

Which Dimension Is Special Depends on the Question

— The Area Count Allows **14** and Only **12** Fit — The Icosahedral Twelve Have **3.434949** Degrees to Spare and
a Thirteenth Still Does Not Fit — [Paper 315]

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— *From thinking to syncing.* —

Abstract

Paper 59 showed that the volume of the unit ball is greatest in 5 dimensions, and Paper 64 concluded that no single magic dimension exists. This paper asks which dimension the packing question picks—the answer is 8 and 24. No new mathematical theorem and no new law is claimed. First, the condition is a single angle. The centres of touching spheres must be at least 60 degrees apart as seen from the middle (Section 2). Second, this is the core of the paper. The area count allows **14**, and only **12** fit (Section 3). Third, there is room and still no thirteenth. Twelve on an icosahedron are **63.434949** degrees apart, **3.434949** degrees more than needed (Section 4). Fourth, settling it took 259 years. Newton and Gregory disagreed in 1694; it closed in 1953 (Section 4). Fifth, only six dimensions are known exactly. 1, 2, 3, 4, 8 and 24, and $8 \rightarrow 24$ jumps by 819 (Section 5). Sixth, the separator is what is being maximised. Volume picks 5, closed orbits pick 3, packing picks 8 and 24 (Section 6).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed——kissing numbers, packing densities, the E_8 and Leech lattices, the Schutte--van der Waerden settlement and Viazovska's proof are all standard. We do not build geometry——all we use is one angle and one ratio of areas. We do not prove the kissing number——the proof that 12 is the maximum is not entered. That 12 fit is checked here by construction; that 13 do not is quoted. We do not build lattices——the constructions of E_8 and Leech are not entered. We write no number for the unsettled dimensions——for $d \geq 5$ (apart from 8 and 24) the upper and lower bounds do not meet, and no figure is given. We do not enter coding theory——the correspondence between packings and error-correcting codes is not treated. Relation to earlier papers: Paper 59 showed the unit ball's volume peaks in 5 dimensions——this paper asks the same “which dimension is special” with a different question, and the answer is 8 and 24, not 5. Paper 64 concluded that no single magic dimension exists——this paper adds one more independent instance to that conclusion. Paper 69 read $3 + 1$ as an over-determination by independent roots——packing does not join that over-determination, because it does not pick 3. Paper 182 showed the curse of dimensionality is not one curse——this paper shows the gifts of dimension are likewise several. Paper 279 showed the cost of self-avoidance vanishing above four dimensions——it runs the same way as this paper: high dimensions do not simply get easier. What is added is computing that the area count allows 14, measuring the icosahedral configuration at 63.434949 degrees and the slack at 3.434949, listing the six dimensions that are settled, and setting volume, closed orbits and packing side by side to show they pick different dimensions.

1. Notation and the Question

Setting: Place one sphere, then place equal spheres touching it. How many fit——this count is the kissing number.

Observation 1.1

Paper 59 showed that the unit ball's volume peaks in 5 dimensions, and Paper 64 concluded that no single magic dimension exists. Paper 69 read $3 + 1$ as an over-determination by independent roots.

Question: Which dimension, then, does the packing question pick?

Observation 1.2

The answer is 8 and 24. Neither the 5 of volume nor the 3 of orbits——change the question and the special dimension changes.

2. The Condition Is a Single Angle

Check 2.1: Put a unit sphere at the centre. A touching sphere’s centre is at distance 2, and two neighbours’ centres must also be at least 2 apart or they overlap.

Observation 2.2

Two sides of 2 with an opposite side of at least 2—the cosine rule gives an angular separation of at least **60** degrees as seen from the centre.

Quantity	Value
Distance to a neighbour’s centre	2
Minimum distance between neighbours	2
Minimum angular separation required	60.000000 degrees

Observation 2.3

The problem has turned into one about points on a sphere. Radius and volume have gone, and only an angle is left—the same place where Paper 31 wrote that π carries no dimension: the scale has dropped out.

3. The Core: The Area Count Allows 14 and Only 12 Fit

Setting: Each neighbour occupies a cap of angular radius 30 degrees on the middle sphere (half of the 60). Count how many such caps the sphere could hold.

Check 3.1:

Quantity	Value
Cap area / sphere area = $(1 - \cos 30^\circ)/2$	0.06698730
Its reciprocal	14.928203
Count allowed by area	14
Count that actually fits ($d = 3$)	12

Observation 3.2

This is the core of the paper. The area count allows **2** too many—because caps cannot tile a sphere without gaps.

Observation 3.3

That gap of **2** is the difficulty, measured (Section 4). With an upper count of **14** and a truth of **12**, an argument is needed that rules out **13**—area does not reach it.

Observation 3.4

Paper 224 wrote “flat wherever you touch it, and four pi in total.” Here too the local count and the global count disagree—adding areas does not settle whether they fit.

4. There Is Room, and Still No Thirteenth

Setting: There is more than one way to place **12**. Compare the cuboctahedron (vertices at the permutations of $(\pm 1, \pm 1, 0)$) with the icosahedron.

Check 4.1: Computing the minimum separation.

Arrangement	Points	Minimum separation
Cuboctahedron	12	60.000000 degrees (just touching)
Icosahedron	12	63.434949 degrees
Required minimum	—	60 degrees

Observation 4.2

The icosahedral twelve have **3.434949** degrees to spare. None of them touches a neighbour and every one of them can be moved—the room is visibly there.

Observation 4.3

And still a thirteenth does not fit. The room exists but cannot be gathered into one place—existing and being usable are different.

Check 4.4: The history.

Year	Event
1694	Newton said 12 , Gregory thought 13
1953	Schutte and van der Waerden settled it at 12
Gap	259 years

Observation 4.5

Gregory erred because he saw the room. The **3.434949** degrees of slack are real—he mistook one thing only: that visible room need not be usable room.

Observation 4.6

A point to write honestly——this paper does not prove that 13 cannot fit. That 12 fit was checked here from the coordinates above——the upper bound is quoted.

5. Only Six Dimensions Are Known Exactly

Check 5.1:

d	Kissing number	Ratio to the row above
1	2	—
2	6	3.0000
3	12	2.0000
4	24	2.0000
8	240	10.0000
24	196560	819.0000

Observation 5.2

Outside these six the bounds do not meet. Even $d = 5$ is not settled——the ground next to 3 is empty.

Check 5.3: The packing densities that are proved.

d	Density	Settled
1	1.0000000000	trivial
2	0.9068996821	Thue 1910
3	0.7404804897	Hales 1998
8	0.2536695079	Viazovska 2016
24	0.0019295743	Cohn et al. 2016

Observation 5.4

The same list—1, 2, 3, 8, 24. The dimensions where the kissing number is known and where the density is known nearly coincide—not by accident but because the same lattices (E_8 and Leech) supply both answers.

Observation 5.5

Density falls with dimension. At $d = 24$ it is 0.0019295743—high-dimensional space is almost entirely gap. The same phenomenon Paper 182 counted as the curse, seen from another side.

6. Separator: What Is Being Maximised

Observation 6.1

Separator—which quantity the question asks to be greatest.

Question	Dimension picked	Source
Volume of the unit ball is greatest	5	Paper 59
Orbits close, waves travel sharply	3	Papers 64, 69
$4\pi = 2\pi\chi$ holds	3	Paper 128
The cost of self-avoidance vanishes	4 and above	Paper 279
Spheres pack specially	8 and 24	this paper

Observation 6.2

Five questions return four different answers. Paper 64’s conclusion that no single magic dimension exists gains one more independent instance—and this answer overlaps none of the others.

Observation 6.3

Applying Paper 300’s criteria.

Criterion	Result
Is there a correspondence preserving identities?	no. Nothing links volume to packing
Does the agreement persist under motion?	no. Move <i>d</i> and the answers part
Do the same quantities appear?	no. An integral against a lattice

Observation 6.4

All three say no——“special dimension” is one phrase with several distinct roots. The same form as Paper 16’s six roots under “=” and Paper 313’s four under “synchrony”.

Observation 6.5

One last thing——8 and 24 are special because special lattices live there. E_8 and Leech——it is not the dimension that is special but the object able to live in it. Not mistaking the subject is the discipline here.

7. Roll Call and Closing

1. **60 degrees is required** (Check 2.1): **Radius and volume drop out.**
2. **Area allows 14** (Check 3.1): **The reciprocal of 0.06698730.**
3. **Only 12 fit** (Check 3.1): **Two too many.**
4. **The cuboctahedron sits at 60** (Check 4.1): **Just touching.**
5. **The icosahedron at 63.434949** (Check 4.1): **3.434949 to spare.**
6. **And no thirteenth** (Observation 4.3): **Existing is not being usable.**
7. **259 years to settle** (Check 4.4): **1694 to 1953.**
8. **Six dimensions known** (Check 5.1): **8 → 24 jumps by 819.**
9. **Density gives the same list** (Check 5.3): **E_8 and Leech supply both.**
10. **Five questions, four answers** (Observation 6.1): **No single special dimension.**

Closing: Paper 59 showed the unit ball's volume peaks in 5 dimensions and Paper 64 concluded that no single magic dimension exists. The packing question picks 8 and 24——neither the 5 of volume nor the 3 of orbits. The condition is a single angle——neighbouring centres must be at least 60 degrees apart, and at that point radius and volume drop out and the problem becomes one about points on a sphere. And the area count allows 14 while only 12 fit——caps cannot tile a sphere, and that gap of 2 is the difficulty itself. Twelve placed on an icosahedron are **63.434949** degrees apart, **3.434949** degrees more than the 60 required——none of them touches, every one can be moved, and still a thirteenth does not fit. Gregory erred because he saw the room——the 1694 dispute closed in 1953, **259** years later. Only six dimensions are settled——1, 2, 3, 4, 8, 24, with a factor of **819** from 8 to 24; the proved densities give nearly the same list. One thing separates them——which quantity the question asks to be greatest. Volume says 5, closed orbits say 3, $4\pi = 2\pi\chi$ says 3, self-avoidance says 4 and above, packing says 8 and 24——five questions, four answers. One last thing——8 and 24 are special because E_8 and the Leech lattice live there. It is not the dimension that is special but the object able to live in it, and the subject must not be mistaken.

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