

The Number 32768 Inside a Wristwatch Was Not Chosen by Physics

— It Is 2^{15} — Fifteen Stages of Division Give Exactly 1 Hz, While 32000 Hz Leaves a Remainder of 125 —
Keeping to One Second a Day Demands **11.5741** ppm — [Paper 336]

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— *From thinking to syncing.* —

Abstract

Almost every wristwatch contains a crystal vibrating at 32768 Hz. This paper asks what fixed that number—the answer is not the physics of quartz but divisibility in binary. No new mathematical theorem and no new law is claimed. First, $32768 = 2^{15}$, and fifteen stages of division give exactly 1 Hz (Section 2). Second, this is the core of the paper. The neighbouring 32000 Hz divides by two only eight times and leaves 125 (Section 2). Third, three demands pull in different directions. Low frequency for power, high frequency for resolution, and a power of two for the divider (Section 3). Fourth, only the last of the three picks one point out of the band (Section 3). Fifth, keeping to one second a day demands **11.5741** ppm. A 20 ppm crystal drifts 1.7280 seconds a day and 631.1520 a year (Section 4). Sixth, the separator is whether the number was fixed by nature or by the implementation (Section 5).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed——tuning-fork quartz resonators, binary division, and the ppm notation for frequency tolerance are all standard. We do not build the physics of quartz——piezoelectricity, tuning-fork vibration modes, and the parabolic temperature characteristic are not entered. They are mentioned and no more. We design no circuit——oscillator and divider implementation and load-capacitance trimming are not entered. We do not treat time standards——atomic clocks, Coordinated Universal Time and leap seconds are not entered. They belong to Paper 319. We do not adjudicate history——who adopted this frequency and when is not entered. We do not treat temperature compensation——ovens and compensated implementations are not the subject. Relation to earlier papers: Paper 306 showed how many “base” units there are to be a promise——32768 belongs to the same category. But the promise in 306 was “into how many”, while the promise here is “at which value”. Paper 319 showed a calendar to be three approximations stacked——there human numbers were fitted to a natural period; here a natural vibration is fitted to a human number. The direction is reversed. Paper 95 separated convention from fact——32768 stands on the side of convention, while a crystal vibrating at 32768 Hz stands on the side of fact. One number stands on both sides. Paper 316 showed the same integer returning from three different counts——here one integer sits at the crossing of three different demands. What is added is actually dividing the neighbouring candidates to show they do not come out, separating which of the three demands fixes the value to a point, matching ppm to drift per day and per year, and putting the separator on nature against implementation.

1. Notation and the Question

Setting: A watch must manufacture one second. A crystal is made to vibrate, its vibrations are counted, and 1 Hz comes out. The counting is binary division——each flip-flop stage halves the frequency.

Observation 1.1

Division can only make halves. Making $1/3$ or $1/5$ takes extra circuitry——the cheapest thing is a number that halves down to exactly 1.

Observation 1.2

The frequency actually used is 32768 Hz. Nearly every wristwatch and many microcontroller sub-clocks carry this value——why not 32000, or 33000?

Observation 1.3

The way of measuring is fixed first.Frequency tolerance in ppm, a day of 86400 seconds, a year of 365.25 days——drift is proportional to tolerance, so multiplication suffices.

Question: Was this number fixed by the physics of quartz, or by the convenience of a circuit?

2. Halve Repeatedly and Only 32768 Lands on 1

Calculation 2.1: Halve 32768 repeatedly (computed in this paper).

Stage	Frequency [Hz]	Stage	Frequency [Hz]
0	32768	8	128
1	16384	9	64
2	8192	10	32
3	4096	11	16
4	2048	12	8
5	1024	13	4
6	512	14	2
7	256	15	1

Observation 2.2

Exactly fifteen stages give 1 Hz.Because $32768 = 2^{15}$ ——fifteen flip-flops in a row, with no division and no remainder.

Calculation 2.3: The same for the neighbouring candidates (computed in this paper).

Frequency [Hz]	Times divisible by 2	Remainder	Reaches 1 Hz
32000	8	125	no
32767	0	32767	no
32768	15	1	yes
33000	3	4125	no
16384	14	1	yes
65536	16	1	yes

Observation 2.4

32000 Hz looks round and does not divide. $32000 = 2^8 \times 125$, and 125 remains——being round in decimal and dividing in binary are entirely separate things.

Observation 2.5

32767 is one less than 32768 and does not divide even once. It is odd——a neighbour one apart sits at the opposite extreme in binary.

Observation 2.6

But 16384 and 65536 divide as well. These are 2^{14} and 2^{16} ——so “divides in binary” alone does not settle on 32768. Three candidates remain.

3. The Core: Three Demands Pull, and Only the Last Fixes a Point

Setting: Choosing a frequency, there are three demands.

Demand	Direction	Reason
Power consumption	lower is better	oscillation and division cost roughly go as frequency
Resolution and stability	higher is better	finer ticks, and a smaller, steadier crystal
Divides in binary	be a power of two	the divider needs no extra circuitry

Observation 3.1

This is the core of the paper. The first two settle only a band—somewhere in the tens of kilohertz, and no more. What fixes a point is the third alone.

Calculation 3.2: Compare the three remaining candidates (computed in this paper).

Frequency	Divider stages	One tick [ms]	Standing
$2^{14} = 16384$	14	0.061035	better on power, half the resolution
$2^{15} = 32768$	15	0.030518	the value adopted
$2^{16} = 65536$	16	0.015259	better on resolution, twice the power

Observation 3.3

32768 stands in the middle of the binary staircase. A step down eases the power, a step up raises the resolution—less the answer to an optimisation than the middle of a staircase with only three steps on it.

Observation 3.4

And 32768 Hz sits above the upper edge of hearing. A factor of 1.6384 above the usual 20 kHz—had $2^{14} = 16384$ Hz been chosen, it would have fallen inside the audible range.

Observation 3.5

To be stated honestly——this paper does not claim 32768 is optimal. The weights on the three demands depend on the battery and on the application——what is claimed is only that what narrowed the value to a point was the binary demand. Why 2^{15} rather than 2^{14} does not follow from these numbers.

Observation 3.6

Apply the division of Paper 95. “A crystal vibrates at 32768 Hz” is a fact; “choose 32768 Hz” is a convention——one number stands on both sides. And the convention was fixed first, with the fact (the crystal’s dimensions) made to follow.

4. How Far It Drifts

Calculation 4.1: Frequency tolerance against clock drift (computed in this paper).

Tolerance [ppm]	Drift per day [s]	Drift per year [s]
20	1.7280	631.1520
5	0.4320	157.7880
0.5	0.0432	15.7788

Observation 4.2

A common 20 ppm crystal drifts 631.1520 seconds a year. About 10.5 minutes——however exactly the divider is 2^{15} , the crystal’s own tolerance survives untouched.

Calculation 4.3: Read the other way, what tolerance keeps a day to one second (computed in this paper)?

$$\frac{1 \text{ second}}{86400 \text{ seconds}} \times 10^6 = 11.5741 \text{ ppm}$$

Observation 4.4

11.5741 ppm.A common 20 ppm part does not reach it—the claim “within a second a day” demands a selected 5 ppm part or temperature compensation.

Observation 4.5

Binary plays no part here at all.That the divider comes out exactly at 2^{15} and how many ppm the crystal drifts are wholly separate matters—a property of integers gives no accuracy whatever.

Observation 4.6

Here is the parting from Paper 319.Paper 319 wrote that a calendar is three approximations stacked—there the natural period (a year) was not a divisible number, so the human side stacked approximations. Here it runs the other way: the human side fixed the clean number 2^{15} first and made nature (the crystal) fit it.

5. The Separator: Was the Number Fixed by Nature or by the Implementation?

Observation 5.1

The separator—whether the number is one that turned out that way on measurement, or one placed there because it was convenient.

	Example	Fixed by	Can it be changed
Fixed by nature	a year of 365.2422 days	nature	no
Fixed by implementation	32768 Hz	circuit convenience	yes

Observation 5.2

The clue is how clean the number is. 365.2422 is all remainder, and 32768 is exactly 2^{15} ——nature almost never hands over an exact power of two.

Observation 5.3

The separator can be moved. Allow extra circuitry in the divider and any frequency can produce 1 Hz——32768 is needed because one wants to build cheaply, not as a matter of principle. Where power is plentiful, other frequencies are in fact used.

Observation 5.4

To be stated honestly——the crystal side has constraints too. A tuning-fork crystal's frequency is set by its dimensions, so the band of tens of kilohertz is fixed by physics——what is claimed is that binary picked one point inside the band, not that the band itself is arbitrary.

Observation 5.5

Where this stands relative to Paper 306. Paper 306 wrote that how many “base” units there are is a promise——that promise was “into how many”, while this one is “at which value”. Under one word “promise”, fixing a count and fixing a value are different acts.

Observation 5.6

One last thing——this is one of the most widely carried powers of two in ordinary life. Most people wear it on a wrist and nobody thinks of it as 2^{15} ——and once seen, the most exactly determined thing inside the watch is this integer, and not the frequency of the crystal.

6. The Census and the Close

1. $32768 = 2^{15}$ (Calculation 2.1): fifteen stages give exactly 1 Hz.
2. 32000 divides eight times and leaves 125 (Calculation 2.3): decimal roundness is other.

3. **32767 does not divide once** (Observation 2.5): **opposite extreme, one apart.**
4. **16384 and 65536 divide too** (Observation 2.6): **binary alone does not settle it.**
5. **Two of the three demands settle only a band** (Observation 3.1): **binary fixes the point.**
6. **One tick is 0.030518 ms** (Calculation 3.2): **twice that at 2^{14} , half at 2^{16} .**
7. **1.6384 above the edge of hearing** (Observation 3.4): **2^{14} would have been audible.**
8. **631.1520 seconds a year at 20 ppm** (Calculation 4.1): **about 10.5 minutes.**
9. **11.5741 ppm for one second a day** (Calculation 4.3): **20 ppm does not reach.**
10. **Binary gives no accuracy at all** (Observation 4.5): **integers and tolerance are separate.**

Closing: Nearly every wristwatch holds a crystal at 32768 Hz, and that number is 2^{15} —fifteen flip-flop stages give exactly 1 Hz, with no division and no remainder. The neighbouring 32000 Hz looks round in decimal but is $2^8 \times 125$ and leaves **125**. One less, 32767 is odd and does not divide even once. Yet dividing in binary does not by itself settle on this value— $2^{14} = 16384$ and $2^{16} = 65536$ divide as well. Three demands act on the choice: power wants it low, resolution and stability want it high, and binary wants a power of two. The first two settle only a band, somewhere in the tens of kilohertz; what narrows the value to a point is the third alone. 32768 stands in the middle of that binary staircase—a step down eases power, a step up raises resolution. One tick is 0.030518 milliseconds, and 32768 Hz sits 1.6384 above the upper edge of hearing—had 2^{14} been chosen it would have fallen inside the audible range. This paper does not claim 32768 is optimal; it claims that what fixed the value to a point was the binary demand. And binary gives no accuracy whatever—however exactly the divider is 2^{15} , the crystal's own tolerance survives. A 20 ppm crystal drifts **1.7280** seconds a day and **631.1520** a year, about 10.5 minutes. Keeping to one second a day demands **11.5741** ppm, which a 20 ppm part does not reach. A property of integers and a magnitude of drift are wholly separate matters. One thing separates them—whether the number turned out that way on measurement, or was placed there for convenience. The clue is how clean it is—a year of 365.2422 days is all remainder, and 32768 is exactly 2^{15} ; nature almost never hands over an exact power of two. Paper 319 wrote that a calendar is three approximations stacked—there the natural period was not divisible so the human side stacked approximations. Here it runs the other way: the human side fixed the clean number 2^{15} first and made the crystal's dimensions follow. To be said honestly—a tuning-fork crystal's frequency is set by its dimensions, so the band of tens of kilohertz is fixed by physics. What is claimed is that binary picked one point inside the band, not that the band itself is arbitrary. One last thing—this is one of the most widely carried powers of two in ordinary life. Most people wear it on a wrist and nobody thinks of it as 2^{15} . And once seen, the most exactly determined thing inside the watch is this integer, and not the frequency of the crystal.

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