

One and the Same Sensor Error Reaches Position by Three Different Powers

— t^1 , $t^{1.5}$ and t^2 — From One Minute to One Hour They Open by **60**, **464.76** and **3600** — And One of Them Does Not Grow at All — [Paper 322]

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— *From thinking to syncing.* —

Abstract

Paper 312 showed that averaging for longer stops helping past a certain point. This paper asks how the other side, the growing side, grows—the answer is that one and the same error arrives split into three powers. No new mathematical theorem and no new law is claimed. First, an accelerometer bias is integrated twice and becomes t^2 . **635.471** m in one hour at $10\mu g$ (Section 2). Second, white noise goes as $t^{1.5}$. **103.923** m in one hour (Section 3). Third, a velocity bias reaches only t^1 (Section 3). Fourth, this is the core of the paper. From one minute to one hour they open by **60**, **464.76** and **3600** (Section 4). Fifth, the bias overtakes the noise after **96.2797** seconds (Section 4). Sixth, the error at the Schuler period, **84.4058** minutes, does not grow at all (Section 5).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed——inertial-navigation error propagation, the classification of noise by Allan variance, and Schuler tuning are all standard. We do not design navigation systems——all we use is three powers and one period. We do not treat Kalman filtering——fusion with external observations is not entered. This paper looks only at uncorrected dead reckoning. We do not assess instruments—— $10\mu g$, 1 mg and 10 mg are values standing for grades, not particular products. We do not separate attitude error——the path by which a gyro bias reaches position (t^3) is only mentioned. We do not treat the shape of the Earth——gravity anomalies, flattening and Coriolis corrections are not entered. Relation to earlier papers: Paper 312 located where averaging stops helping——this paper is its other face: if 312 is the floor below which it will not improve, this is the ceiling above which it worsens. The two ends of one Allan variance. Paper 289 showed the error-doubling time setting the limit of forecasting——289's error grows exponentially and this paper's grows as a power. The presence of chaos is the difference. Paper 320 showed the bifurcation ratio not depending on the shape of the map——the exponents here likewise do not depend on the grade of the instrument, the same form. Paper 276 showed the decibel is a ratio and not a quantity——an exponent is dimensionless too, and is the same across the decades. Paper 321 showed one length answering two questions——this paper extends it: one error carrying three answers. What is added is computing position error against time for three grades of accelerometer, putting the three powers in one table, finding the moment the bias overtakes the noise, and setting the Schuler period beside them as an error that does not grow.

1. Notation and the Question

Setting: Dead reckoning means integrating measured acceleration twice, without looking outside.

$$x(t) = x_0 + v_0 t + \iint a_{\text{measured}} dt dt$$

Observation 1.1

Integration does not remove error. Averaging shrinks it to $1/\sqrt{N}$, but integration adds it up——the same operation facing the other way.

Question: So how does the error grow?

Observation 1.2

The answer is not one thing——each kind of error takes a different power of time.

2. A Bias Grows as t^2

Setting: A constant accelerometer bias b gives a position error $\frac{1}{2}bt^2$.

Check 2.1: Three grades (computed here).

Grade	b [m/s ²]	1 min	1 hour	1 day
$10\mu g$	9.8067×10^{-5}	0.177 m	635.471 m	366.03 km
1 mg	9.8067×10^{-3}	17.652 m	63.547 km	36603 km
10 mg	9.8067×10^{-2}	176.520 m	635.471 km	366031 km

Observation 2.2

In one day it passes the circumference of the Earth. At 1 mg, 36603 km is 0.91 of the equator——without correction, a day makes the position meaningless.

Observation 2.3

The difference between grades carries straight through. $10\mu g$ and 10 mg differ by 1000, and so does the error at every instant——time does not change the grade.

Check 2.4: How far you get before 1 km of error.

Grade	Time to reach 1 km
$10\mu g$	4516.01 s = 1.2544 hours
1 mg	451.60 s = 0.1254 hours
10 mg	142.81 s = 0.0397 hours

Observation 2.5

An instrument a thousand times better buys only thirty-two times the time. $1.2544/0.0397 = 31.6$ —because the inverse of t^2 is a square root. Money buys the coefficient; the time it buys is only its square root.

3. Noise Is $t^{1.5}$, a Velocity Bias Is t^1

Setting: White noise on acceleration makes velocity a random walk, and position grows as $qt^{1.5}/\sqrt{3}$. Take $q = 0.05$ m/s/ $\sqrt{h}$.

Check 3.1:

Time	Velocity error	Position error
1 min	0.0065 m/s	0.224 m
1 hour	0.0500 m/s	103.923 m
1 day	0.2449 m/s	12218.805 m

Check 3.2: An initial velocity off by 0.1 m/s.

Time	Position error
1 min	6.00 m
1 hour	360.00 m
1 day	8640.00 m

Observation 3.3

At one minute the velocity offset is the largest of the three. 6.00 m against 0.224 m against 0.177 m—the error you notice first is not the error that ends up mattering.

Observation 3.4

At one day the order is reversed. 366031 m against 12219 m against 8640 m—the ranking has turned over.

4. The Core: Three Powers Opening Across the Same Interval

Check 4.1: From one minute to one hour (60 times), each error grows by:

Kind of error	Grows as	Over 60 times the time
Velocity bias	t^1	60.00
White noise	$t^{1.5}$	464.76
Accelerometer bias	t^2	3600.00

Observation 4.2

This is the core of the paper. The same imperfection in the same sensor is reaching position by three different powers. The gap between 60 and 3600 is itself a factor of 60—which power an error arrives by matters more than how good the instrument is.

Check 4.3: When the bias term equals the noise term.

Grade	Crossover
10 μg	96.2797 s (1.6047 min)
1 mg	0.0096 s
10 mg	0.0001 s

Observation 4.4

Even the best grade is overtaken in **96** seconds. Below it t^2 dominates from the start—there is no interval in which the noise is the problem at all.

Observation 4.5

This is the other face of Paper 312.312 wrote that averaging for longer stops helping past a certain point—— here that point is 96.2797 seconds.

	What is being looked at	Power
Paper 312	where averaging works	$\tau^{-1/2}$ (falling)
Paper 312	after it stops	$\tau^{+1/2}$ (rising)
This paper	position, after integrating	$t^{1.5}$ and t^2

Observation 4.6

Integration lifts each power by one. $\tau^{-1/2} \rightarrow t^{1.5}$, a constant $\rightarrow t^2$ ——the floor of the Allan variance stops being a floor once you are looking at position.

5. Separator: Whether the Error Grows or Swings

Setting: Every error so far has grown.There is one that does not.

Observation 5.1

Schuler tuning.An error in altitude changes the estimate of gravity, which changes the altitude again—— when that loop closes, the error swings instead of growing.

Check 5.2:

$$T_S = 2\pi\sqrt{R/g} = 2\pi\sqrt{6371000/9.80665} = 5064.35 \text{ s} = 84.4058 \text{ min}$$

Observation 5.3

84.4058 minutes is also the period of a circular orbit grazing the surface.The same $\sqrt{R/g}$ ——by Paper 300’s criteria they share a root, and the translation can be constructed.

Observation 5.4

Separator——whether there is a loop returning the error to itself.

The loop is	The error	Time dependence
absent	grows	$t^1, t^{1.5}, t^2$
present	swings	t^0 (bounded)

Observation 5.5

The power falls to 0 not because the instrument got better. It is because the system closed——the same hardware, and only the way the error grows has changed.

Observation 5.6

A point to write honestly——Schuler tuning works only horizontally. The vertical error instead diverges exponentially——inside one instrument, the power depends on the direction.

Observation 5.7

One last thing——the exponent does not depend on the grade. $10\mu g$ or 10 mg, it is t^2 ——the same shape as Paper 320's “the bifurcation ratio does not depend on the shape of the map”. Money buys the coefficient, not the exponent.

6. Roll Call and Closing

1. 635.471 m in an hour at $10\mu g$ (Check 2.1): Growing as t^2 .
2. 36603 km in a day (Observation 2.2): 0.91 of the equator.
3. A thousand-fold instrument buys thirty-two-fold time (Observation 2.5): The inverse of t^2 is a root.
4. Noise gives 103.923 m in an hour (Check 3.1): $t^{1.5}$.
5. The ranking reverses between a minute and a day (Observation 3.4): Different powers.

6. **60, 464.76 and 3600** (Check 4.1): **Across the same 60-fold interval.**
7. **Crossover at 96.2797 s** (Check 4.3): **Even at the best grade.**
8. **Integration lifts the power by one** (Observation 4.6): **312's floor disappears.**
9. **Schuler period 84.4058 min** (Check 5.2): **t^0 , it does not grow.**
10. **The exponent ignores the grade** (Observation 5.7): **Only the coefficient is for sale.**

Closing: Paper 312 showed that averaging for longer stops helping past a certain point. This paper measures the other face—in dead reckoning, one and the same sensor error reaches position by three different powers. An accelerometer bias is integrated twice into t^2 : even at $10\mu g$ that is **635.471 m in an hour, and at **1 mg 36603 km** in a day, 0.91 of the equator—and an instrument a thousand times better buys only thirty-two times the time, because the inverse of t^2 is a square root. White noise gives $t^{1.5}$ and **103.923 m** in an hour; an initial velocity offset gives t^1 and **360 m**—at one minute the velocity offset is the largest, and at one day the ranking has completely reversed. Across the same 60-fold interval the three open by **60, 464.76 and 3600**—which power an error arrives by matters more than how good the instrument is. The bias overtakes the noise after **96.2797** seconds even at the best grade, and below it there is no interval in which the noise is the problem at all—integration lifts each power by one, and the floor of the Allan variance stops being a floor once you are looking at position. And one error does not grow—under Schuler tuning the error returns to itself in a loop and swings, bounded, with period $2\pi\sqrt{R/g} = \mathbf{84.4058}$ minutes. The power falls to 0 not because the instrument got better but because the system closed. One thing separates them—whether there is a loop returning the error to itself. And it works only horizontally; the vertical error instead diverges. One last thing—the exponent does not depend on the grade. At $10\mu g$ or at 10 mg it is t^2 , and money buys the coefficient, not the exponent.**

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