenomenon.

7. Roll Call and Closing

1. Four existing senses (Check 2.1): **Convention, equals, requirement, metaphor.**
2. No two share a status (Observation 2.2): **One word covered them.**
3. Distinct on all criteria (Check 3.1): **No translating construction.**
4. None has a threshold (Observation 3.4): **No “gradually coming into step.”**
5. $K_c = 2\gamma$ (Check 4.1): **Only the fifth has a threshold.**
6. Double gives 0.707107 (Check 4.3): **It rises as a square root.**
7. The exponent is exactly $1/2$ (Observation 4.5): **Agreeing to six decimals.**
8. Half the spread equals double the coupling (Observation 4.6): **Two ways to do it.**
9. $r = 0.99$ costs 50.25 (Check 5.1): **0.999 costs 500.25.**

10. Only the fifth says yes (Observation 6.1): It can fail, so it is a phenomenon.

Closing: This corpus has used “synchrony” in 20 papers, 78 times, and put it in the lab’s motto. Yet it has meant four different things—the readings of distant clocks (Paper 24, a convention), an equality of four physical quantities (Paper 16, an equals sign), a phase gone round once (Paper 57, a requirement), position matched with velocity (Paper 76, a declared metaphor). Paper 300’s criteria make all four distinct roots, with not one translating construction between them—the same shape as Paper 16’s six roots under “=”. And none of the four has a threshold—a convention is chosen, a requirement imposed, a metaphor is not a claim, and the state of “gradually coming into step” is nowhere. A fifth sense has it—the entrainment of coupled oscillators, where for a Lorentzian $K_c = 2\gamma$ is exact and below it nobody comes into step. The rise is a square root—double the coupling past threshold and r reaches only **0.707107**; ten times gives 0.948683. The exponent is exactly $1/2$, agreeing to the sixth decimal. And the last stretch is expensive— $r = 0.99$ costs **50.25** times and $r = 0.999$ costs **500.25**. $r = 1$ would take infinite coupling. One thing separates them—whether a state of not being in step can exist. Because failing can happen, succeeding is a claim—in the first four, failing is not even defined. That the four are not phenomena is no defect—a convention works correctly as a convention. All that is said is that calling all five by one word hides their status.

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