

The Size of the Hole Is Not in the Stress Concentration

— Two Elliptical Holes Differing **10000**-Fold in Area Carry the Same $K_t = \mathbf{21.0}$ — What Enters Is the Sharpness a/b Alone: **3.0** for a Circular Hole and **201.0** at $a/b = 100$ — [Paper 343]

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— *From thinking to syncing.* —

Abstract

Cut a hole in a plate and the stress rises at its edge. This paper asks what decides how far it rises—the answer is not the size of the hole but only its sharpness. No new mathematical theorem and no new law is claimed. First, the stress concentration factor of an elliptical hole is $K_t = 1 + 2a/b$, and size does not enter (Section 2). Second, this is the core of the paper. Two holes differing **10000**-fold in area carry the same $K_t = \mathbf{21.0}$ (Section 2). Third, for a circular hole $K_t = 3.0$, independent of its diameter entirely (Section 2). Fourth, multiply a/b by 100 and K_t multiplies by 67. From 3.0 to **201.0** (Section 2). Fifth, turn the same ellipse through 90 degrees and K_t falls from **21.0** to **1.2** (Section 4). Sixth, the separator is area against sharpness (Section 5).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed——Inglis’s solution for the elliptical hole, the factor $K_t = 1 + 2a/b$, and its form $K_t = 1 + 2\sqrt{a/\rho}$ in terms of curvature are all standard. We do not build elasticity——the derivation, complex stress functions and boundary conditions are not entered. The formula is used as given. We do not build fracture mechanics——the stress intensity factor K_I and crack growth laws are not entered. They are mentioned and no more. We do not treat plasticity——real materials yield, so $K_t = 201$ does not mean stress rises 201-fold. Only the elastic range is looked at. We do not treat finite width——the infinite-plate solution is used; a hole large against the plate needs a correction. We do not treat fatigue——notch factors under cyclic loading are not entered. Relation to earlier papers: Paper 325 showed the size of the drop is not in the angle of the rainbow——this paper has exactly that form, with the size of the hole not in the stress concentration. In both, there is no route by which the other party’s size can enter. Paper 326 showed the square-cube “law” to be an identity——there too how dimensions act was settled by geometry. Paper 344 shows strength to be a function of volume and not a constant of the material——there size acts and here it does not. Within one subject, the failure of materials, a quantity where size acts stands beside one where it does not. Paper 342 showed the same two materials differing 16.93-fold by direction alone——there and here alike, direction changes the answer. What is added is confirming that the two expressions (a/b and curvature) agree exactly, exhibiting two holes 10^4 apart in area with the same K_t , giving the drop on turning the same ellipse through 90 degrees, and putting the separator on area against sharpness.

1. Notation and the Question

Setting: Pull an infinite plate uniformly and cut in it an elliptical hole of semi-axes a and b , with a perpendicular to the pull. The maximum stress at the edge is

$$\sigma_{\max} = \sigma_0 \left(1 + \frac{2a}{b} \right), \quad K_t = 1 + \frac{2a}{b}$$

(Inglis, 1913).

Observation 1.1

On the right, a and b appear only as a ratio. Scale a and b together and K_t does not move——the size of the hole is not in the formula.

Observation 1.2

Naively one expects the bigger hole to be the more dangerous. Removing more section does weaken the plate——but that is the average stress, not the concentration. They are two different quantities.

Observation 1.3

The way of measuring is fixed first. Infinite plate, linear elasticity, uniaxial tension——no finite-width correction and no plasticity.

Question: What property of the hole decides K_t ?

2. Two Holes 10000-Fold Apart in Area Carry the Same K_t

Calculation 2.1: Give K_t against a/b and check it against the curvature form (computed in this paper).

a/b	$K_t = 1 + 2a/b$	Curvature $\rho = b^2/a$ (with $a = 1$)	$1 + 2\sqrt{a/\rho}$
1 (circular)	3.0	1.000000	3.0000
2	5.0	0.250000	5.0000
5	11.0	0.040000	11.0000
10	21.0	0.010000	21.0000
100	201.0	0.000100	201.0000

Observation 2.2

Two checks.(a) A circular hole gives $K_t = 3.0$, the classical known value.(b) The two expressions agree exactly on every row (largest discrepancy 0)——one independently known case and one agreement between independent formulas, before the table is read.

Calculation 2.3: Hold a/b fixed and change only the size (computed in this paper).

a	b	Hole area πab	K_t
1	0.1	0.314159	21.0
100	10	3141.592654	21.0
Ratio		10000	1.0000

Observation 2.4

This is the core of the paper. Two holes 10000-fold apart in area carry the same 21.0—a flaw of 0.3 mm^2 and a hole of 3000 mm^2 raise the edge stress by the same factor.

Observation 2.5

The 3.0 of a circular hole is independent of diameter entirely. A round hole the width of a hair and one that admits a fist both treble the stress at the edge——“it is small, so it is safe” does not hold for concentration.

Observation 2.6

Multiply a/b by 100 and K_t multiplies by 67. From 3.0 to 201.0——keep the area and only elongate the shape, and the stress rises 67-fold.

3. Sharpness Measured as Curvature Says the Same Thing

Observation 3.1

The radius of curvature at the end of the major axis is $\rho = b^2/a$. With it, $K_t = 1 + 2\sqrt{a/\rho}$ ——and the two expressions agreed on every row of Section 2.

Observation 3.2

But $\sqrt{a/\rho}$ still shows an a . On its face, size seems to have entered——substitute $\rho = b^2/a$ and the a cancels. They are one quantity written two ways.

Observation 3.3

The rewriting has a practical point. ρ can be measured even for shapes where a and b are undefined — a notch, a weld toe, a fillet——which is why design uses the curvature form.

Observation 3.4

As $b \rightarrow 0$, $K_t \rightarrow \infty$. Sharpness can be made infinite, so a crack carries infinite stress within elasticity—and there the quantity K_t itself ceases to serve; fracture mechanics changes to another quantity, the stress intensity factor.

Observation 3.5

And there, at last, size enters. The stress intensity factor is $K_I = \sigma \sqrt{\pi a}$, with a standing alone—the size that could not enter K_t does enter K_I . Which one is used decides whether size acts.

Observation 3.6

To be stated honestly—no real material turns $K_t = 201$ into 201 times the stress. Reach the yield point and plasticity blunts the tip— K_t is an elastic solution, giving a bound and not a measurement.

4. Turn the Same Ellipse Through 90 Degrees and 21.0 Becomes 1.2

Calculation 4.1: Rotate the same elliptical hole 90 degrees relative to the pull (computed in this paper).

Orientation	a/b	K_t
Major axis across the pull	10	21.0
Major axis along the pull	0.1	1.2
Ratio		17.5

Observation 4.2

The same hole, the same area, the same material, and 17.5-fold from orientation alone. A K_t of 1.2 is very nearly no concentration at all—a slender hole laid along the pull is close to harmless.

Observation 4.3

So orientation is a design variable. The same saving of weight can be had in a dangerous or a safe orientation——aircraft windows have rounded corners to keep a/b small.

Observation 4.4

It pairs with Paper 342. Paper 342 wrote that the same two materials differ 16.932376-fold by direction alone——here the same hole differs 17.5-fold by direction alone. The numbers happen to be close and the roots are different: one is a ratio of stiffness bounds, the other of stress concentrations.

Observation 4.5

It has the form of Paper 325. Paper 325 wrote that the size of the drop is not in the angle of the rainbow——here the size of the hole is not in the stress concentration. In both, similar figures return the same answer.

5. The Separator: Area, or Sharpness?

Observation 5.1

The separator——whether what measures the danger is the section removed (area) or the curvature at the edge (sharpness).

	What acts	Does size enter	Example
Rise in average stress	area removed	yes	a large round hole
Concentration K_t	a/b (sharpness)	no	a small sharp flaw

Observation 5.2

The two rank dangers in opposite directions. By area a large round hole is worse; by concentration a small sharp flaw is——which is fatal depends on whether the material is brittle or ductile.

Observation 5.3

The separator can be moved. Drill a round hole at the tip of a sharp flaw and a/b falls, and K_t with it—the apparently contradictory remedy of adding area to gain safety works. It is a real technique, the stop hole.

Observation 5.4

This is the practical core of the paper. Drilling a hole makes things safer because the quantity measuring the danger is not area—mistake the separator and the remedy cannot be understood.

Observation 5.5

To be stated honestly—a stop hole is not always effective. In fatigue the cycling of stress acts, and a static K_t does not settle it—what is claimed is that K_t falls, not that life is extended.

Observation 5.6

One last thing—what this formula says is that danger cannot be measured by how much material was lost. With the same area lost, the shape of the loss changes it 67-fold—it is not the amount but the outline that decides.

6. The Census and the Close

1. Size does not enter $K_t = 1 + 2a/b$ (Observation 1.1): a and b appear only as a ratio.
2. $K_t = 3.0$ for a circular hole (Calculation 2.1): the classical value.
3. The two expressions agree exactly (Observation 2.2): largest discrepancy 0.
4. 10000-fold in area, the same 21.0 (Calculation 2.3): the core of the paper.
5. The 3.0 is independent of diameter (Observation 2.5): “small, so safe” fails.
6. a/b by 100, K_t by 67 (Observation 2.6): from 3.0 to 201.0.
7. $K_t \rightarrow \infty$ as $b \rightarrow 0$ (Observation 3.4): there K_t ceases to serve.
8. $K_I = \sigma\sqrt{\pi a}$ does carry a (Observation 3.5): the choice decides.

9. **21.0 becomes 1.2 on turning** (Calculation 4.1): **a factor of 17.5.**

10. **A stop hole adds area to gain safety** (Observation 5.3): **because the separator is not area.**

Closing: At the edge of an elliptical hole the stress rises by $K_t = 1 + 2a/b$, and on the right a and b appear only as a ratio—the size of the hole is not in the formula. Two elliptical holes **10000-fold** apart in area carry the same $K_t = \mathbf{21.0}$ —a flaw of area 0.314159 and a hole of 3141.592654 raise the edge stress by the same factor. The 3.0 of a circular hole likewise does not depend on diameter at all: a round hole the width of a hair and one that admits a fist both treble the stress. “It is small, so it is safe” does not hold for concentration. What enters is sharpness alone—multiply a/b by 100 and K_t goes from 3.0 to **201.0**, a factor of 67. Rewritten by curvature, $K_t = 1 + 2\sqrt{a/\rho}$ agreed with $1 + 2a/b$ on every row— $\sqrt{a/\rho}$ seems to carry an a , but $\rho = b^2/a$ cancels it: one quantity written two ways. And as $b \rightarrow 0$, $K_t \rightarrow \infty$ —a crack carries infinite stress within elasticity, and there the quantity K_t itself ceases to serve. The stress intensity factor that fracture mechanics turns to, $K_I = \sigma\sqrt{\pi a}$, carries a standing alone—the size that could not enter K_t enters K_I , and which one is used decides whether size acts. Turn the same ellipse through 90 degrees and 21.0 becomes 1.2—the same hole, area and material, 17.5-fold from orientation. A slender hole laid along the pull is close to harmless, and aircraft windows have rounded corners to keep a/b small. One thing separates them—whether the danger is measured by the section removed or by the curvature at the edge. By area a large round hole is worse; by concentration a small sharp flaw is. And the separator can be moved—drill a round hole at the tip of a sharp flaw and a/b falls with K_t : adding area to gain safety works, and that remedy cannot be understood if the separator is mistaken. To be said honestly—no real material turns $K_t = 201$ into 201 times the stress; yielding blunts the tip and K_t is an elastic bound. Nor is a stop hole always effective: in fatigue the cycling of stress acts, and what is claimed is that K_t falls, not that life is extended. One last thing—what this formula says is that danger cannot be measured by how much material was lost. With the same area lost, the shape of the loss changes it 67-fold. It is not the amount but the outline that decides.

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