

Mix the Same Two Materials in the Same Proportion and the Stiffness Differs by 16.93

— Only the Direction Decides — And the Ratio of Bound to Bound Has a Closed Form, Exactly Symmetric in v_f and $1 - v_f$, Maximal at $v_f = 0.5$ with the Value $\frac{1}{2} + \frac{1}{4}(E_f/E_m + E_m/E_f) = \mathbf{16.932376}$ — [Paper 342]

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— *From thinking to syncing.* —

Abstract

Mix carbon fibre and epoxy half and half. This paper asks what the stiffness of that composite is—the answer is that it is not one number but a range of **16.932376**. No new mathematical theorem and no new law is claimed. First, at $v_f = 0.5$ the parallel bound is 116.7500 GPa and the series bound 6.8951 GPa (Section 2). Second, this is the core of the paper. The ratio of the bounds has a closed form and is exactly symmetric in v_f and $1 - v_f$ (Section 3). Third, the ratio is $v_f^2 + (1 - v_f)^2 + v_f(1 - v_f)(E_f/E_m + E_m/E_f)$ (Section 3). Fourth, it is maximal at $v_f = 0.5$ with the value $\frac{1}{2} + \frac{1}{4}(E_f/E_m + E_m/E_f) = \mathbf{16.932376}$ (Section 3). Fifth, the two materials enter only through the single symmetric combination $E_f/E_m + E_m/E_f$. Which one is called the fibre does not move the ratio (Section 4). Sixth, the separator is whether strain is shared or stress is (Section 5).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed—the Voigt (parallel, iso-strain) and Reuss (series, iso-stress) averages, the rule of mixtures, and the Hashin-Shtrikman bounds are all standard. We do not build mechanics of materials—the fibre-matrix interface, debonding, and residual stress are not entered. They are mentioned and no more. We do not treat strength—only stiffness (Young’s modulus) is treated; fracture strength and fatigue are not the subject. Strength belongs to Paper 344. We claim no optimality for the bounds—Voigt and Reuss are loose bounds; for an isotropic two-phase material the Hashin-Shtrikman bounds are narrower. The two loose ones are what is treated here. We design no laminate—classical laminate theory and weave patterns are not entered. We do not treat anisotropy in general—only the longitudinal and transverse directions of a unidirectional material. Relation to earlier papers: Paper 326 showed the square-cube “law” to be an identity and not a law—the ratio here is likewise not something measured but something that follows from algebra. The same move, performed on materials. Paper 330 showed the squaring of the condition number to be an identity—there too what was taken for “it usually comes out this way” followed from a definition. Paper 344 shows strength to be a function of volume and not a constant of the material—this paper says stiffness is not one number, and 344 says the same of strength. They make a pair. Paper 300 showed whether two things share a root is decidable—parallel and series have distinct roots by that test. What is added is deriving the closed form and showing the symmetry and the maximum algebraically, noting that the two materials enter only through one symmetric combination, confirming as a counter-case that $E_f = E_m$ gives a ratio of 1 for every v_f , and putting the separator on what is shared.

1. Notation and the Question

Setting: Mix fibre of modulus E_f with matrix of modulus E_m at volume fraction v_f . There are two extremes of mixing.

	What is shared	Modulus of the composite
Voigt (parallel)	strain	$E = v_f E_f + (1 - v_f) E_m$
Reuss (series)	stress	$1/E = v_f / E_f + (1 - v_f) / E_m$

Observation 1.1

Parallel means pulling along the fibres. Fibre and matrix stretch by the same amount—series means pulling across them, with one stress passing through both.

Observation 1.2

What was mixed is the same. The same two materials at the same volume fraction——only the direction of the load changed.

Observation 1.3

The way of measuring is fixed first. Carbon fibre $E_f = 230$ GPa and epoxy $E_m = 3.5$ GPa——a ratio $E_f/E_m = 65.714286$.

Question: How far does direction alone open it?

2. At $v_f = 0.5$, 116.7500 Against 6.8951

Calculation 2.1: Compute both against volume fraction (computed in this paper).

v_f	Voigt (parallel) [GPa]	Reuss (series) [GPa]	Upper/lower
0.00	3.5000	3.5000	1.0000
0.20	48.8000	4.3584	11.1967
0.40	94.1000	5.7747	16.2951
0.50	116.7500	6.8951	16.9324
0.60	139.4000	8.5547	16.2951
0.70	162.0500	11.2666	14.3832
1.00	230.0000	230.0000	1.0000

Observation 2.2

A check——at $v_f = 0$ both give 3.5000 and at $v_f = 1$ both give 230.0000. At the unmixed extremes the manner of mixing means nothing, so they must agree. They do.

Observation 2.3

At $v_f = 0.5$, 116.7500 against 6.8951. The same materials in the same proportion, apart by 16.9324——“what is the Young’s modulus of this composite” has no single answer.

Observation 2.4

The ratios at $v_f = 0.4$ and $v_f = 0.6$ agree to four decimals. Both are 16.2951——it does not look accidental.

Observation 2.5

And at both ends the ratio returns to 1.1.0000 at $v_f = 0$ and at $v_f = 1$ ——the ratio is a hill with its peak in the middle.

3. The Core: The Ratio Has a Closed Form, and It Is Symmetric

Setting: Write the ratio in symbols.

Observation 3.1

Put $a = E_f$, $b = E_m$, $u = v_f$; then

$$R(u) = (ua + (1 - u)b) \left(\frac{u}{a} + \frac{1 - u}{b} \right) = u^2 + (1 - u)^2 + u(1 - u) \left(\frac{a}{b} + \frac{b}{a} \right)$$

——expanded and collected, with no approximation.

Observation 3.2

This is the core of the paper. All three terms are invariant under swapping u for $1 - u$ ——both $u^2 + (1 - u)^2$ and $u(1 - u)$ are symmetric. So $R(u) = R(1 - u)$ holds exactly.

Observation 3.3

So the agreement at $v_f = 0.4$ and 0.6 was no accident. What looked equal to four decimals in Section 2 is equal to eight——16.29508075 against 16.29508075, because it is an identity.

Calculation 3.4: Check the closed form against direct computation (computed in this paper).

u	Direct	Closed form
0.10	6.73565528	6.73565528
0.30	14.38319565	14.38319565
0.40	16.29508075	16.29508075
0.50	16.93237578	16.93237578
0.60	16.29508075	16.29508075
0.90	6.73565528	6.73565528

Observation 3.5

Sweeping u from 0 to 1 in steps of 0.01, the largest discrepancy is 7.11×10^{-15} . Within double-precision rounding——the identity holds.

Observation 3.6

A symmetric quadratic has its maximum in the middle. Setting $u = 1/2$,

$$R_{\max} = \frac{1}{2} + \frac{1}{4} \left(\frac{a}{b} + \frac{b}{a} \right) = \mathbf{16.932376}$$

——the 16.9324 read off the table in Section 2 has come out of the algebra.

Observation 3.7

This is the move of Paper 326. Paper 326 wrote that the square-cube “law” is a geometric identity——16.932376 here is likewise not a number obtained by measuring materials but one that follows from the definitions of two averages. It is not something to confirm by experiment.

4. The Two Materials Enter Through One Symmetric Combination

Observation 4.1

In the closed form the two materials appear only as $a/b + b/a$. Swap a and b and that combination is unchanged——which one is called “fibre” and which “matrix” does not move the ratio at all.

Calculation 4.2: Does the closed form hold for other pairs (computed in this paper)?

Pair	a/b	$R_{\max}$ (closed form)	Direct at $u = 0.5$
Carbon fibre/epoxy	65.714286	16.932376	16.932376
200/70 GPa	2.857143	1.301786	1.301786
1000/1 GPa	1000.000000	250.500250	250.500250

Observation 4.3

For materials of similar stiffness there is almost no range. 200 against 70 GPa gives 1.301786, only thirty per cent——direction matters only when the two stiffnesses differ by decades.

Observation 4.4

If $E_f = E_m$ the ratio is 1 for every v_f . With $a/b + b/a = 2$,
 $R(u) = u^2 + (1 - u)^2 + 2u(1 - u) = (u + (1 - u))^2 = 1$ ——mix a material with itself and direction cannot matter. Obvious, and the formula says it.

Observation 4.5

When a/b is large, $R_{\max} \approx a/(4b)$. Because b/a drops out: $1000/1$ gives 250.500250 , nearly $1000/4 = 250$ ——the range is set by a quarter of the stiffness ratio.

Observation 4.6

To be stated honestly——Voigt and Reuss are loose bounds. For an isotropic two-phase material the Hashin-Shtrikman bounds are narrower——the 16.932376 here is not “the range achievable by direction” but “the range spanned by two simple averages”. Measured longitudinal and transverse values fall inside it without reaching either end.

5. The Separator: Is Strain Shared, or Stress?

Observation 5.1

The separator——whether what the two phases share is strain or stress.

	Shared	Direction of load	E at $v_f = 0.5$ [GPa]
Voigt	strain	along the fibres	116.7500
Reuss	stress	across the fibres	6.8951

Observation 5.2

The separator is not on the side of the material. The same panel, the same composition, the same volume fraction——what makes the difference is only which way one pulls.

Observation 5.3

So “the Young’s modulus of this material” is an incomplete question. Without a direction the answer swings by 16.932376——a word that could be omitted for isotropic materials cannot be omitted here.

Observation 5.4

The separator can be moved. Weave the fibres, or stack plies at angles, and any point between the extremes can be chosen——design is the act of deciding where in this 16.932376 range to sit.

Observation 5.5

It pairs with Paper 344. Paper 344 says strength is a function of volume and not a constant of the material——this paper says stiffness is a function of direction and not a constant of the material. What is called “a constant of the material” turned out twice not to be one.

Observation 5.6

One last thing——the upper bound 116.7500 is about half the fibre’s own 230. Half of it is fibre, so that looks natural; but the lower bound 6.8951 is not even twice the matrix’s own 3.5——placed in series the fibre barely works. The same quantity of carbon fibre is very nearly wasted, according to direction.

6. The Census and the Close

1. 116.7500 against 6.8951 at $v_f = 0.5$ (Calculation 2.1): a factor of 16.9324.
2. Both agree at $v_f = 0$ and 1 (Observation 2.2): unmixed, mixing means nothing.
3. $v_f = 0.4$ and 0.6 agree (Observation 2.4): not an accident.
4. $R = u^2 + (1 - u)^2 + u(1 - u)(a/b + b/a)$ (Observation 3.1): no approximation.
5. $R(u) = R(1 - u)$ exactly (Observation 3.2): all three terms symmetric.
6. 16.29508075 against 16.29508075 (Observation 3.3): equal to eight decimals.
7. $R_{\max} = \frac{1}{2} + \frac{1}{4}(a/b + b/a) = 16.932376$ (Observation 3.6): out of the algebra.
8. Materials enter only as $a/b + b/a$ (Observation 4.1): which is the fibre does not matter.
9. $E_f = E_m$ gives 1 for every v_f (Observation 4.4): the formula says it.
10. The lower bound is not twice the matrix (Observation 5.6): the fibre barely works.

Closing: Mix carbon fibre and epoxy half and half, ask what the Young’s modulus is, and there is no single answer——along the fibres 116.7500 GPa, across them 6.8951, a factor of 16.9324. The same panel, the same composition, the same volume fraction, and only the direction of the load changed. And that ratio has a closed form

——with $a = E_f$, $b = E_m$, $u = v_f$, $R(u) = u^2 + (1 - u)^2 + u(1 - u)(a/b + b/a)$, expanded and collected with no approximation. All three terms are invariant under $u \leftrightarrow 1 - u$, so $R(u) = R(1 - u)$ holds exactly. The agreement at $v_f = 0.4$ and 0.6 was therefore no accident: they are equal to eight decimals, 16.29508075. A symmetric quadratic has its maximum in the middle——at $u = 1/2$, $R_{\max} = \frac{1}{2} + \frac{1}{4}(a/b + b/a) = \mathbf{16.932376}$. The number read off the table came out of the algebra. As with the square-cube law in Paper 326, this is not a number obtained by measuring materials but one that follows from the definitions of two averages. And the two materials enter only through the single symmetric combination $a/b + b/a$ ——swapping a and b leaves it unchanged, so which one is called the fibre does not move the ratio at all. With $E_f = E_m$ the formula collapses to $(u + (1 - u))^2 = 1$: mix a material with itself and direction cannot matter. For large a/b , $R_{\max} \approx a/(4b)$, so the range is a quarter of the stiffness ratio. One thing separates them——whether the two phases share strain or share stress. The separator is not on the side of the material——so “the Young’s modulus of this material” is an incomplete question: without a direction the answer swings by 16.932376, and a word omissible for isotropic materials cannot be omitted here. To be said honestly——Voigt and Reuss are loose bounds, and for an isotropic two-phase material the Hashin-Shtrikman bounds are narrower. The 16.932376 is the range spanned by two simple averages, not the range achievable by direction; measured values fall inside it without reaching either end. One last thing——the upper bound 116.7500 is about half the fibre’s own 230, which looks natural for a half-and-half mixture. But the lower bound 6.8951 is not even twice the matrix’s own 3.5. Placed in series the fibre barely works — the same quantity of carbon fibre is very nearly wasted, according to direction.

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