

The Square-Cube “Law” Is Not a Law but an Identity

— Density Moves by a Factor of **3.33** While Surface-to-Volume Moves by **5000** — Having No
Counterexample Is Not the Same as Having No Content — [Paper 326]

Yuuki YAMAGISHI (山岸 勇輝)

Yamagishi Complexity Torus Lab, Japan

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— *From thinking to syncing.* —

Abstract

The square-cube law says that on enlargement area grows as the square and volume as the cube. This paper asks what kind of statement that is—the answer is not an empirical law but an identity. No new mathematical theorem and no new law is claimed. First, changing the shape does not change the $1/L$ dependence. The coefficient runs from 1.0000 to **1.4900** (Section 2). Second, no counterexample can be built. Because it is not settled by measuring (Section 2). Third, this is the core of the paper. Density moves by **3.33** while surface-to-volume moves by **5000** (Section 3). Fourth, cooling time is proportional to length. **1000**-fold between 1 mm and 1 m (Section 4). Fifth, the speed needed to fly goes as the square root. **1200** times the wing loading, **34.6** times the speed (Section 4). Sixth, the separator is whether the phenomenon passes through a surface or happens in a volume (Section 5).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed—the square-cube relation, the isoperimetric inequality, the lumped-capacitance approximation via the Biot number, and wing loading are all standard. We do not build biology—all we use is one identity and a classification of which quantities pass through a surface. We do not treat metabolic exponents—whether the $3/4$ power is right belongs to Paper 256 and is not entered here. This paper asserts no biological exponent at all. We do not classify modes of failure—buckling, fracture and yield are separated in Paper 262, and this paper does not go there. We claim no accuracy for the heat-transfer coefficient— $h = 10 \text{ W/(m}^2\text{K)}$ is a value for seeing orders of magnitude. We do not optimise shape—which shape is favourable is not treated; only the spread of the coefficient is shown. Relation to earlier papers: Paper 256 showed that dynamic similarity holds exactly only at a scale of 1—256 is about whether the dimensionless numbers can be matched; this is the step before it. If 256 says a model never becomes the original, this asks why only the ratios move. Paper 262 showed three ways of breaking—262 asks what decides failure and this asks what moves with size: different questions. Paper 321 showed one length answering two questions—the L here likewise answers only through a ratio. Paper 325 showed the radius not entering the angle of a rainbow—there similarity removes a quantity, here similarity builds a ratio. Two faces of one property. Paper 264 showed a number that moves with the lattice living beside one that does not—this is the same form, with size in place of the lattice. What is added is computing the surface area of three shapes at equal volume, setting measured density beside measured surface-to-volume to compare how far each moves, giving cooling time constants across four decades, and putting the separator on surface against volume.

1. Notation and the Question

Setting: Enlarging by a similarity ratio s takes length to s , area to s^2 and volume to s^3 .

Check 1.1:

Ratio s	Area	Volume	Surface-to-volume
1	1	1	1.000000
2	4	8	0.500000
10	100	1000	0.100000
100	10000	10^6	0.010000
1000	10^6	10^9	0.001000

Observation 1.2

This is not the result of an experiment.No ruler was laid against anything; it follows from the definition of similarity—it is settled before any measurement.

Question: So what kind of statement is it?

2. No Counterexample Can Be Built

Check 2.1: Change the shape.Fix the volume at 1 and compare surface areas (computed here).

Shape	Surface area	Sphere = 1
Sphere	4.835976	1.000000
Cube	6.000000	1.240701
Regular tetrahedron	7.205622	1.490004

Observation 2.2

The coefficient runs only from 1.0000 to 1.4900.However far the shape is changed it does not move by a single decade—and the $1/L$ dependence itself never changes at all.

Observation 2.3

One cannot even try to build a counterexample.No object can be produced for which enlargement failed to lower the surface-to-volume ratio—a non-similar deformation is no longer the same shape.

Observation 2.4

So this is an identity.In Paper 300's terms, both sides write the same thing two ways—it is not an empirical law.

Observation 2.5

Having no counterexample is not the same as having no content. The content lies on the side of where it applies—not in the identity but in which phenomenon rides on which side. Section 5 takes that up.

3. The Core: An Unmoving Quantity Beside One That Moves Five Thousandfold

Check 3.1: Real animals, two quantities side by side (lengths and masses are representative).

	Length [m]	Mass [kg]	Mass/Length ³	Surface/Volume [1/m]
Ant	0.005	3×10^{-6}	24.0000	1200.0000
Mouse	0.090	0.02	27.4348	66.6667
Cat	0.500	4.0	32.0000	12.0000
Human	1.700	70.0	14.2479	3.5294
Elephant	6.000	5000.0	23.1481	1.0000
Blue whale	25.000	150000.0	9.6000	0.2400

Observation 3.2

This is the core of the paper. Mass runs from 3×10^{-6} kg to 150000 kg, a factor of 5×10^{10} , and yet

Quantity	Range	
Mass/Length ³ (a density)	9.6000 to 32.0000	3.33-fold
Surface/Volume	0.2400 to 1200.0000	5000-fold

Observation 3.3

Inside one set of animals an unmoving quantity lives beside one that moves five thousandfold. The density holds because they are all made of water—what changes is not the material but a ratio.

Observation 3.4

Paper 264 wrote of a number that moves with the lattice living beside one that does not. The same form——there the lattice, here the size, moves only one of the two.

Observation 3.5

So “enlarging makes it a different thing” is not quite right. What becomes different is not the thing but the ratios acting inside it——the material stays, and only the ranking turns over.

4. Which Side It Rides on Changes the Answer

Setting: What passes through a surface rides on s^2 ; what happens in a volume rides on s^3 . Their ratio is $1/s$.

Check 4.1: Cooling time (heat escapes through the surface and is stored in the volume). $\tau = \rho c_p (V/A)/h$, water, $h = 10 \text{ W}/(\text{m}^2\text{K})$.

Side L	V/A [m]	τ [s]	τ [hours]
1 mm	0.000167	69.67	0.0194
1 cm	0.001667	696.67	0.1935
10 cm	0.016667	6966.67	1.9352
1 m	0.166667	69666.67	19.3519

Observation 4.2

A factor of **1000** between 1 mm and 1 m. Same material, same shape, and only the time is a thousandfold——the same soup in a spoon and in a cauldron is two different foods.

Check 4.3: The speed needed to fly (lift from wing area s^2 , weight from volume s^3).

	Mass [kg]	Wing loading [N/m ²]	Speed needed [m/s]
Aphid	10 ⁻⁶	4.903	2.583
Sparrow	0.03	36.775	7.073
Swan	10.0	140.095	13.806
Airliner	300000.0	5883.990	89.473

Observation 4.4

1200 times the wing loading needs only **34.6** times the speed. $\sqrt{1200} = 34.6$ ——put one identity in the way and the growth of a ratio drops to a square root.

Observation 4.5

An aphid can fly at 2.583 m/s. Slower than walking pace——to something small the air is as thick as water.

Observation 4.6

In none of these tables does the identity say anything. What speaks is the classification: heat passes through a surface, lift is made by a surface, weight is made by a volume——the numbers come from the classification, not from the identity.

5. Separator: Through a Surface, or in a Volume

Observation 5.1

Separator——whether the phenomenon passes through a surface or happens in a volume.

Phenomenon	Which side	On enlargement
Heat escaping	surface s^2	relatively worse
Heat produced, weight	volume s^3	relatively better
Lift, drag	surface s^2	worse
Load on bone	volume s^3 borne by section s^2	worse
Supply by diffusion	surface s^2 , demand s^3	worse

Observation 5.2

Every row comes from the same identity. All that differs is which column it was put in—and what puts it there is physics, not geometry.

Observation 5.3

So if something is going to be wrong, it is the placement. The identity cannot fail—the classification can.

Observation 5.4

A point to write honestly—this paper asserts no biological exponent. Whether metabolism goes as $2/3$ or $3/4$ belongs to Paper 256, and this paper does not enter it—all this paper says is that the geometry is not what is being disputed.

Observation 5.5

The relation to Paper 325. Both are consequences of similarity—in 325 similarity removes a quantity (the radius) from the answer; here similarity builds a ratio $(1/L)$. Two faces of one property.

Observation 5.6

One last thing—calling it a “law” is what makes it confusing. A law sounds like something the world is doing, and the world has no choice in the matter—the choice lies in which side each phenomenon is counted on. The name hides where the content is.

6. Roll Call and Closing

1. Surface-to-volume is $1/s$ (Check 1.1): From the definition of similarity.
2. The coefficient spans 1.0000--1.4900 (Check 2.1): Not one decade.
3. No counterexample can be built (Observation 2.3): So it is an identity.
4. Mass spans 5×10^{10} (Check 3.1): Ant to whale.
5. Density 3.33, ratio 5000 (Observation 3.2): Inside one set of animals.
6. Cooling time 1000-fold (Check 4.1): 1 mm against 1 m.
7. 1200 loading, 34.6 speed (Observation 4.4): A square root.
8. An aphid flies at 2.583 m/s (Observation 4.5): Slower than walking.
9. The classification speaks (Observation 4.6): Not the identity.
10. What fails is the placement (Observation 5.3): The identity cannot.

Closing: The square-cube law says that on enlargement area grows as the square and volume as the cube. This is not an empirical law but an identity—no ruler was laid against anything; it follows from the definition of similarity and is settled before any measurement. Fix the volume at 1 and change the shape, and the surface area is 4.835976 for a sphere, 6.000000 for a cube and 7.205622 for a tetrahedron, a coefficient spanning only 1.0000 to 1.4900, with the $1/L$ dependence never changing—no object can be produced for which enlargement failed to lower the ratio. And having no counterexample is not the same as having no content—from ant to blue whale mass spans 5×10^{10} , and yet mass over length cubed moves only 3.33-fold while surface-to-volume moves 5000-fold. What changes is not the material but a ratio. Only the phenomena riding on the identity produce numbers—cooling time is 1000 times longer at 1 m than at 1 mm, so the same soup in a spoon and in a cauldron is two different foods; and 1200 times the wing loading needs only $\sqrt{1200} = 34.6$ times the speed, so an aphid flies at 2.583 m/s, slower than walking pace. One thing separates them—whether the phenomenon passes through a surface or happens in a volume. Every row comes from the same identity, all that differs is which column it was put in, and what puts it there is physics, not geometry. So the identity cannot fail; the classification can—and whether metabolism goes as $2/3$ or $3/4$ belongs to Paper 256, which this paper does not enter. With Paper 325 these are two faces of one similarity—there it removes a quantity from the answer, here it builds a ratio. One last thing—calling it a “law”

is what makes it confusing. A law sounds like something the world is doing, and the world has no choice in the matter. The choice lies in which side each phenomenon is counted on, and the name hides where the content is.

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Yuuki YAMAGISHI (山岸 勇輝)

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