

The Shorter Path Is Not the One a Navigator Steers

— Tokyo to London, the Rhumb Line Is **18.1512** Per Cent Longer Than the Great Circle, While Tokyo to Sydney Differs by **0.0043** Per Cent — Halfway Round the World at **70** Degrees North It Is **53.9091** — [Paper 340]

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— *From thinking to syncing.* —

Abstract

On a sphere the shortest path between two points is a great circle. Navigators nonetheless chose the longer one. This paper asks how the longer path can be the one chosen—the answer is that what is held constant is not distance but heading. No new mathematical theorem and no new law is claimed. First, Tokyo to London the rhumb line is **18.1512** per cent longer than the great circle (Section 2). Second, from the same Tokyo, the route to Sydney differs by **0.0043** per cent (Section 2). Third, this is the core of the paper. What sets the difference is not distance but how far the route runs across latitude (Section 3). Fourth, along one meridian and along the equator the difference is exactly zero (Section 3). Fifth, halfway round the world at **70** degrees north the difference reaches **53.9091** per cent (Section 3). Sixth, the separator is whether heading is held constant or distance minimised (Section 5).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed——great-circle distance, the rhumb line (loxodrome), the Mercator projection, and spherical trigonometry are all standard.We do not build geodesy——the Earth is taken as a sphere of radius 6371.0088 km; geodesics on an ellipsoid (Vincenty) are not entered. Flattening changes things by an order of magnitude less than the differences here, though it matters in precise navigation.We do not teach navigation——voyage planning, the placing of waypoints, and corrections for current and wind are not entered. They are mentioned and no more.We do not discuss projections in general——properties of the Mercator projection are used, but no general theory of projections is entered.We do not adjudicate history——who first used the loxodrome and when is not entered.We do not treat aviation——real flight paths with upper winds and jet streams are not the subject.Relation to earlier papers: Paper 234 showed “distance” to be six things, one of which satisfies the triangle inequality backwards——its question was what may pass as a distance. This paper holds one distance fixed on one sphere and asks why the longer route is chosen. Not a question about definitions but about choosing.Paper 300 showed whether two things share a root is decidable——by that test the two routes here have distinct roots.Paper 205 showed the shortest enclosure is not always a disc——there too “shortest” departed from naive intuition.Paper 328 showed the same ellipse being three months out——both belong to the sea and sky the author began from.What is added is computing both for five real routes and tabulating the differences, identifying the two cases in which the difference is exactly zero, showing that east-west travel at high latitude reaches 53.9091 per cent, and putting the separator on what is held constant.

1. Notation and the Question

Setting: Two ways of drawing a path between two points on a sphere.

Name	Definition	How the helm is used
Great circle	the shortest path on the sphere	the heading changes continually
Rhumb line (loxodrome)	the line crossing every meridian at one angle	the heading is held constant

Observation 1.1

That the great circle is shortest is a theorem of spherical geometry.So a rhumb line is never shorter——when equality holds is half of the question here.

Observation 1.2

And still the sailing ships ran rhumb lines. Because one need only hold the compass needle on a single mark——the reason for choosing a path that is not shortest lies somewhere other than in its length.

Observation 1.3

The way of measuring is fixed first. The Earth is a sphere of radius $R = 6371.0088$ km; the great circle from the spherical law of cosines, the rhumb line as a straight line on the Mercator projection——flattening is ignored, being an order of magnitude below the subject.

Question: How much is lost, and when is nothing lost?

2. Two Lengths for Five Routes

Calculation 2.1: Compute both for five real routes (computed in this paper).

Route	Great circle [km]	Rhumb line [km]	Difference
Tokyo → Sydney	7825.83	7826.17	0.0043 per cent
Tokyo → Singapore	5311.23	5321.24	0.1884 per cent
Lisbon → New York	5422.04	5551.91	2.3952 per cent
Tokyo → San Francisco	8274.63	8730.02	5.5034 per cent
Tokyo → London	9558.57	11293.57	18.1512 per cent

Observation 2.2

Checks——the great circle Tokyo to San Francisco, 8274.63 km, agrees with the usual figure of about 8300 km to 0.31 per cent. And half the equator, πR , comes to 20015.1144 km, near half the defining 40000——the tool is tried on two known cases before the table is read.

Observation 2.3

The differences span more than four thousandfold, from 0.0043 to 18.1512 per cent. Several of these leave the same port——leave from one place and the price differs by decades according to where you are going.

Observation 2.4

The 18.1512 per cent Tokyo to London is 1735 km.11293.57 against 9558.57——running the rhumb line adds about the distance from Tokyo to Shanghai (1754.27 km).

Observation 2.5

Tokyo to Sydney has almost no difference.7825.83 against 7826.17, a matter of 0.34 km——run 7800 km and lose 340 metres.

3. The Core: What Decides Is Not Distance but Running Across Latitude

Setting: Order the table of Section 2 by distance and it does not order by difference. The longest route (Tokyo to London, 9558.57 km) does carry the largest difference, but Tokyo to Sydney, shorter than Tokyo to San Francisco, carries the smallest.

Observation 3.1

This is the core of the paper. What acts is not the total distance but how far the route runs east and west at high latitude——Tokyo to London has a great circle sweeping far north, and the rhumb line cuts straight across it in latitude, so the loss is large.

Calculation 3.2: Identify two cases where the difference is exactly zero (computed in this paper).

Case	Great circle [km]	Rhumb line [km]	Difference
One meridian (35°N → 35°S)	7783.655616	7783.655616	0
Along the equator (0° → 90°E)	10007.557221	10007.557221	0

Observation 3.3

In these two, the great circle is itself a line of constant heading. A meridian always points true north and the equator always true east—the two paths coincide only when the great circle happens also to be a rhumb line.

Observation 3.4

And among great circles on a sphere, only the meridians and the equator are rhumb lines. Every other great circle changes heading as one advances—so “no loss” is the exception and not the rule.

Calculation 3.5: Swing the other way and find the maximum (computed in this paper).

Route	Great circle [km]	Rhumb line [km]	Difference
Half the world at 70°N (0° → 180°)	4447.80	6845.57	53.9091 per cent

Observation 3.6

Here the rhumb line is 1.54 times the great circle. Running due east at 70 degrees north takes one halfway round the world along a parallel—while the great circle crosses near the pole and arrives far sooner.

Observation 3.7

A parallel is not a great circle, except the equator. The parallel at 70 degrees north looks like “straight east” and is curved on the sphere—holding a constant heading and going straight have become two different things.

4. The Mercator Projection Made the Rhumb Line a Straight Line

Setting: The reason the rhumb line held practice lies in the chart.

Observation 4.1

On the Mercator projection a rhumb line is exactly a straight line. Because latitude is stretched by $\ln \tan(\pi/4 + \varphi/2)$ —lay a ruler from origin to destination, read its slope, and that is the heading to hold.

Observation 4.2

On the same chart the great circle is a curve. The shortest path looks like a detour and the detour looks straight—the projection was deciding which path got chosen.

Observation 4.3

The price rises with latitude. Mercator stretching is $1/\cos \varphi$, which at 70 degrees north is 2.92—where the chart distorts most, the rhumb line loses most. One and the same $\cos \varphi$ acts on both.

Observation 4.4

Practice mixed the two. Divide the great circle into legs and sail each leg as a rhumb line—most of the loss goes and the heading stays constant within each leg. Not a choice between two but a question of how finely to divide.

Observation 4.5

To be stated honestly—the Earth is taken as a sphere here. The real Earth is flattened and its geodesics depart from spherical great circles by up to about 0.5 per cent—the differences here (up to 53.9091 per cent) are far larger so the conclusion stands, but for the 0.0043 per cent of Tokyo to Sydney the flattening is the larger effect.

5. The Separator: What Is Held Constant?

Observation 5.1

The separator——whether what is held constant along the route is the heading, or the property of being shortest.

	Held constant	The helm	On the chart
Great circle	being shortest	changed continually	a curve
Rhumb line	the heading	set once and left	a straight line

Observation 5.2

The separator is neither in the sea nor in the sphere. The same sea, the same two points, the same sphere——what makes the difference is what one keeps hold of aboard the ship.

Observation 5.3

The difference from Paper 234, stated. Paper 234 counted six “distances” and asked what may pass as one——here the distance is fixed to one. Under one and the same distance, why is the longer path chosen? Not a question about definitions but about choosing.

Observation 5.4

And this separator is still moving. Satellite navigation and autopilots have all but removed the labour of holding a heading, so the reason for choosing a rhumb line has gone——nothing about the paths changed, and the chosen path changed. The separator was on the side of the instruments.

Observation 5.5

To be stated honestly——heading is not the only reason to take a rhumb line. Currents, winds, ice, depth and ports of refuge all count——what is shown here is the size of the price, not whether it should be paid.

Observation 5.6

One last thing——Tokyo to Sydney the difference was 340 metres. On a route running nearly north and south, the shortest path and the easy path are effectively the same thing——the size of the loss is decided by the destination, not by the navigator's skill or judgement.

6. The Census and the Close

1. **18.1512 per cent Tokyo to London** (Calculation 2.1): **a loss of 1735 km.**
2. **0.0043 per cent Tokyo to Sydney** (Calculation 2.1): **340 metres.**
3. **More than four thousandfold between them** (Observation 2.3): **same port, different destinations.**
4. **Tokyo to San Francisco is 8274.63 km** (Observation 2.2): **0.31 per cent from the usual figure.**
5. **Zero along a meridian and the equator** (Calculation 3.2): **the great circle is itself constant-heading.**
6. **Only meridians and the equator are both** (Observation 3.4): **zero is the exception.**
7. **53.9091 per cent halfway round at 70°N** (Calculation 3.5): **a factor of 1.54.**
8. **A parallel is no great circle** (Observation 3.7): **straight east is not straight.**
9. **Mercator stretching is $1/\cos \varphi$** (Observation 4.3): **one $\cos \varphi$ acts on both.**
10. **Practice divided the great circle into rhumb legs** (Observation 4.4): **not a choice between two.**

Closing: On a sphere the shortest path between two points is a great circle and a rhumb line is never shorter——and still the sailing ships ran rhumb lines, because one need only hold the compass needle on a single mark. The price differs by decades according to the destination——Tokyo to London the rhumb line is 11293.57 km against the great circle's 9558.57, longer by **18.1512 per cent**, that is 1735 km. From the same Tokyo to Sydney it is 7825.83 against 7826.17: run 7800 km and lose 340 metres. The differences span more than four thousandfold. What acts is not the total distance——it is how far the route runs east and west at high latitude. Along one meridian and along the equator the difference is exactly zero——a meridian always points true north and the equator always true east, so there the great circle is itself a line of constant heading. And among great circles only meridians and the equator are rhumb lines, so “no loss” is the exception, not the rule. Swing the other way and halfway round the world at 70

degrees north the difference is **53.9091** per cent, the rhumb line 1.54 times the great circle. The reason the rhumb line held practice lies in the chart——on the Mercator projection it is exactly a straight line, and the slope of a ruled line is the heading to hold. On the same chart the great circle is a curve: the shortest path looked like a detour and the detour looked straight. The stretching is $1/\cos \varphi$, so where the chart distorts most the rhumb line loses most — one and the same $\cos \varphi$ acting on both. One thing separates them——whether what is held constant is the heading or the property of being shortest. The separator is neither in the sea nor in the sphere——the same sea, the same two points, the same sphere; what makes the difference is what one keeps hold of aboard the ship. Paper 234 counted six “distances”——its question was what may pass as a distance. Here the distance is fixed to one, and the question is why, under that one distance, the longer path is chosen. Not a question about definitions but about choosing. And the separator is still moving——satellite navigation and autopilots have all but removed the labour of holding a heading, so the reason for choosing a rhumb line has gone. Nothing about the paths changed, and the chosen path changed. The separator was on the side of the instruments. One last thing——Tokyo to Sydney the difference was 340 metres. On a route running nearly north and south, the shortest path and the easy path are effectively the same thing. The size of the loss is decided by the destination, not by the navigator’s skill or judgement.

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