

A Calendar Is Three Approximations Stacked

— The Julian Slips a Day in **128** Years and the Gregorian in **3226** — **97** Leap Days per **400** Years Give
365.242500 — [Paper 319]

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— *From thinking to syncing.* —

Abstract

A year is not **365** days. This paper asks what a calendar is doing—the answer is approximating with an integer, correcting the residue with another integer, and correcting that. No new mathematical theorem and no new law is claimed. First, a year is **365.24219** days (Section 2). Second, this is the core of the paper. The Julian slips a day in **128** years, the Gregorian in **3226**—better by **25.19** (Section 3). Third, the rule is three nested corrections. Divisible by 4, by 100, by 400 (Section 3). Fourth, **97** leap days per 400 years give **365.242500** (Section 3). Fifth, the length of a year is itself moving. What is being corrected does not hold still (Section 4). Sixth, the separator is where to stop, and that is a promise (Section 5).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed—the length of the tropical year, the Julian and Gregorian leap rules, continued-fraction approximation and leap seconds are all standard. We do not build astronomy—all we use is one subtraction and one division. We do not write calendar history—the politics of reform and the dates of adoption are not entered. We do not forecast the tropical year—tidal slowing of the rotation is quoted as fact, and no future value is computed. We do not discuss leap-second policy—the debate about ending them is not entered. We do not compare other calendars—lunar, lunisolar and Islamic calendars are not treated. Relation to earlier papers: Paper 306 showed that how many base units there are is a promise—the leap rule is a promise too, and where it is cut has been decided. Paper 295 showed the molar mass of carbon-12 is no longer exactly 12—the same shape, of the fixed definition and the moving reality, appears in the calendar. Paper 314 showed there is only one kind of constant one may ask “has it changed” about—the tropical year is dimensionful, so its change cannot be separated from the choice of unit. This paper writes everything as the dimensionless ratio of day to year. Paper 82 read time as an accumulation of events—a calendar is exactly a vessel for counting events (sunrises). Paper 123 showed that what differs between the twins is a worldline, not a lagging clock—all time here is terrestrial coordinate time and no relativistic effect is treated. What is added is computing the years to a day’s slip from each calendar’s mean year, counting the leap days per 400 years to confirm the mean, listing the continued-fraction convergents and showing that $97/400$ is not among them, and putting the separator at where the approximation is cut.

1. Notation and the Question

Setting: The time for the Earth to go once round the Sun and the seasons to return is the tropical year. One rotation of the Earth is a day.

Observation 1.1

The two do not divide. Revolution and rotation are separate motions with no reason to stand in an integer ratio—every calendar problem comes from here.

Question: What, then, is a calendar doing?

2. A Year Is 365.24219 Days

Check 2.1:

Quantity	Value
Tropical year	365.24219 days
Excess over 365	0.24219 d = 5 h 48 m 45 s

Observation 2.2

About six hours are left over each year. Left alone that is about a day every four years—a month in 120 years, half the seasons in 720.

Observation 2.3

0.24219 is nearly 1/4. So the first approximation is “add a day every four years”—the Julian calendar.

3. The Core: 128 Years and 3226

Check 3.1: Each calendar’s mean year against the tropical year.

Calendar	Mean year	Error	A day slips in
Julian	365.250000 d	+0.007810	128.0 yr
Gregorian	365.242500 d	+0.000310	3225.8 yr

Observation 3.2

This is the core of the paper. 128 years became 3226—better by **25.19**.

Check 3.3: The Gregorian rule has three stages.

Stage	Rule	Per 400 years
First	divisible by 4 is a leap year	+100
Second	divisible by 100 is not	−4
Third	divisible by 400 is	+1
Total	—	97

Observation 3.4

97 per 400 years. The mean year is $(400 \times 365 + 97)/400 = \mathbf{365.242500}$ days—only 0.00031 from the tropical year.

Observation 3.5

Every stage only adds or subtracts. A hundred were put in, four taken out, one put back—each stage picks up the error of the one before.

Check 3.6: The continued-fraction convergents of 0.24219.

Convergent	Decimal	Error	Meaning
1/4	0.25000000	+0.00781000	Julian
7/29	0.24137931	−0.00081069	—
8/33	0.24242424	+0.00023424	the Persian 33 -year cycle
31/128	0.24218750	−0.00000250	—
97/400	0.24250000	+0.00031000	Gregorian (not on the list)

Observation 3.7

97/400 is not on that list. The convergents are $1/4$, $7/29$, $8/33$, $31/128$, and 97/400 appears nowhere among them—the Gregorian rule stands off the ladder of best approximations.

Observation 3.8

And two rungs back, $8/33$ is closer. The errors are 0.00023424 against 0.00031000—the Persian 33-year cycle is more accurate than the Gregorian calendar.

Observation 3.9

$31/128$ is in another class again. An error of 0.00000250, so a day slips in 400,000 years—31 leap days per 128 years, that is “every fourth year, except drop one every 128”.

Observation 3.10

And 400 was chosen anyway. 400 is a multiple of 100 and rides straight on decimal counting—33 and 128 both need a separate decision about where to start counting. What was chosen was not accuracy but ease of use.

4. What Is Being Corrected Does Not Hold Still

Observation 4.1

The tropical year is itself changing. Tidal friction slows the rotation and lengthens the day—so the answer to “how many days in a year” drifts.

Observation 4.2

So the figure of 3226 years is not good for 3226 years. The denominator moves, so it is an extrapolation from today’s value—this paper does not assert that extrapolation.

Observation 4.3

A leap second is a correction at another layer.

Correction	What it reconciles	Timing
Leap year	year with day	fixed by rule
Leap second	day with second	no rule; decided by observation

Observation 4.4

The second row has no rule. When and by how much the Earth's rotation will lag cannot be predicted, so it is measured and then decided—a leap year is computed, a leap second is observed.

Observation 4.5

Paper 295 wrote that the molar mass of carbon-12 is no longer exactly 12. The same shape—beside the side fixed by definition, the real thing moves.

5. Separator: Where to Stop

Observation 5.1

Separator—where the approximation is cut. And nature does not decide it.

Check 5.2: More stages make it as good as one likes.

Rule	Mean year	A day slips in
Every fourth year only	365.250000	128 yr
+ drop at 100	365.240000	456.6 yr
+ restore at 400	365.242500	3226 yr
+ drop at 3200 (hypothetical)	365.242188	400,000 yr

Observation 5.3

The fourth row does not exist. It can be written down and nobody has adopted it——because 3226 years was judged enough.

Observation 5.4

Paper 306 wrote that how many base units there are is a promise. The number of leap stages is a promise too——three is not correct; three is where it was decided to stop.

Observation 5.5

Applying Paper 300's criteria to the two corrections.

Criterion	Leap year against leap second
Do they reconcile the same quantities?	no. Year with day, day with second
Are they decided the same way?	no. Computed, observed
Can they be predicted?	no. One can, one cannot
Do both correct calendar drift?	yes

Observation 5.6

Three noes and one yes—two distinct roots under the one phrase “calendar correction”.

Observation 5.7

One last thing—calendars are not hard because astronomy is hard. Both the tropical year and the day are measured to ample precision—what is hard is having to approximate a non-integer with a rule made of integers. There is no error on nature’s side; the error is on the side of the promise.

6. Roll Call and Closing

1. **365.24219** days (Check 2.1): **5 h 48 m 45 s** left over.
2. **Julian: a day in 128 years** (Check 3.1): **Error +0.007810**.
3. **Gregorian: a day in 3226** (Check 3.1): **Better by 25.19**.
4. **97 per 400 years** (Check 3.3): **100 − 4 + 1**.
5. **Mean year 365.242500** (Observation 3.4): **Off by 0.00031**.
6. **97/400 is not a convergent** (Observation 3.7): **31/128 gives 400,000 years**.
7. **Ease of use was chosen** (Observation 3.10): **Not accuracy**.
8. **The day is lengthening** (Observation 4.1): **3226 is an extrapolation**.
9. **Leap seconds are observed** (Observation 4.4): **Leap years are computed**.
10. **Three noes, one yes** (Check 5.5): **Corrections have distinct roots**.

Closing: Revolution and rotation are separate motions, so there is no reason for a year to be a whole number of days. The tropical year is **365.24219** days and leaves **5 h 48 m 45 s** over every year. The Julian calendar slips a day in **128** years, the Gregorian in **3226**—better by **25.19**. The rule is three nested corrections—add at 4 (+100), drop at 100 (−4), restore at 400 (+1), giving **97** leap days per 400 years and a mean year of **365.242500**, each stage picking up the error of the one before. And yet 97/400 is not a continued-fraction convergent at all—the convergents are 1/4, 7/29, 8/33, 31/128. 8/33, two rungs back, is already closer (0.00023424 against 0.00031000), and 31/128 would slip a day only in **400,000** years. 400 was chosen because it rides on decimal counting: what was chosen was ease of use, not accuracy. And what is being corrected does not hold still—tidal friction lengthens the day, so the figure of 3226 is itself an extrapolation from today. A leap second is a correction at another layer—a leap year reconciles year with day by rule; a leap second reconciles day with second with no rule, by observation.

One is computed, the other observed, and by Paper 300’s criteria they are distinct roots. One thing separates them —where the approximation is cut. And nature does not decide it. One last thing—calendars are not hard because astronomy is hard. Both quantities are measured to ample precision; what is hard is approximating a non-integer with a rule made of integers, and the error lies not on nature’s side but on the side of the promise.

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