

The Round Trip Changes the Exponent

— Communication Goes as $1/r^2$ and Radar as $1/r^4$ — Doubling the Range Costs **4** Times the Power for One and **16** for the Other — Doubling the Power Buys Only **18.9207** Per Cent More Range — [Paper 335]

Yuuki YAMAGISHI (山岸 勇輝)

Yamagishi Complexity Torus Lab, Japan

September 2026

— *From thinking to syncing.* —

Abstract

Radio is transmitted and has to reach something far away. This paper asks what fixes the exponent on distance—the answer is whether the trip is one way or two, and not the frequency nor any property of space. No new mathematical theorem and no new law is claimed. First, doubling the range costs **4** times the power for communication and **16** for radar (Section 2). Second, this is the core of the paper. The exponent goes from two to four because the same spreading happens twice (Section 3). Third, so power does not buy range. Doubling the power adds only **18.9207** per cent (Section 2). Fourth, the way to buy range is aperture, not power. Four times the aperture area doubles the range (Section 4). Fifth, the same exponent works on the side that hides. Cutting the radar cross-section to a hundredth cuts detection range only to **0.316228** (Section 4). Sixth, the separator is one way against two (Section 5).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed—the Friis transmission formula, the radar equation, and the relation between aperture and gain are all standard. We do not build propagation theory—atmospheric attenuation, ground reflection, diffraction and refraction are not entered. Only free space. We do not treat noise—receiver noise figures, integration gain, and the statistics of detection probability are not entered. They are mentioned and no more. We do not treat the physics of cross-section—computing a cross-section from shape and material is not entered. It is taken as given. We design no weapon—only where the exponent comes from is looked at. We do not treat bistatic geometry—configurations separating transmitter from receiver are not entered. Relation to earlier papers: Paper 322 showed one and the same sensor error reaching position by three different powers—there one cause splits into three exponents; here one mechanism splits by how many times it acts. Both count exponents rather than reading them. Paper 326 showed the square-cube law to be an identity and not a law— $1/r^2$ is the same, coming from the geometry of a sphere having area $4\pi r^2$. It is not a property of radio. Paper 321 showed one length fixing both how thick an atmosphere is and whether it stays—one quantity settling two things, and here one spreading acting twice. Paper 300 showed whether two things share a root is decidable—communication and radar share a root by that test. Only the count differs. What is added is tabulating required power and achievable range in both directions, comparing the aperture route against the power route in one table, giving in numbers what a reduced cross-section buys, and putting the separator on one way against two.

1. Notation and the Question

Setting: Radio is sent to something at distance r . Two cases stand side by side.

Case	How power arrives	Example
One way	$P_r \propto 1/r^2$	communication, broadcast, satellite downlink
Round trip	$P_r \propto 1/r^4$	radar, sonar, LiDAR

Observation 1.1

$1/r^2$ is not a property of radio. A sphere of radius r has area $4\pi r^2$, so the same power thins over an area going as r^2 —this is geometry, and stands where Paper 326 stood on the square-cube law.

Observation 1.2

On a round trip the same thinning happens twice. It thins by $1/r^2$ on the way out, scatters at the target, and thins by $1/r^2$ again coming back— $1/r^2 \times 1/r^2 = 1/r^4$.

Observation 1.3

The way of measuring is fixed first. Free space, isotropic scattering, no atmospheric loss—real systems are worse, so these numbers are an upper bound.

Question: When the exponent goes from two to four, what changes and by how much?

2. Doubling the Range Costs 16 Times the Power

Calculation 2.1: The power needed to multiply the range by k (computed in this paper).

Range factor k	Communication k^2	Radar k^4
2	4	16
3	9	81
10	100	10000
100	10000	100000000

Observation 2.2

Ten times the range costs ten thousand times the power. Where communication would need a hundred—a factor of 100 between them is two more decades of power.

Calculation 2.3: The same seen from the other side: what range does k times the power buy (computed in this paper)?

Power factor k	Communication $k^{1/2}$	Radar $k^{1/4}$
2	1.414214 (41.4214 per cent)	1.189207 (18.9207 per cent)
4	2.000000	1.414214
10	3.162278	1.778279
16	4.000000	2.000000

Observation 2.4

Double the power and radar gains only 18.9207 per cent. Doubling power is a large matter for the hardware——cooling and supply nearly double, and what arrives is under a fifth.

Observation 2.5

The same doubling gives communication 41.4214 per cent. Exactly twice the gain——one and the same investment returns more than twice as much on a one-way path.

Observation 2.6

$k^{1/4}$ stops paying quickly. Ten times the power gives 1.778279, not even a doubling——buying range with power runs into a fourth root.

3. The Core: The Exponent Counts How Many Times the Same Spreading Happens

Observation 3.1

This is the core of the paper. Between $1/r^2$ and $1/r^4$ there is no new physics whatever——it is the same spherical spreading, once or twice.

Observation 3.2

So only the exponents 2 and 4 occur. There is no half of a one-way trip——no free-space system in this counting gives $1/r^3$.

Observation 3.3

Apply the criterion of Paper 300.

Criterion	Result
Is there a correspondence preserving an identity	Yes. A round trip is one way done twice
Does the same quantity appear	Yes. The sphere's $4\pi r^2$ in both
Is the relation kept as variables move	Yes. However r moves, the exponents stay 2 and 4

Observation 3.4

All three rows point to a common root. Communication and radar look like different technologies——on paper they are the same thing done once or twice.

Observation 3.5

Where this stands relative to Paper 322. Paper 322 wrote that one sensor error reaches position by three different powers——there one cause splits into three; here one mechanism splits by a count. Both count how many exponents stand, rather than reading their values.

Observation 3.6

To be stated honestly——real systems show exponents other than four. Near the ground, interference with the reflected ray can give $1/r^4$ on a one-way path, and atmospheric absorption falls exponentially——what is claimed here is that counting geometry alone in free space gives 2 and 4, not that observed exponents are always 2 or 4.

4. Range Is Bought with Aperture, Not with Power

Setting: Antenna gain G is proportional to aperture area A . In radar it acts on transmission and reception alike, so range goes as $(G_t G_r)^{1/4}$.

Calculation 4.1: Range for k times the aperture (computed in this paper).

Aperture area factor	Range factor	The same range bought with power
2	1.414214	4 times the power
4	2.000000	16 times the power
16	4.000000	256 times the power

Observation 4.2

Aperture acts as a square root, power as a fourth root. Because the aperture counts twice, on transmit and on receive—four times the area (twice the diameter) doubles the range, and doing that with power takes 16 times.

Observation 4.3

This is why radars are large. An instrument that must see far grows its dish rather than its transmitter—power at a fourth root, aperture at a square root: the same doubling has two different prices.

Calculation 4.4: The same exponent works on the side that hides. Detection range against reduced cross-section (computed in this paper).

Cross-section factor	Detection range factor	Reduction
1/2	0.840896	15.91 per cent
1/10	0.562341	43.77 per cent
1/100	0.316228	68.38 per cent
1/1000	0.177828	82.22 per cent

Observation 4.5

Cut the cross-section to a hundredth and the range falls only to a third. A factor of 0.316228—the fourth root discounts the effort of hiding by exactly as much.

Observation 4.6

Communication does not behave this way. On a one-way $1/r^2$ path, a hundredth gives 0.100000, a tenth—the very same “one hundredth” splits into a third and a tenth on the exponent alone.

Observation 4.7

The asymmetry acts identically on both sides. The seeker needs 16 times the power to double the range; the hider needs a sixteenth of the cross-section to halve it—both efforts are discounted by a fourth root.

5. The Separator: One Way, or Two?

Observation 5.1

The separator—whether the spherical thinning was traversed once or twice.

	Exponent	Range from double power	Power to double range
One way (communication)	$1/r^2$	41.4214 per cent	4 times
Round trip (radar)	$1/r^4$	18.9207 per cent	16 times

Observation 5.2

The separator is neither in the hardware nor in the frequency. The same transmitter, the same antenna, the same frequency—and the exponent changes according to whether the far end returns the signal or receives it.

Observation 5.3

The separator can be moved. Put a transponder on the target and the round trip becomes two one-way legs, and $1/r^4$ returns to $1/r^2$ —this is why an aircraft transponder helps at long range, and why primary and secondary radar carry different exponents.

Observation 5.4

The same separator works for sound and for light. Sonar and LiDAR make round trips and so go as $1/r^4$ —what sets the exponent is not the kind of wave but the fact of returning.

Observation 5.5

To be stated honestly—a target that is not a point does not give $1/r^4$. Weather radar, whose target volume grows as r^2 , tends effectively toward $1/r^2$ —the exponent depends on how the target extends. A point target is what was treated here.

Observation 5.6

One last thing—calling radar “an instrument that sees far” hides the exponent. At equal power, radar reaches only the square root of the distance communication reaches—it is not an instrument that sees far; it is an instrument for which seeing far is squarely harder.

6. The Census and the Close

1. **16 times the power to double the range** (Calculation 2.1): **communication needs 4.**
2. **Ten thousand times for ten times the range** (Observation 2.2): **communication needs a hundred.**
3. **Double the power gives 18.9207 per cent** (Calculation 2.3): **under a fifth.**
4. **The same doubling gives communication 41.4214 per cent** (Observation 2.5): **more than twice the return.**
5. **$1/r^2$ is geometry, not radio** (Observation 1.1): **the area of a sphere.**
6. **Only 2 and 4 occur** (Observation 3.2): **there is no half a trip.**
7. **Four times the aperture doubles the range** (Calculation 4.1): **16 times with power.**

8. A hundredth of cross-section gives **0.316228** (Calculation 4.4): **only a third.**
9. Communication would give **0.100000** (Observation 4.6): **on the exponent alone.**
10. A transponder restores $1/r^2$ (Observation 5.3): **the separator can be moved.**

Closing: That radio thins as $1/r^2$ over a sphere is not a property of radio but the geometry of a sphere of radius r having area $4\pi r^2$ —it stands where Paper 326 stood on the square-cube law. And on a round trip the same thinning happens twice, giving $1/r^4$ —between $1/r^2$ and $1/r^4$ there is no new physics whatever. The price is counted in the exponent—doubling the range costs 4 times the power for communication and **16** for radar, and seeing ten times as far costs ten thousand. Read the other way, doubling the power buys radar only **18.9207** per cent—the same doubling buys communication **41.4214** per cent, so one investment returns more than twice as much. Ten times the power gives **1.778279**, not even a doubling. So range is bought with aperture, not with power—aperture counts twice and so acts as a square root, while power acts as a fourth root. Four times the area, twice the diameter, doubles the range, and doing that with power takes 16 times. That is why an instrument meant to see far grows its dish and not its transmitter. And the same exponent discounts the side that hides, by exactly as much—cutting the radar cross-section to **ahundredth** cuts detection range only to **0.316228**, a third. On a one-way $1/r^2$ path the same hundredth gives **0.100000**, a tenth. Seeker and hider alike have their efforts taken to a fourth root. One thing separates them—whether the spherical thinning was traversed once or twice. The separator is neither in the hardware nor in the frequency—the same transmitter and frequency give different exponents according to whether the far end returns the signal or receives it. And it can be moved—a transponder on the target turns the round trip into two one-way legs and restores $1/r^2$. Sonar and LiDAR go as $1/r^4$ too: what sets the exponent is not the kind of wave but the fact of returning. To be said honestly—what was treated here is a point target in free space. A weather radar, whose target volume grows as r^2 , tends effectively toward $1/r^2$, and propagation near the ground or through absorbing air falls off otherwise. Only the geometry was counted. One last thing—calling radar “an instrument that sees far” hides the exponent. At equal power it reaches only the square root of what communication reaches. It is not an instrument that sees far; it is an instrument for which seeing far is squarely harder.

References

1. Friis, H. T. (1946). A Note on a Simple Transmission Formula. *Proceedings of the IRE*, 34(5).
2. Watson-Watt, R. A. (1945). Radar in War and in Peace. *Nature*, 156.
3. Skolnik, M. I. (2008). *Radar Handbook*, 3rd edition. New York: McGraw-Hill.
4. Balanis, C. A. (2016). *Antenna Theory: Analysis and Design*, 4th edition. Hoboken: Wiley.
5. Richards, M. A., Scheer, J. A., Holm, W. A. (2010). *Principles of Modern Radar: Basic Principles*. Raleigh: SciTech.

6. Yamagishi, Y. (2026). *The Three Hundredth Census: Whether Two Things Share a Root Is Decidable* [Paper 300]. Zenodo. 10.5281/zenodo.22171914
7. Yamagishi, Y. (2026). *The Length That Sets How Thick an Atmosphere Is Also Sets Whether It Stays* [Paper 321]. Zenodo. 10.5281/zenodo.22209154
8. Yamagishi, Y. (2026). *One and the Same Sensor Error Reaches Position by Three Different Powers* [Paper 322]. Zenodo. 10.5281/zenodo.22209240
9. Yamagishi, Y. (2026). *The Square-Cube “Law” Is Not a Law but an Identity* [Paper 326]. Zenodo. 10.5281/zenodo.22209992

— *From thinking to syncing.* —

Yuuki YAMAGISHI (山岸 勇輝)

© 2026 Yamagishi Complexity Torus Lab, Japan · Licensed under Creative Commons Attribution 4.0 International (CC BY 4.0)