

The Yamagishi Unified Budget Canvas Theory — The $32\pi^2$ shared by the QCD topological charge and the gravitational Pontryagin density, read as an index normalisation — [Paper 32]

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— *From thinking to syncing.* —

ABSTRACT

This paper examines the geometric constant $32\pi^2$ common to the instanton topological charge density $q(\mathbf{x})$ of quantum chromodynamics and the Pontryagin density $q_{\text{grav}}(\mathbf{x})$ of general relativity. We give the factorisation $32\pi^2 = 2 \times (4\pi)^2$ under the Yamagishi mass patch $\mathbf{m} = 4\pi$ and ask why the same constant occurs in both.

Important confirmation in the 2nd edition: this coincidence is genuine. Where $16\pi^2$ or $32\pi^2$ recur across this corpus, the recurrences are usually **numerical coincidences of distinct origin** that must not be merged (error pattern C; see Papers 22, 29, 30, 45, 48). **The coincidence treated here is an exception, and merging is justified.** The QCD instanton number and the gravitational Pontryagin index are **both normalisations of Chern–Pontryagin characteristic classes to integer values**, so their shared $32\pi^2$ descends from one and the same mathematical structure (Section 5). **It is the only legitimate unification identified in this review.**

Withdrawal in the 2nd edition: On the other hand the first edition's substitution $32\pi^2 \rightarrow 2c^2n$ (a "VRAM rendering budget") is **withdrawn**. The paper's own synchronisation relation yields $2n$, not $2c^2n$; the c^2 is a **spurious factor left behind when $1/v^2$ was dropped**. In addition $32\pi^2$ is dimensionless while $2c^2n$ is not (Section 4).

Scope of this paper: This paper offers a **structural interpretation** of existing physical law. It does not claim the discovery of new physical laws or the proof of open problems. **Note on units:** Substitutions such as $\mathbf{m} = 4\pi$ are a **choice of natural units**. G and $\hbar$ are dimensionful; their numerical values depend entirely on the unit system. Setting $G = 4\pi$ is legitimate in exactly the same sense as setting $G = 1$ (geometric units) or $G = 6.674 \times 10^{-11}$ (SI); **physics does not single out one of these as "correct."** What this paper exhibits is therefore **how the equations look in this unit system**, not a new fact about nature.

Introduction

QCD topological charge density

The instanton number of a gauge field in QCD is the spacetime integral of the topological charge density

$$q(x) = \frac{g^2}{32\pi^2} G_{\mu\nu}^a \tilde{G}^{a\mu\nu}$$

Gravitational Pontryagin density

In general relativity the topological invariant of spacetime curvature (the Pontryagin class) is expressed via the Riemann tensor and its dual as

$$q_{\text{grav}}(x) = \frac{1}{32\pi^2} R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma}$$

Factorisation of $32\pi^2$

The decomposition $32\pi^2 = 2 \times (4\pi)^2$

$$32\pi^2 = 2 \times 16\pi^2 = 2 \times (4\pi)^2$$

Origin of the factor 2 (consistent with Paper 48)

The factor **2** in $32\pi^2 = 2 \times 16\pi^2$ originates in the **normalisation of the generators** of a non-abelian gauge theory, $\text{Tr}(T^a T^b) = \frac{1}{2} \delta^{ab}$. This was **settled in Paper 48** and is carried over here. **No new origin is introduced.**

[2nd Edition] Withdrawal of $32\pi^2 \rightarrow 2c^2n$

The first edition took the master synchronisation relation $p = 4\pi v = c\sqrt{n}$, set $v = c$ to obtain $4\pi = \sqrt{n}$, and substituted

$$32\pi^2 \longrightarrow 2c^2n$$

presenting this as a "Yamagishi unified budget isomorphism theorem". **This substitution is withdrawn.**

Correction 1: the paper's own relation gives $2n$, not $2c^2n$

Working within the first edition's own setup: at $v = c$, from $4\pi c = c\sqrt{n}$ we get $4\pi = \sqrt{n}$, i.e.

$$n = (4\pi)^2 = 157.9137 \dots$$

Substituting into the factorisation,

$$32\pi^2 = 2(4\pi)^2 = 2(\sqrt{n})^2 = 2n = 315.8273\dots$$

so that **no c^2 appears** (both sides were checked numerically to equal **315.8273**).

The source of the spurious c^2 is clear. For general v the relation reads $4\pi = c\sqrt{n}/v$, whence

$$32\pi^2 = \frac{2c^2n}{v^2},$$

and the first edition dropped the $1/v^2$ while retaining the c^2 . Setting $v = c$ makes $c^2/v^2 = 1$ and returns $2n$. The c^2 is a leftover.

Correction 2: the dimensions do not match

More fundamentally, $32\pi^2$ is **dimensionless** — as it must be, being the constant that normalises a topological index. But $2c^2n$ has $[\mathbf{m}^2/\mathbf{s}^2] \cdot [\mathbf{m}^{-3}] = [\mathbf{m}^{-1}\mathbf{s}^{-2}]$ and is **not dimensionless**. Quite apart from the algebra, the equation cannot hold.

Correction 3: demotion of the "isomorphism theorem"

Accordingly the first edition's "Yamagishi unified budget isomorphism theorem" is withdrawn. **No quantity corresponding to a "rendering budget" $2c^2n$ has been established in this paper.**

[2nd Edition] The QCD–gravity coincidence is genuine

Although the substitution above is withdrawn, **the observation from which this paper started is correct, and important.** We restate it as the principal content of the second edition.

Both descend from the same mathematical structure

The QCD instanton number and the gravitational Pontryagin index both **integrate to an integer** over spacetime:

$$\nu = \int d^4x \frac{g^2}{32\pi^2} G_{\mu\nu}^a \tilde{G}^{a\mu\nu} \in \mathbb{Z} \quad (1)$$

$$P = \int d^4x \frac{1}{32\pi^2} R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} \in \mathbb{Z} \quad (2)$$

Both are **Chern–Pontryagin characteristic classes**, and $32\pi^2$ appears in each as **the constant normalising the index to integer values**. The shared $32\pi^2$ is therefore **not a coincidence but a consequence of one and the same mathematical structure**.

This is exceptional within the present corpus (relation to error pattern C)

Throughout this framework $16\pi^2$ and $32\pi^2$ appear in many contexts, but **most of these have distinct origins**. The decompositions identified up to the third review:

Context	Origin	Merge?
32 (this paper: QCD / gravity)	Chern–Pontryagin index normalisation	yes
29 (chiral anomaly)	the same (Atiyah–Singer index)	yes
22 (tokamak)	toroidal $\times$ poloidal directions	no
30 (stellar)	square of the ball-volume factor $(4\pi/3)^2$	no
08, 45 (Hopf)	volume of the radius-2 S^3	no
46, 47	4D momentum measure	no
48 (Gauss–Bonnet)	$n = 2$ term of $(4\pi)^2 n!$	no

Only this paper and Paper 29 may be merged. Both play the identical role of **normalising a topological index to integer values**, and by the discipline established in Paper 48 this is a **legitimate unification**. It must not be confused with the other decompositions.

The extent of the claim, stated precisely

This coincidence is **known mathematics**. That the normalisation of Chern–Pontryagin classes appears in both field theory and gravity is standard, and this paper did not discover it. **The contribution here is to have discriminated, among the several $16\pi^2$ -type decompositions in this corpus, which may be merged and which may not.**

Explicit Scope

- **What this paper establishes:** (i) the arithmetic $32\pi^2 = 2 \times (4\pi)^2$ (correct); (ii) that the factor **2** arises from generator normalisation (consistent with Paper 48); (iii) **that the $32\pi^2$ shared by QCD and gravity rests on a common origin — normalisation of a topological index**. Point (iii) is the core of this paper.
- **What it does not establish:** (i) $32\pi^2 = 2c^2n$ (fails both algebraically and dimensionally); (ii) any physical quantity corresponding to a "rendering budget"; (iii) a *physical* unification of QCD and gravity — what is shown is a **shared mathematical normalisation**, not a unification of forces.
- **Character of this paper:** a restatement of known mathematics, valuable for **supplying a criterion that separates the coincidences within this corpus that may legitimately be merged from those that may not**.

Conclusion

This paper records the factorisation $32\pi^2 = 2 \times (4\pi)^2$ for the constant common to the QCD topological charge density and the gravitational Pontryagin density, with the factor **2** arising from generator normalisation.

The first edition's substitution $32\pi^2 \rightarrow 2c^2n$ (a VRAM rendering budget) is withdrawn. The paper's own synchronisation relation yields **$2n$** ; the **c^2** is a leftover from dropping $1/v^2$. Moreover **$32\pi^2$** is dimensionless while **$2c^2n$** is not. **On the other hand the observation from which the paper began is correct.** The two occurrences of **$32\pi^2$** share an origin: both **normalise a Chern–Pontryagin index to integer values**. **This is one of the few coincidences in this corpus for which merging is justified** — the other being Paper 29.

References

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