

The Length That Sets How Thick an Atmosphere Is Also Sets Whether It Stays

— Earth's Scale Height Is **7.319** km, **0.1149%** of the Radius — The Number Deciding Escape Is the Reciprocal of That Ratio — [Paper 321]

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— *From thinking to syncing.* —

Abstract

An atmosphere has one length in it, $H = kT/(mg)$. This paper asks what that length decides—the answer is both the thickness and whether the atmosphere stays. No new mathematical theorem and no new law is claimed. First, on Earth it is **7.319** km, **0.1149%** of the **6371** km radius (Section 2). Second, pressure falls by $1/e$ every H . At Everest's height, **0.2985** (Section 3). Third, **63.2121%** of the mass sits below one H (Section 3). Fourth, this is the core of the paper. The number deciding escape is R/H itself (Section 4). Fifth, the Moon is at **25.20** and leaks. Earth **871.65**, Titan **124.90** (Section 4). Sixth, the separator is which of gravity and heat wins, and it can only be written as a ratio (Section 5).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed——scale height, the isothermal barometric law, and the Jeans parameter as an escape criterion are all standard. We do not build atmospheric science——all we use is one length and one ratio. We assume isothermality——a real atmosphere changes temperature with height, and H with it. We compute at one representative temperature. We do not treat escape mechanisms——hydrodynamic escape, non-thermal escape and solar-wind stripping are not entered. λ is a guide, not a verdict. We do not follow compositional evolution——which gas leaves first, and when, is not treated. We claim no accuracy for the representative values—— T and mean molecular weight are numbers for seeing orders of magnitude. Relation to earlier papers: Paper 309 showed that $\ell_c = \sqrt{\gamma/\rho g}$ is the length dividing gravity from surface tension——this is a third of the same kind, dividing gravity from heat this time. Paper 291 showed three lengths inside one ocean——this paper is the reverse: one length answering two questions. Paper 268 showed that the triple point's being a “point” is settled by the phase rule before measurement——the same territory, pressure, and this paper looks at the distribution inside a single phase. Paper 276 showed the decibel is a ratio and not a quantity—— R/H is a ratio too, and carries no dimension. Paper 316 showed the same integer returning from three counts——this paper is its mirror: one ratio answering two questions. What is added is computing the scale height for five bodies, tabulating H/R as a percentage, giving the pressure profile and the cumulative mass in numbers, and showing that the Jeans parameter equals R/H .

1. Notation and the Question

Setting: In an isothermal atmosphere, gravity balancing the pressure gradient gives

$$p(z) = p_0 e^{-z/H}, \quad H = \frac{kT}{mg}$$

with k Boltzmann's constant, T the temperature, m the molecular mass and g the gravitational acceleration.

Observation 1.1

H is a ratio of heat to gravity. A molecule carries kT of energy and is pulled by a weight mg ——their ratio comes out as a length.

Question: What does that length decide?

2. On Earth It Is 7.319 km

Check 2.1: Five bodies (T and mean molecular weight are representative values).

Body	T [K]	μ	g [m/s ²]	H [km]	H/R
Earth	250	28.96	9.81	7.319	0.1149%
Mars	210	43.34	3.72	10.830	0.3195%
Venus	730	43.45	8.87	15.749	0.2602%
Titan	94	28.00	1.35	20.676	0.8030%
Jupiter	165	2.22	24.79	24.928	0.0357%

Observation 2.2

Earth's atmosphere is **0.1149%** of the radius. On a 30 cm globe that is 0.17 mm—thinner than a sheet of paper.

Observation 2.3

Venus is far hotter than Earth and yet its H is barely twice as large. The temperature is 2.9 times higher but the molecules are 1.5 times heavier—three quantities cancelling against each other.

Observation 2.4

Jupiter is thickest, because hydrogen is light. $\mu = 2.22$ is a thirteenth of Earth's—and even with 2.5 times the gravity, H comes out 3.4 times Earth's.

Check 2.5: The same length, written in macroscopic quantities.

Expression	Value
$kT/(mg)$ (molecular)	7319.054158 m
$RT/(Mg)$ (molar)	7319.054156 m

Observation 2.6

They agree to the sixth decimal. k and R , m and M are the same quantities across Avogadro's number, so it cancels in the ratio—Paper 306's “the count of base units is a convention” surfacing here.

3. Pressure Falls by $1/e$ Every H

Check 3.1: Earth ($H = 7.319$ km).

Height z [km]	z/H	p/p_0
1	0.137	0.87229320
5	0.683	0.50502448
8.848 (Everest)	1.209	0.29852566
10	1.366	0.25504972
20	2.733	0.06505036
50	6.831	0.00107926

Observation 3.2

At the summit of Everest it is **0.2985**. Below a third—8.8 km of climbing removes seventy per cent of the air.

Check 3.3: How much of the atmosphere lies below a given height.

Height	Mass below it
$1H$ (7.3 km)	63.2121%
$2H$ (14.6 km)	86.4665%
$3H$ (22.0 km)	95.0213%
$5H$ (36.6 km)	99.3262%

Observation 3.4

63.2121% sits below a single H . That is $1 - 1/e$ —and it depends on neither T nor g nor μ .

Observation 3.5

It is **63.2121%** on every body. Venus, Jupiter, Titan alike—measured in H , every atmosphere has the same shape.

4. The Core: The Number Deciding Escape Is R/H

Setting: Whether an atmosphere escapes is gauged by the Jeans parameter

$$\lambda = \frac{GMm}{kTR}$$

the ratio of the energy needed to escape to a molecule’s thermal energy.

Observation 4.1

This is the core of the paper. Substituting $g = GM/R^2$,

$$\lambda = \frac{GMm}{kTR} = \frac{(GM/R^2)mR}{kT} = \frac{mgR}{kT} = \frac{R}{H}$$

λ is R/H itself—the length that measures thickness is the one that decides whether the atmosphere stays.

Check 4.2:

Body	$\lambda = GMm/(kTR)$	R/H	The atmosphere
Earth	871.65	870.47	stays
Mars	313.60	313.02	stays
Titan	124.90	124.54	stays
Moon	25.20	—	leaks
Jupiter	—	2804.51	stays

Observation 4.3

The two columns agree. The slight difference (871.65 against 870.47) is because the tabulated g includes rotation——algebraically they are exactly equal.

Observation 4.4

The Moon is at **25.20**. Below the usual guide of 30, and the Moon has no atmosphere——its H is too large against its R .

Observation 4.5

Titan stays, at 124.90. It is smaller than the Moon yet holds an atmosphere, because T is only 94 K——cold compensates for small.

Observation 4.6

A point to write honestly—— λ is a guide, not a verdict. Jeans escape is one mechanism among several, and non-thermal processes such as solar-wind stripping exist——Mars's thin atmosphere is not explained by λ alone.

5. Separator: Which of Gravity and Heat Wins

Observation 5.1

Separator—which is larger, kT or mgR . And that can only be written as a ratio— H alone settles nothing.

Check 5.2:

Quantity	Dimension	What it says
H	a length	nothing (it needs something to be compared with)
H/R	dimensionless	how thin the atmosphere is
R/H	dimensionless	whether it stays

Observation 5.3

Only the top row carries a dimension, and only it settles nothing. Jupiter’s $H = 24.928$ km is larger than Earth’s, and yet at $R/H = 2804.51$ it is far safer than Earth—the ordering by length runs opposite to the ordering by safety.

Observation 5.4

Paper 309 wrote that the length dividing gravity from surface tension is not the size of the material. The same holds here— H is not the size of the body but what the body is compared with. A third of the same kind.

	The length	What it divides
Paper 309	$\sqrt{\gamma/\rho g}$	gravity from surface tension
Paper 291	three lengths	distinct mechanisms in one ocean
This paper	$kT/(mg)$	gravity from heat

Observation 5.5

Applying Paper 300's criteria to H/R and R/H . They are reciprocals, and they share a root—a translation exists (take the reciprocal) and it keeps agreeing as the variables move. This is a rare paper that answers “same root”.

Observation 5.6

One last thing—one ratio is answering two questions. “How thin is the atmosphere” and “does it stay” look like different questions—and yet the answers are the two faces of one number. A thin atmosphere is an atmosphere in the act of leaving.

6. Roll Call and Closing

1. Earth is **7.319 km** (Check 2.1): **0.1149%** of the radius.
2. **0.17 mm** on a globe (Observation 2.2): **Thinner than paper**.
3. Jupiter is **thickest** (Observation 2.4): **Because hydrogen is light**.
4. Two expressions agree to six decimals (Check 2.5): $kT/(mg) = RT/(Mg)$.
5. **0.2985** at Everest (Check 3.1): **Seventy per cent gone in 8.8 km**.
6. **63.2121%** below **1H** (Check 3.3): **$1 - 1/e$, on every body**.
7. $\lambda = R/H$ (Observation 4.1): **Algebraically exact**.
8. **The Moon is at 25.20** (Observation 4.4): **Below the guide of 30**.
9. **Titan compensates with cold** (Observation 4.5): **94 K**.
10. **H alone settles nothing** (Check 5.2): **It needs a comparison**.

Closing: An isothermal atmosphere has one length in it, $H = kT/(mg)$, a ratio of heat to gravity that has come out as a length. On Earth it is **7.319 km**—only **0.1149%** of the 6371 km radius, 0.17 mm on a 30 cm globe, thinner than a sheet of paper. Pressure falls by $1/e$ every H , reaching **0.2985** at the summit of Everest—and **63.2121%** of the mass sits below a single H . That $1 - 1/e$ depends on neither T nor g nor μ , and is the same on Venus, Jupiter and Titan. And the number deciding whether the atmosphere escapes comes out of that same H —putting $g = GM/R^2$ into the Jeans parameter $\lambda = GMm/(kTR)$ makes it exactly $\lambda = R/H$. Earth stays at 871.65, Mars at 313.60, Titan at 124.90, and the Moon leaks at **25.20**—Titan is smaller than the Moon and keeps its atmosphere because 94 K of cold compensates for the smallness. One thing separates them—which is larger, kT or mgR . And

that can only be written as a ratio. H alone settles nothing——Jupiter’s H exceeds Earth’s, and at $R/H = 2804.51$ it is far safer, so the ordering by length runs opposite to the ordering by safety. This is a third of the kind Paper 309 began with “the length dividing gravity from surface tension is not the size of the material”—— H is not the size of the body but what the body is compared with. One last thing——one ratio is answering two questions. “How thin is it” and “does it stay” look different, and the answers are the two faces of one number. A thin atmosphere is an atmosphere in the act of leaving.

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