

What Is Being Added Is Not the Values but the Precisions

— Fusing Three Sensors with $\sigma = (1, 2, 5)$ Gives **0.880451**, and Discarding the Worst Costs Only **1.5874** Per Cent — But If They Are Perfectly Correlated, Three Sensors Are One — [Paper 334]

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— *From thinking to syncing.* —

Abstract

Several sensors of differing accuracy are combined into one answer. This paper asks what is being added when that happens—the answer is not the values, but the reciprocals of the variances. No new mathematical theorem and no new law is claimed. First, fusing $\sigma = (1, 2, 5)$ gives **0.880451**, better than the best single sensor by 11.9549 per cent (Section 2). Second, this is the core of the paper. The weights go as the reciprocal of the variance, and the $\sigma = 5$ sensor carries only 3.1008 per cent (Section 3). Third, so discarding the worst sensor costs only **1.5874** per cent (Section 3). Fourth, n sensors of equal accuracy give $1/\sqrt{n}$, exactly 0.500000 at $n = 4$ (Section 2). Fifth, correlation erases the benefit. Two sensors at $\rho = 0.99$ give 0.997497, effectively one sensor (Section 4). Sixth, the separator is whether the errors are independent or correlated (Section 5).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed——inverse-variance weighting, the best linear unbiased estimator, and the variance formula under correlation are all standard. We do not build estimation theory——the general theory of maximum likelihood and of Bayesian estimation is not entered. Only one formula, and what that formula ignores. We do not implement a Kalman filter——sequential estimation with time evolution is not entered. Only fusion at a single instant. It is mentioned and no more. We do not treat outliers——robust statistics and the detection of failed sensors are not entered. Every sensor is taken to work to specification. We do not treat bias——each sensor is taken to be unbiased; the combination of systematic errors is not the subject. We design no sensor——the physics of individual sensors is not entered. Relation to earlier papers: Paper 322 showed one and the same sensor error reaching position by three different powers——its scope note states explicitly that Kalman filtering is not treated and that fusion with external observations is not entered, and this paper takes that seat. Where 322 followed one error to its destinations, this paper looks at how several errors are combined. Paper 317 showed a significance level to be a promise per test——there more trials make things worse; here more sensors make things better. Both rest on independence. Paper 300 showed whether two things share a root is decidable——the $1/\sqrt{n}$ here and the multiplicity of Paper 317 come from the same assumption of independence. Paper 221 showed that changing the last bit moves an eigenvalue by 0.16——there error is amplified; here it is suppressed by averaging. What is added is giving the weights as percentages, giving in numbers the cost of discarding the worst sensor, showing correlation erasing the benefit in stages, and putting the separator on independence against correlation.

1. Notation and the Question

Setting: The same quantity x is measured independently by n sensors. Write x_i for the i th reading and σ_i for its standard deviation. Forming $\hat{x} = \sum w_i x_i$ with $\sum w_i = 1$, the weights that minimise the variance of $\hat{x}$ are

$$w_i = \frac{1/\sigma_i^2}{\sum_j 1/\sigma_j^2}, \quad \frac{1}{\sigma_{\text{fused}}^2} = \sum_i \frac{1}{\sigma_i^2}.$$

Observation 1.1

Look at the form of the second equation. What adds is neither σ nor σ^2 but $1/\sigma^2$ ——the reciprocal of the variance is the quantity that adds cleanly.

Observation 1.2

$1/\sigma^2$ has a name. In statistics it is called the precision——so what is being added is neither the readings nor the errors, but the precisions.

Observation 1.3

The way of measuring is fixed first. Every sensor is unbiased and the errors are independent—these two assumptions become the separator later.

Question: When several sensors of differing accuracy are combined, what counts and by how much?

2. Three Sensors Fuse to 0.880451

Calculation 2.1: For three sensors with $\sigma = (1, 2, 5)$ (computed in this paper).

Quantity	Value
$\sum 1/\sigma_i^2$	$1 + 0.25 + 0.04 = 1.29$
σ after fusion	0.880451
Best single sensor ($\sigma = 1$)	1.000000
Improvement from fusing	11.9549 per cent

Observation 2.2

Fusion is always better than the best single sensor. However bad the sensor added, $\sum 1/\sigma_i^2$ only grows—under this formula, adding cannot make things worse.

Observation 2.3

Yet the improvement is only 11.9549 per cent. With three times the sensors—because the second and third are too poor.

Calculation 2.4: For n sensors of equal accuracy (computed in this paper).

Count n	σ after fusion	Improvement
1	1.000000	—
2	0.707107	29.29 per cent
4	0.500000	50.00 per cent
9	0.333333	66.67 per cent
100	0.100000	90.00 per cent

Observation 2.5

At $n = 4$ it is exactly a half. $1/\sqrt{4} = 0.500000$ —halving the error takes four sensors; a tenth of it takes a hundred.

Observation 2.6

This is precisely the standard error of the mean, $\sigma/\sqrt{n}$. 0.333333 at $n = 9$ is $1/3$ —“how many sensors” and “how many measurements” are not distinguished by this formula.

3. The Core: Weights Go as the Reciprocal of Variance, and Poor Sensors Carry Almost Nothing

Calculation 3.1: The weights for $\sigma = (1, 2, 5)$ as percentages (computed in this paper).

Sensor	$1/\sigma^2$	Weight
$\sigma = 1$	1.0000	77.5194 per cent
$\sigma = 2$	0.2500	19.3798 per cent
$\sigma = 5$	0.0400	3.1008 per cent

Observation 3.2

This is the core of the paper. A sensor five times worse in σ carries not a fifth of the weight but a twenty-fifth—the square makes poor sensors ignored quickly.

Calculation 3.3: What happens on discarding the worst ($\sigma = 5$) (computed in this paper).

Configuration	σ after fusion
All three	0.880451
Discard $\sigma = 5$	0.894427
Degradation	1.5874 per cent

Observation 3.4

Discarding it costs only 1.5874 per cent. One of three sensors removed entirely, for that—its wiring, its power, its calibration and its capacity to fail were being bought for 1.5874 per cent of accuracy.

Observation 3.5

Read the other way, adding a poor sensor does no harm. Its weight shrinks by itself—“do not mix in low-accuracy sensors” is, under this formula, wrong. But only when σ_i is known correctly.

Calculation 3.6: What if σ_i is wrong? Suppose the $\sigma = 5$ sensor declares itself to be $\sigma = 1$ (computed in this paper).

	Weight of the third	σ claimed after fusion	Actual σ
Correct declaration	3.1008 per cent	0.880451	0.880451
False declaration	44.4444 per cent	0.666667	2.277100

Observation 3.7

The third sensor's weight leaps from 3.1008 to **44.4444** per cent, a factor of 14.33. And the fused σ is claimed to be 0.666667 while it is actually **2.277100**—worse than claimed by a factor of 3.4157.

Observation 3.8

It is worse than the 0.880451 obtained by doing it correctly. 2.277100 is 2.59 times that—a false declaration makes things worse than never having added the sensor at all. The formula protects you only while each sensor declares its own badness honestly.

Observation 3.9

This takes the seat Paper 322 left open. Paper 322 followed one sensor error to position by three separate powers, and stated in its scope note that fusion with external observations was not entered—322 follows where an error goes; this paper looks at how errors are combined.

4. Correlation Erases the Benefit

Setting: Sections 2 and 3 rested on the errors being independent. For two sensors with $\sigma = 1$ correlated at ρ , the variance of the plain average is $(1 + \rho)/2$.

Calculation 4.1: Move ρ (computed in this paper).

Correlation ρ	σ after fusion	Improvement over one sensor
0.00	0.707107	29.29 per cent
0.50	0.866025	13.40 per cent
0.90	0.974679	2.53 per cent
0.99	0.997497	0.25 per cent
1.00	1.000000	0.00 per cent

Observation 4.2

At $\rho = 1$ the benefit vanishes completely. If two sensors carry the same error, the second adds no information at all—the count is 2 and the effective count is 1.

Observation 4.3

Even at $\rho = 0.9$ the improvement is already down to 2.53 per cent. Against 29.29 per cent when independent——high correlation destroys the point of adding sensors before you get there.

Observation 4.4

This is the dangerous place. The formula returns $1/\sqrt{n}$ without ever looking at ρ ——assume independence, raise n , and you report an accuracy that was never achieved.

Observation 4.5

Correlation creeps in most easily when identical sensors are used. The same thermal drift, the same supply noise, the same calibration habit——the easiest way to add sensors, buying more of the same, is also the easiest way to add correlation.

Observation 4.6

This makes a pair with Paper 317. Paper 317 wrote that a significance level is a promise per test and not per study——there, repeating independent trials piles up false positives; here, repeating independent measurements reduces error. One assumption of independence creates a danger in one place and a benefit in the other.

5. The Separator: Are the Errors Independent or Correlated?

Observation 5.1

The separator——whether what increases with the count is independent information or copies of the same information.

	Effect of n sensors	Two sensors at $\sigma = 1$	What increases
Independent	$1/\sqrt{n}$	0.707107	information
Perfectly correlated	no change	1.000000	only the count

Observation 5.2

The separator is not on the side of sensor accuracy. Every sensor has $\sigma = 1$ and yet ρ alone makes the result 0.707107 or 1.000000——a datasheet gives σ ; it does not give ρ .

Observation 5.3

ρ cannot be measured on one sensor. It appears only when two are run together and compared——however carefully each is calibrated alone, ρ stays unknown.

Observation 5.4

To be stated honestly——correlation is not necessarily harmful. Negative ρ beats independence——two sensors at $\rho = -1$ would give variance 0. Such a pair is hard to build, but the formula says so. It is not correlation that hurts but positive correlation.

Observation 5.5

The separator can be moved. Mixing in a sensor of a different principle lowers ρ —adding one less accurate device of another kind can beat buying a second of the same. Lowering σ and lowering ρ are separate investments.

Observation 5.6

One last thing—what this formula answers is not “which one do I believe”. No sensor is thrown away and none is believed outright; each is given a seat in proportion to its declared badness—it is an allocation and not a selection, and the ground of the allocation is not the readings but their scatter.

6. The Census and the Close

1. Three sensors fuse to **0.880451** (Calculation 2.1): **11.9549** per cent better than the best.
2. Adding cannot make it worse (Observation 2.2): $\sum 1/\sigma^2$ only grows.
3. Exactly **0.500000** at $n = 4$ (Calculation 2.4): **four sensors to halve it**.
4. The same formula as the standard error (Observation 2.6): sensors and repeats are not distinguished.
5. The $\sigma = 5$ sensor weighs **3.1008** per cent (Calculation 3.1): **a twenty-fifth, not a fifth**.
6. Discarding the worst costs **1.5874** per cent (Calculation 3.3): **a whole sensor removed**.
7. A false declaration claims **0.666667** and delivers **2.277100** (Calculation 3.6): **worse than discarding it**.
8. **0.997497** at $\rho = 0.99$ (Calculation 4.1): **effectively one sensor**.
9. Only **2.53** per cent even at $\rho = 0.9$ (Observation 4.3): **29.29 if independent**.
10. ρ is not on the datasheet (Observation 5.2): **and cannot be measured alone**.

Closing: When sensors of differing accuracy are combined, what is added is neither the values nor the errors but the reciprocals of the variances, that is, the precisions—the form $1/\sigma_{\text{fused}}^2 = \sum 1/\sigma_i^2$ says so. Three sensors with $\sigma = (1, 2, 5)$ fuse to **0.880451**, better than the best single one by **11.9549** per cent. However bad the sensor added, the sum only grows, so under this formula adding cannot make things worse. And the weights go as the square—a sensor five times worse in σ carries **3.1008** per cent, a twenty-fifth and not a fifth. So discarding the worst one entirely costs only **1.5874** per cent—its wiring, its power, its calibration and its capacity to fail were being bought for **1.5874** per cent of accuracy. But that protection depends on honest declarations—let the $\sigma = 5$ sensor declare itself $\sigma = 1$ and its weight leaps to **44.4444** per cent; the fused σ is claimed to be **0.666667** while it is actually

2.277100, worse than claimed by **3.4157** and worse than the correct **0.880451** by **2.59**. A false declaration is worse than never having added the sensor. n sensors of equal accuracy give $1/\sqrt{n}$, exactly **0.500000** at $n = 4$ —the same formula as the standard error of the mean, so “how many sensors” and “how many measurements” are not distinguished. All of which stands on the errors being independent—two sensors at $\sigma = 1$ give **0.707107** at $\rho = 0$, **0.974679** at $\rho = 0.9$, **0.997497** at $\rho = 0.99$ and **1.000000** at $\rho = 1$. At $\rho = 1$ the benefit vanishes entirely; the second sensor adds no information. Even at $\rho = 0.9$ the improvement is down to **2.53** per cent. This is the dangerous place—the formula returns $1/\sqrt{n}$ without looking at ρ , so assuming independence and raising n means reporting an accuracy never achieved. And the easiest way to add sensors, buying more of the same, is the easiest way to add correlation. One thing separates them—whether what increases is independent information or copies of the same. The separator is not on the side of accuracy—every sensor has $\sigma = 1$ and ρ alone makes the result **0.707107** or **1.000000**; a datasheet gives σ and not ρ , and ρ cannot be measured on one sensor. To be said honestly—correlation is not itself the harm. Negative ρ beats independence and $\rho = -1$ would give variance zero. It is positive correlation that hurts. Mixing in a sensor of a different principle lowers ρ , so adding one less accurate device of another kind can beat buying a second of the same. Lowering σ and lowering ρ are separate investments. One last thing—what this formula answers is not “which one do I believe”. No sensor is thrown away and none is believed outright; each is given a seat in proportion to its declared badness. It is an allocation and not a selection, and the ground of the allocation is not the readings but their scatter.

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