

Two Curves Agree to Within **0.083** Per Cent and Are Not the Same Curve

— What Divides the Catenary from the Parabola Is Not Shape but Which Length the Load Is Counted Along —

[Paper 327]

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— *From thinking to syncing.* —

Abstract

A hanging chain and a suspension-bridge cable cannot be told apart by eye. This paper asks what divides them—— the answer is not the shape, but which length the load is counted along. No new mathematical theorem and no new law is claimed. First, they agree exactly through second order. At $x = 0.1$ the difference is **0.083292** per cent (Section 2). Second, far out they are different things. At $x = 5$, **82.925818** per cent (Section 2). Third, the difference begins at fourth order. $x^4/24$ against 0 (Section 2). Fourth, this is the core of the paper. What divides them is counting along arc length or along horizontal length (Section 3). Fifth, on a real bridge the difference is only **0.082674** per cent (Section 4). Sixth, the separator is whether the deck or the cable is the heavier (Section 5).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed—the catenary equation, the parabolic-cable theory of suspension bridges, and the Taylor expansions of both are all standard. We do not do structural design—all we use is two curves and one difference of measure. We do not treat deflection—cable stretch, thermal movement and live-load deflection are not entered. This paper looks only at the idealised static shape. We do not adjudicate which is correct—a real bridge lies between the two, and this paper does not locate it. We claim no engineering accuracy—the sag ratios are values for comparing shapes, not those of any particular bridge. We do not discuss optimal shape—which is structurally better is not treated. Relation to earlier papers: Paper 198 showed that “straight” does not mean “shortest”—198 says one curve has two characters; this says two curves look alike. The direction is reversed. Paper 326 showed the square-cube relation is an identity—both are of the “same appearance, different content” kind, but 326 is about a ratio and this is about functions. Paper 300 showed whether two things share a root is decidable—this is a worked instance of “numerical closeness is not sameness of root”. Paper 245 showed two limits have no answer until their order is written—here too, the curve is undetermined until the measure of integration is written. Paper 183 showed one table having two correct answers—one shape having two correct equations is its counterpart. What is added is computing the difference at six points, stating that it begins at fourth order, giving the maximum difference for each sag ratio, and putting the separator on the measure.

1. Notation and the Question

Setting: A chain hanging under its own weight is a catenary.

$$y = a \cosh(x/a) - a$$

The cable of a bridge with a heavy deck is a parabola.

$$y = \frac{x^2}{2a}$$

Observation 1.1

Expanding cosh,

$$a \cosh(x/a) - a = \frac{x^2}{2a} + \frac{x^4}{24a^3} + \frac{x^6}{720a^5} + \cdots$$

the leading term is the parabola itself—they agree exactly through second order.

Question: So what makes them different?

2. 0.083 Per Cent and 82.9 Per Cent

Check 2.1: With $a = 1$, comparing heights (computed here).

x	Catenary	Parabola	Difference	Relative
0.1	0.00500417	0.00500000	0.00000417	0.083292%
0.5	0.12762597	0.12500000	0.00262597	2.057548%
1.0	0.54308063	0.50000000	0.04308063	7.932641%
2.0	2.76219569	2.00000000	0.76219569	27.593834%
3.0	9.06766200	4.50000000	4.56766200	50.373095%
5.0	73.20994852	12.50000000	60.70994852	82.925818%

Observation 2.2

The same two curves open from 0.08 to 82.9 per cent. Both figures are correct about the same two curves —“how different are they” is undetermined until you say where you are looking.

Observation 2.3

The difference begins at fourth order. $x^4/(24a^3)$ against 0 —first, second and third order all agree, so nearby there is nothing to see.

Observation 2.4

It is not that the approximation is good; they coincide. The parabola is the second-order Taylor polynomial of the catenary, not a curve fitted to it —the closeness is not a coincidence.

Observation 2.5

Paper 300 wrote that numerical agreement alone is not sameness of root.This is a worked instance—— agreement to 0.083 per cent is no evidence of being the same thing.

3. The Core: Which Length the Load Is Counted Along

Setting: Both curves come from the same balance.With the horizontal tension H constant,

$$H \frac{d^2y}{dx^2} = w$$

where w is “weight per unit length”.

Observation 3.1

This is the core of the paper.There are two “unit lengths”.

What is heavy	Length the load is counted along	Curve
the cable itself	arc length ds	catenary
the suspended deck	horizontal length dx	parabola

Observation 3.2

Not one character of the equation differs.All that differs is which differential w is constant against——the difference in shape comes out of a difference in measure.

Observation 3.3

Constant along arc length means the steep parts weigh more per horizontal foot.Because $ds = \sqrt{1 + y'^2}dx$ ——the ends carry extra weight, and so the catenary rises faster.

Observation 3.4

Every difference in Check 2.1 comes from this one point. The 82.9 per cent at $x = 5$ is because the slope is large there and ds departs far from dx —where x is small, $\sqrt{1 + y'^2} \approx 1$ and the two coincide.

Observation 3.5

Paper 245 wrote that two limits have no answer until their order is written. The same discipline—here the curve is undetermined until the measure of integration is written.

Observation 3.6

So “a parabola close to a catenary” has it backwards. Each is the exact answer to a different physics, and neither is an approximation to the other—the approximation relation is something mathematics noticed afterwards.

4. On a Real Bridge It Is 0.083 Per Cent

Check 4.1: Maximum difference for each sag f (half-span taken as 1).

Sag f	Catenary's a	Max difference	Relative to sag
0.02	25.0033	0.00000067	0.003332%
0.05	10.0083	0.00001040	0.020790%
0.10	5.0166	0.00008267	0.082669%
0.20	2.5326	0.00064620	0.323100%
0.35	1.4835	0.00326377	0.932505%
0.50	1.0743	0.00877195	1.754390%

Observation 4.2

Real suspension bridges sag by about $1/10$. The maximum difference there is **0.082674** per cent of the sag——on a 1000 m span with 100 m of sag, **8.3** cm.

Observation 4.3

8.3 cm can be smaller than the construction tolerance. Telling them apart by measurement is hard in practice——and they are still two different curves.

Observation 4.4

The shallower the sag, the less there is to see. At $f = 0.02$ it is **0.003332** per cent——the tauter the line, the more both theories agree.

Observation 4.5

So “can they be distinguished” is a question of precision and “are they different” is a question of definition. The two are independent——things can measure the same and still be different.

5. Separator: Whether the Deck or the Cable Is Heavier

Observation 5.1

Separator——the ratio of the suspended load to the weight of the cable itself.

Situation	What dominates	Shape
a chain carrying nothing	its own weight (arc)	catenary
a bridge with an overwhelming deck	the deck (horizontal)	parabola
both comparable	both	neither

Observation 5.2

The third row is the real bridge. Neither ideal shape——and this paper does not locate it.

Observation 5.3

Paper 183 wrote that one table has two correct answers. This is its counterpart——one shape with two correct equations.

Observation 5.4

The direction against Paper 198.

What is one and what is two	
Paper 198	one curve with two characters (straight / shortest)
This paper	two curves with one appearance

Observation 5.5

The direction is exactly reversed. 198 splits one into two; this paper has two looking like one——both obey the same discipline: appearance is not evidence.

Observation 5.6

A point to write honestly——this paper does not say which is structurally better. The difference between the curves and the merits of a design are different questions——the second is not entered.

Observation 5.7

One last thing—Galileo wrote that a hanging chain is a parabola. He was wrong, by 0.083 per cent—not an error the eye could correct, and with no cosh yet in existence there was nothing to correct it to. Writing the right shape needs the words for it to exist first.

6. Roll Call and Closing

1. Exact agreement to second order (Observation 1.1): The parabola is the second-order term.
2. 0.083292 per cent at $x = 0.1$ (Check 2.1): Nothing to see.
3. 82.925818 per cent at $x = 5$ (Check 2.1): About the same two curves.
4. The difference begins at fourth order (Observation 2.3): $x^4/24a^3$ against 0.
5. Not one character of the equation differs (Observation 3.2): Only the measure.
6. Arc length makes the ends heavy (Observation 3.3): $ds = \sqrt{1 + y'^2} dx$.
7. 0.082674 per cent on a real bridge (Observation 4.2): 8.3 cm on 1000 m.
8. Distinguishable and different are independent (Observation 4.5): Precision and definition.
9. Reversed against 198 (Observation 5.4): One into two, two into one.
10. Galileo's error was 0.083 per cent (Observation 5.7): The words did not exist.

Closing: A hanging chain and a suspension-bridge cable cannot be told apart by eye. Expanding cosh shows the leading term to be the parabola itself, so they agree exactly through second order—with $a = 1$ the difference is 0.083292 per cent at $x = 0.1$ and 82.925818 per cent at $x = 5$, and both figures are correct about the same two curves. “How different are they” is undetermined until you say where you are looking. The difference begins at fourth order—first, second and third all agree, and the parabola is the second-order Taylor polynomial of the catenary, not a curve fitted to it. And what divides them is not shape but measure—both come from the same balance $Hy'' = w$, and all that differs is whether w is constant against arc length ds or horizontal length dx . Constant along the arc makes the steep ends weigh more per horizontal foot, and so the catenary rises faster. So “a parabola close to a catenary” has it backwards—each is the exact answer to a different physics, and the approximation relation is something mathematics noticed afterwards. At the 1/10 sag of a real suspension bridge the maximum difference is 0.082674 per cent of the sag—8.3 cm on a 1000 m span, sometimes smaller than the construction tolerance. And they are still two different curves. One thing separates them—whether the suspended deck or the cable itself is heavier. A real bridge is neither ideal, and this paper does not locate it. Against Paper 198 the direction is exactly reversed—198 says one curve has two characters, this says two curves have one appearance, and both obey the same discipline: appearance is not evidence. One last thing—Galileo wrote that a

hanging chain is a parabola. He was wrong, by 0.083 per cent. Not an error the eye could correct, and with no cosh yet in existence there was nothing to correct it to. Writing the right shape needs the words for it to exist first.

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