

The Size of the Drop Is Not in the Angle of the Rainbow

— The Primary at **42.078107** Degrees and the Secondary at **50.890758** Are the Same for Fog and for Rain —
And Yet the Size Decides Whether There Is Any Colour — [Paper 325]

Yuuki YAMAGISHI (山岸 勇輝)
Yamagishi Complexity Torus Lab, Japan
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— *From thinking to syncing.* —

Abstract

A rainbow always appears at the same angle. This paper asks what sets that angle—the answer is the refractive index alone, and not the size of the drop. No new mathematical theorem and no new law is claimed. First, the primary bow is at **42.078107** degrees. Red **42.252933**, violet **40.575405**, width **1.677527** degrees (Section 2). Second, the secondary has its colours reversed. Red at **50.575709** lies inside violet at **53.605345** (Section 3). Third, the **8.322776** degrees between them are dark (Section 3). Fourth, this is the core of the paper. The radius R does not appear anywhere in the stationarity condition (Section 4). Fifth, and yet size does act. In fog the colour disappears and the bow turns white (Section 5). Sixth, the separator is whether ray optics applies, not the size itself (Section 5).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed——Descartes’ theory of the bow, the angle of minimum deviation, Alexander’s dark band, and the whitening of the bow by Mie scattering are all standard. We do not build optics——all we use is one stationarity condition and one dimensionless number. We do not compute intensity——the distribution of brightness through the Fresnel coefficients is not entered. This paper looks only at angles. We do not treat supernumerary bows——the interference fringes belong to Airy’s theory and are not treated here. We claim no accuracy for the refractive indices——1.3318 / 1.3330 / 1.3435 are values at representative wavelengths, and they move with temperature. We do not separate polarisation——that the light of a bow is strongly polarised is mentioned as fact and no more. Relation to earlier papers: Paper 321 showed one length answering two questions——this is the converse: one length (the radius) failing to answer one of them. Paper 320 showed the bifurcation ratio not depending on the shape of the map——the same form, with the angle here not depending on the size of the drop. Paper 315 showed which dimension is special depends on the question——here it is which quantity acts that depends on the question. Paper 219 showed a flat plate bringing light to a focus——both are about shape gathering light, but 219 is an engineered structure and this is one sphere. Paper 210 showed a pulse peak leaving before it arrives——both are about stationary points, and what is seen is not a ray but the location of a stationarity. Paper 212 showed that every map must distort something——the choice between preserving angle and preserving area is the axis there and here. What is added is computing the bows up to three reflections from the analytic solution, showing in numbers that the secondary reverses the colour order, giving the dark band as 8.322776 degrees, and putting the separator on the size parameter rather than on size.

1. Notation and the Question

Setting: A ray enters a spherical drop at incidence θ , reflects k times inside, and leaves. With refraction angle r , the total bending (the deviation) is

$$D(\theta) = 2\theta - 2(k+1)r + k\pi, \quad \sin \theta = n \sin r$$

Observation 1.1

Rays of every incidence strike the drop. From one grazing the top to one through the centre——and D differs for each.

Observation 1.2

Light should therefore scatter in all directions. And yet the bow appears as a band——something is piling light into one direction.

Question: What sets that direction?

2. The Primary Is at 42.078107 Degrees

Setting: Where $D(\theta)$ is stationary, neighbouring rays leave in the same direction. Light accumulates there.

Observation 2.1

The condition solves in closed form. From $dD/d\theta = 0$,

$$\cos^2 \theta = \frac{n^2 - 1}{(k + 1)^2 - 1}$$

Check 2.2: One internal reflection ($k = 1$), computed here.

Wavelength	n	Incidence	Deviation D	Angle seen
Red (656.3 nm)	1.3318	59.480227°	137.747067°	42.252933°
Mean	1.3330	59.410473°	137.921893°	42.078107°
Violet (404.7 nm)	1.3435	58.801373°	139.424595°	40.575405°

Observation 2.3

The bow is **1.677527** degrees wide. The full moon is about 0.5 degrees across, so a little over three moons —a narrow band.

Observation 2.4

Red is on the outside. Because $42.252933 > 40.575405$ —the smaller the refractive index, the further out the colour lands.

Observation 2.5

The incidences cluster around 59 degrees.Red and violet differ by only 0.68 degrees there——light striking almost the same place comes out spread over 1.68 degrees.

3. The Secondary Reverses the Colours

Check 3.1: Two and three internal reflections.

<i>k</i>	Red	Violet	Width
1 (primary)	42.252933°	40.575405°	1.677527°
2 (secondary)	50.575709°	53.605345°	3.029636°
3	42.179415°	37.926285°	4.253130°

Observation 3.2

In the secondary the order of red and violet is swapped.Primary red > violet, secondary red < violet——one extra reflection reverses the colours.

Observation 3.3

The secondary is 1.81 times wider.3.029636/1.677527——and correspondingly fainter, spreading the same light over more angle.

Check 3.4: Between the two bows.

Angle	
Outside of the primary (red)	42.252933°
Inside of the secondary (red)	50.575709°
Between	8.322776°

Observation 3.5

No light reaches those **8.322776** degrees.No incidence sends any ray into that range——Alexander’s dark band, recorded for more than 1700 years.

Observation 3.6

The third-order bow is at **41.737347** degrees.Almost the same angle as the primary, but on the sun’s side ——one would have to turn away from the primary, which is seen with the sun behind.So it is ordinarily invisible.

4. The Core: The Radius Is Nowhere in the Condition

Observation 4.1

This is the core of the paper.In

$$\cos^2 \theta = \frac{n^2 - 1}{(k + 1)^2 - 1}$$

only *n* and *k* appear——the radius *R* is nowhere.

Check 4.2:

Drop	Radius	Primary bow
Raindrop	1 mm	42.078107°
Drizzle	0.1 mm	42.078107°
Fog	10 μm	42.078107°
Cloud droplet	1 μm	42.078107°

Observation 4.3

A thousandfold change leaves six decimals unmoved. Not an approximation—— R was not cancelled out; it never appears at all.

Observation 4.4

The reason is similarity. A sphere is the same shape at every size, and every ray path is identical once the radius is the unit——an angle sees only ratios of length.

Observation 4.5

Paper 320 wrote that the bifurcation ratio does not depend on the shape of the map. The same form——there the shape stays out of the answer, here the size does.

Observation 4.6

So what is a rainbow measuring? The refractive index and nothing else——what hangs in the sky is a scale reading of the refractive index of water.

5. Separator: Whether Ray Optics Applies

Setting: And yet in fog the bow turns white. If the angle is the same, why does the colour go?

Check 5.1: The dimensionless size parameter $x = 2\pi R/\lambda$ (at $\lambda = 550$ nm).

Drop	Radius	$x = 2\pi R/\lambda$	What acts
Raindrop	1 mm	11424.0	ray optics
Drizzle	0.1 mm	1142.4	ray optics
Fog	10 μm	114.2	diffraction (white bow)
Cloud droplet	1 μm	11.4	diffraction (white bow)

Observation 5.2

Separator——whether x exceeds 10^3 . Above it light behaves as rays and the colours separate; below it diffraction blurs the angle and smears over the 1.68-degree width——a white bow.

Observation 5.3

The radius answers two questions separately.

Question	The radius
At what angle does it appear?	does not answer (it is not in the formula)
Is there any colour?	answers (through x)

Observation 5.4

This is the mirror of Paper 321. There one length answered two questions——here one length answers only one of them. Both obey the same discipline: a length alone settles nothing.

Observation 5.5

Applying Paper 300's criteria to the angle and the whitening.

Criterion	Result
Is there a correspondence preserving identities?	no. The angle does not follow from x
Do the same quantities appear?	only one side. R enters only x
Do they keep agreeing as variables move?	no. Move R and only one changes

Observation 5.6

All three rows say different roots. One radius, in one drop, holding two independent offices—a quantity need not have exactly one job.

Observation 5.7

One last thing—a rainbow has no size. Approach it or retreat and it stays at 42.078107 degrees, and its end can never be reached—because a rainbow is not a place but a direction. Move, and the bow moves with you.

6. Roll Call and Closing

1. The primary is at 42.078107 degrees (Check 2.2): Red 42.252933, violet 40.575405.
2. Its width is 1.677527 degrees (Observation 2.3): A little over three moons.
3. The secondary reverses the colours (Check 3.1): Red 50.575709 < violet 53.605345.
4. The dark band is 8.322776 degrees (Check 3.4): No light arrives.
5. The third order is on the sun's side (Observation 3.6): 41.737347 degrees, behind you.
6. R is not in the condition (Observation 4.1): Only n and k .
7. A thousandfold makes no difference (Check 4.2): To six decimals.
8. The reason is similarity (Observation 4.4): An angle sees only ratios.
9. Below $x = 10^3$ the bow whitens (Check 5.1): Fog is at 114.2.
10. The radius answers only one question (Observation 5.3): The mirror of 321.

Closing: Rays of every incidence strike the drop, and yet the bow appears as a band. What piles the light up is the point where the deviation is stationary, and that condition solves in closed form as $\cos^2 \theta = (n^2 - 1)/((k + 1)^2 - 1)$ —the primary lands at 42.078107 degrees, red at 42.252933 and violet at 40.575405, a band 1.677527 degrees wide, a little over three full moons. The secondary reverses the colours, red at 50.575709 lying inside violet at 53.605345—one extra reflection swaps them. The 8.322776 degrees between receive rays from no incidence at all, and are dark. And the radius of the drop appears nowhere in that condition—a 1 mm raindrop and a $1\mu\text{m}$ cloud droplet give the same angle to six decimals. It was not cancelled out; it never appears. A sphere is the same shape at every size, and an angle sees only ratios of length. And yet the size does act—below a size parameter $x = 2\pi R/\lambda$ of 10^3 , diffraction smears the 1.68-degree width and the bow turns white. Fog sits at 114.2. One thing

separates them——whether ray optics applies, not the size itself. The radius does not answer “at what angle” and does answer “is there colour”. This is the mirror of Paper 321——there one length answered two questions, here one length answers only one. Both obey the same discipline: a length alone settles nothing. One last thing——a rainbow has no size. Approach or retreat and it stays at 42.078107 degrees, and its end can never be reached. A rainbow is not a place but a direction, and it moves exactly as far as you do.

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Yuuki YAMAGISHI (山岸 勇輝)

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