

The Road into Chaos Carries a Number the Map Does Not

— A Parabola and a Sine Curve Head for the Same **4.669201609** — **4.666483** and **4.658764**, Computed Here —

[Paper 320]

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— *From thinking to syncing.* —

Abstract

Paper 289 showed that one number, the error-doubling time, sets the limit of forecasting. This paper asks how a system gets into that chaos—the answer is by doubling its period again and again, and those steps carry a number the map does not. No new mathematical theorem and no new law is claimed. First, the period doubles. Period 2 at $r = 2.999913835$, 4 at **3.449455935**, 8 at **3.544076323** (Section 2). Second, the steps crowd together. The ratios run **4.751007** → **4.655336** → **4.666483** (Section 3). Third, this is the core of the paper. A sine curve gives the same number—**4.470421** → **4.627798** → **4.658764** (Section 4). Fourth, the shape of the formula is nowhere in the answer. Parabola or sine does not enter (Section 4). Fifth, this measures something other than Paper 289. 289 is the cost inside chaos; this is the doorway (Section 5). Sixth, the separator is whether there is one hump with a smooth top (Section 6).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed—the logistic map, period-doubling bifurcation, Feigenbaum’s constant and its universality, and the classification of unimodal maps are all standard. We do not build dynamical systems—all we use is two maps and one ratio. We do not prove universality—the renormalisation-group account is not entered. We only show numerically that two maps give the same number. We claim no value for the constant—what we obtained is 4.666483 and 4.658764, values still converging. The published 4.669201609 is quoted. We do not define chaos—Lyapunov exponents and topological transitivity are not entered. We do not treat other routes—intermittency, crises and the rest are not treated. Relation to earlier papers: Paper 289 showed that one number, the error-doubling time, sets the limit of forecasting—289 measures the cost once inside chaos; this paper measures the steps just before. The same phenomenon, before and after. Paper 271 showed mean field not seeing the dimension while the dimension alone settles the error—the same word, universality, and the 4.669 here likewise survives the discarding of detail. Paper 264 showed a number that moves with the lattice living beside one that does not—this paper sets a number that moves with the map (the bifurcation points) beside one that does not (the ratio). Paper 316 showed three counts returning one integer—this paper shows two maps returning one constant, the same form. Paper 180 showed “the classical limit” is not one limit—the limit here (the accumulation of bifurcations) is one more. What is added is locating five bifurcation points of the logistic map, giving the ratios as $4.751007 \rightarrow 4.655336 \rightarrow 4.666483$, repeating the computation for the sine map to obtain 4.658764, and putting the separator at one hump with a quadratic top.

1. Notation and the Question

Setting: Take $x_{n+1} = rx_n(1 - x_n)$, with x between 0 and 1 and r from 0 to 4.

Observation 1.1

The formula is one quadratic. Two multiplications and no complexity anywhere in it—and yet as r rises it stops being predictable.

Question: How does it get there?

Observation 1.2

The answer is by doubling its period again and again—and those steps carry a number that is not written in the formula.

2. The Period Doubles

Check 2.1: Orbits after 20000 iterations (computed here).

r	Period	First terms
2.80	1	0.642857
3.20	2	0.799455, 0.513045
3.50	4	0.874997, 0.382820, 0.826941, 0.500884
3.56	8	0.889892, 0.348824, 0.808639, 0.550881
3.90	none found	0.166950, 0.542403, 0.967988, 0.120852

Observation 2.2

It goes $1 \rightarrow 2 \rightarrow 4 \rightarrow 8$. Three, five and six never appear on this road—the way it grows is fixed.

Check 2.3: The bifurcation values, by bisection (computed here).

Period	Begins at r
2	2.999913835
4	3.449455935
8	3.544076323
16	3.564401469
32	3.568757029

Observation 2.4

The steps crowd together fast. $0.4495 \rightarrow 0.0946 \rightarrow 0.0203 \rightarrow 0.0044$ —the room in r runs out.

3. The Steps Crowd Together

Check 3.1: Ratios of successive steps.

Ratio	Value
$(b_2 - b_1)/(b_3 - b_2)$	4.751007
$(b_3 - b_2)/(b_4 - b_3)$	4.655336
$(b_4 - b_3)/(b_5 - b_4)$	4.666483
Published δ	4.669201609

Observation 3.2

It settles towards a fixed ratio. 4.751007 $\rightarrow$ 4.655336 $\rightarrow$ 4.666483——overshooting and coming back, closing on 4.669.

Observation 3.3

A fixed ratio means a geometric series. So the sum is finite——infinitely many bifurcations fit inside a finite range of r .

Observation 3.4

A point to write honestly——4.666483 is still converging. Taking further bifurcations moves it towards 4.669201609——this paper claims no value for the constant, only that it is approaching.

4. The Core: A Sine Curve Gives the Same Number

Setting: Repeat the computation on a map of a different shape, $x_{n+1} = r \sin(\pi x_n)$. It is not even a polynomial.

Check 4.1:

Period	Begins at τ (sine map)	For comparison: logistic
2	0.719941554	2.999913835
4	0.833257388	3.449455935
8	0.858605301	3.544076323
16	0.864082617	3.564401469
32	0.865258319	3.568757029

Check 4.2: The ratios.

Ratio	Sine map	Logistic
First	4.470421	4.751007
Second	4.627798	4.655336
Third	4.658764	4.666483

Observation 4.3

This is the core of the paper. Not one bifurcation point agrees——0.7199 against 2.9999, 0.8333 against 3.4495. And yet the ratios head for the same number.

Observation 4.4

The shape of the formula is nowhere in the answer. Parabola or sine, τ running to 4 or to 1——none of it enters the ratio.

Observation 4.5

Paper 264 wrote of a number that moves with the lattice living beside one that does not. They live together here too——the bifurcation points move with the map and the ratio does not.

Observation 4.6

Paper 271 wrote that mean field does not see the dimension while the dimension alone settles the error. The same word, universality—past the discarded detail, a number that cannot be discarded remains.

5. This Measures Something Other Than Paper 289

Check 5.1:

	What is measured	When
Paper 289	the error-doubling time τ_d	inside chaos
This paper	the ratio of bifurcation steps δ	just before it

Observation 5.2

The two are the same phenomenon, before and after. δ says how it got in, τ_d says what it costs once inside—knowing one does not give the other.

Observation 5.3

Applying Paper 300’s criteria.

Criterion	Result
Is there a correspondence preserving identities?	no. τ_d does not follow from δ
Do the same quantities appear?	no. A step in a parameter against a time
Is it universal?	only one. δ is; τ_d belongs to the system

Observation 5.4

The third row is the sharpest. δ is the same for every unimodal map, while τ_d is 1.5 days for the atmosphere and something else elsewhere—the universal and the system-owned part company.

6. Separator: One Hump with a Smooth Top

Observation 6.1

Separator—whether the map has one hump whose top is quadratic.

Property of the map	δ is	Why
One hump, quadratic top	4.669201609	the same universality class
One hump, quartic top	a different value	the class changes
Two or more humps	not on this road	it bifurcates otherwise

Observation 6.2

Only the shape of the top acts. A parabola and a sine are both quadratic near their peak, and that is the only place they agree—what agrees is not the function but the order of its top.

Observation 6.3

So the shared number is not a coincidence. Paper 300 wrote that numerical agreement alone is not sameness of root—this paper meets that criterion, because the reason for the agreement is identified and breaks when moved.

Observation 6.4

A point to write honestly——this paper does not prove universality. It shows two maps approaching one number; why they do belongs to the renormalisation-group argument——which is not entered here.

Observation 6.5

One last thing——4.669201609 is written in neither formula. Neither $rx(1-x)$ nor $r \sin(\pi x)$ contains it——a number the formula does not hold is held by the formula's behaviour. Numbers do not live only inside formulas.

7. Roll Call and Closing

1. Periods go $1 \rightarrow 2 \rightarrow 4 \rightarrow 8$ (Check 2.1): Three and five never appear.
2. Bifurcation from **2.999913835** (Check 2.3): Five were located.
3. The steps crowd (Observation 2.4): **0.4495** down to **0.0044**.
4. Ratios towards **4.666483** (Check 3.1): Published **4.669201609**.
5. Infinitely many in finite r (Observation 3.3): Because it is geometric.
6. The sine map gives **4.658764** (Check 4.2): Different map, same number.
7. No bifurcation point agrees (Observation 4.3): **0.7199** against **2.9999**.
8. Before and after **289** (Check 5.1): The doorway and the cost.
9. Only δ is universal (Observation 5.4): τ_d belongs to its system.
10. The order of the top acts (Observation 6.2): Not the function.

Closing: Paper 289 showed one number, the error-doubling time, setting the limit of forecasting. This paper measures what comes just before——chaos is entered by doubling the period again and again. In $x_{n+1} = rx_n(1-x_n)$, two multiplications, the period runs $1 \rightarrow 2 \rightarrow 4 \rightarrow 8$ and three and five never appear——the bifurcations sit at $r = \mathbf{2.999913835}$, 3.449455935 , 3.544076323 , 3.564401469 , 3.568757029 , and the steps crowd from **0.4495** down to **0.0044**. Their ratios run $4.751007 \rightarrow 4.655336 \rightarrow \mathbf{4.666483}$ ——a fixed ratio means a geometric series, so infinitely many bifurcations fit inside a finite range of r . And the sine curve $r \sin(\pi x)$ gives the same number——its bifurcation points, **0.7199** against **2.9999**, agree with the logistic map's nowhere, while its ratios run $4.470421 \rightarrow 4.627798 \rightarrow \mathbf{4.658764}$, towards the same place. The shape of the formula is nowhere in the answer. Paper 289 and this one are the same phenomenon before and after—— δ says how it got in, τ_d what it costs

inside, and δ is universal while τ_d belongs to its system. One thing separates them—whether the map has one hump whose top is quadratic. A parabola and a sine are both quadratic near their peak, and what agrees is not the function but the order of its top. So this is not a numerical coincidence—against Paper 300’s “numerical agreement alone is not sameness of root”, the reason is identified and it breaks when moved. One last thing—4.669201609 is written in neither formula. A number the formula does not hold is held by the formula’s behaviour; numbers do not live only inside formulas.

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