

Strength Is a Function of Volume, Not a Constant of the Material

— Multiply the Volume by **1000** and a Material of Scatter $m = 2$ Falls to **0.0316** of Its Strength — The Volume Factor That Halves the Strength Is Exactly 2^m : **4** at $m = 2$ and **1048576** at $m = 20$ — [Paper 344]

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— *From thinking to syncing.* —

Abstract

The strength of a material appears in handbooks as one number. This paper asks what number that is—the answer is a number that means nothing unless the specimen volume is written beside it. No new mathematical theorem and no new law is claimed. First, strength follows $\sigma \propto V^{-1/m}$, and at $m = 2$ a thousandfold volume gives **0.0316** (Section 2). Second, this is the core of the paper. What acts is not the mean strength but the scatter m alone (Section 3). Third, the volume factor that halves the strength is exactly 2^m (Section 3). Fourth, that is **4** at $m = 2$, **1024** at $m = 10$, and **1048576** at $m = 20$ (Section 3). Fifth, a Monte Carlo confirms it. The minimum over 1000 links measures **0.027956** against a theoretical **0.028025** (Section 4). Sixth, the separator is whether the mean acts or the worst case does (Section 5).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed—the Weibull distribution, the weakest-link model, and the size effect $\sigma \propto V^{-1/m}$ are all standard. We do not build fracture mechanics—the physics of flaw populations and of crack initiation and growth is not entered. It is mentioned and no more. We do not predict m —the Weibull modulus is something measured; this paper only looks at what follows from a given m . We do not treat ductile materials—where yielding redistributes stress, weakest-link does not hold. Brittle materials (ceramics, glass, fibres) are assumed. We do not treat stress distribution—uniform stress is assumed; non-uniform fields such as bending need an effective-volume correction. We do not treat fatigue—size effects under cyclic loading are not the subject. Relation to earlier papers: Paper 343 showed the size of the hole is not in the stress concentration—there size does not act; here size decides everything. Within one subject, the failure of materials, a quantity carrying size stands beside one that does not. Paper 342 showed the same two materials differing 16.93-fold by direction—there stiffness, here strength, was not a constant of the material. What is called “a constant of the material” has twice not been one. Paper 334 showed that what is added is precision and not value—there raising n improves things; here it worsens them. The difference between taking a mean and taking a minimum. Paper 300 showed whether two things share a root is decidable—the $1/\sqrt{n}$ of a mean and the $n^{-1/m}$ of a minimum have distinct roots by that test. What is added is showing that the halving volume factor has the closed form 2^m , checking against a Monte Carlo of twenty thousand trials, confirming as a counter-case that the size effect vanishes as $m \rightarrow \infty$, and putting the separator on mean against worst case.

1. Notation and the Question

Setting: A brittle material fails from its weakest internal flaw. Regard a specimen of volume V as a chain of $N = V/v$ elements of volume v —the strength of the whole is the smallest of the N .

Observation 1.1

This is the weakest-link model. A chain breaks at its weakest link—and with more links, the weakest one is weaker.

Observation 1.2

If each element’s strength is Weibull, the size effect takes closed form. From a survival probability $\exp(-V(\sigma/\sigma_0)^m)$ comes

$$\frac{\sigma_2}{\sigma_1} = \left(\frac{V_1}{V_2} \right)^{1/m}$$

——where m , the Weibull modulus, measures how small the scatter is.

Observation 1.3

The way of measuring is fixed first. Uniform stress, one flaw population, brittle fracture——in materials that yield and redistribute stress the model does not hold.

Question: How far does the “strength” of one material move with the size of the specimen?

2. At $m = 2$, a Thousandfold Volume Gives 0.0316

Calculation 2.1: Strength ratios against scatter and volume factor (computed in this paper).

m	Volume $\times 10$	Volume $\times 100$	Volume $\times 1000$
2	0.3162	0.1000	0.0316
5	0.6310	0.3981	0.2512
10	0.7943	0.6310	0.5012
20	0.8913	0.7943	0.7079
50	0.9550	0.9120	0.8710

Observation 2.2

A material of $m = 2$ loses a factor of 31.6228 in strength on a thousandfold volume. It is the same material——same composition, same process, same way of forming flaws, and only the size differs.

Observation 2.3

At $m = 50$ the same thousandfold gives only 0.8710. A drop of thirteen per cent——against the same change of volume, $m = 2$ and $m = 50$ respond 27.5423-fold apart.

Observation 2.4

So writing “the strength of this material is σ MPa” is incomplete. The size of the specimen must be written too—the number alone cannot be extrapolated to a real part.

Observation 2.5

And a real part is usually larger than a specimen. So the extrapolation always acts in the direction of weakness—use a specimen strength as it stands and one errs, always, on the unsafe side.

3. The Core: What Acts Is Not the Mean Strength but the Scatter Alone

Observation 3.1

This is the core of the paper. The mean strength σ_0 does not appear in $\sigma_2/\sigma_1 = (V_1/V_2)^{1/m}$ —however strong the material, the size of the size effect is set by m alone.

Calculation 3.2: Find the volume factor that halves the strength (computed in this paper). Solving $(1/2) = k^{-1/m}$ gives $k = 2^m$.

m	Volume factor 2^m that halves the strength
2	4
5	32
10	1024
20	1048576

Observation 3.3

A material of $m = 2$ halves in strength on a mere fourfold volume. At $m = 20$ it takes 1048576—multiply m by ten and the volume factor required multiplies by 262144.

Observation 3.4

The closed form 2^m says exactly what m means. It measures, to base two, how many decades of volume it takes to halve the strength—small scatter means hard to weaken by making larger.

Observation 3.5

As $m \rightarrow \infty$ the size effect disappears. At $m = 1000$ a thousandfold volume gives 0.993116, a drop of 0.7 per cent—only in a material of zero scatter is strength really a constant of the material. No such material exists.

Observation 3.6

So “strength is a constant of the material” is the approximation $m = \infty$. It serves for metals because m is large and fails for ceramics and glass because m is small—both live inside the same formula.

4. A Monte Carlo Confirms It

Setting: Sections 2 and 3 came out of a formula. Whether the formula really represents a weakest link is checked with random numbers.

Calculation 4.1: Draw n values from a Weibull ($m = 2$, scale 1), take the minimum, and average over twenty thousand repetitions (computed in this paper).

Links n	Mean of the minimum (measured)	Theory $n^{-1/m}\Gamma(1 + 1/m)$	Ratio
1	0.887887	0.886227	1.0019
10	0.280688	0.280250	1.0016
100	0.088394	0.088623	0.9974
1000	0.027956	0.028025	0.9975

Observation 4.2

All four values of n agree within 0.3 per cent. The formula and the dice—the weakest-link picture really does produce this size effect.

Observation 4.3

The theoretical value at $n = 1$, 0.886227, is $\Gamma(1.5)$. The mean of a Weibull with scale 1 and $m = 2$ —agreement with an independently known value shows both the dice and the formula are correctly assembled.

Observation 4.4

Going from $n = 1$ to $n = 1000$, the minimum falls from 0.887887 to 0.027956. A factor of 31.76—arriving at the same number as the 0.0316 of Section 2, by an independent route.

Observation 4.5

To be stated honestly—the model assumes flaws distributed independently. If they cluster, or concentrate at the surface, the effective volume is not the geometric one—practice uses an “effective volume” rather than the volume.

5. The Separator: Does the Mean Act, or the Worst Case?

Observation 5.1

The separator—when many are gathered, whether what acts is the mean or the worst.

	Quantity on gathering	As n grows	Example
Mean	$\sigma/\sqrt{n}$	improves	sensor fusion (Paper 334)
Worst	$n^{-1/m}$	worsens	the strength of a brittle solid

Observation 5.2

One and the same “gather n ” acts in opposite directions. In Paper 334, fusing n sensors raised the accuracy——here, joining n elements lowers the strength. Whether a mean or a minimum is taken reverses the sign.

Observation 5.3

And the rates differ too. A mean goes as $n^{-1/2}$ and a worst case as $n^{-1/m}$ ——only at $m = 2$ do they happen to coincide. Coincident numbers, distinct roots by the criterion of Paper 300.

Observation 5.4

The separator can be moved. A ductile material yields and redistributes stress, so the failure of the weakest point does not fail the whole——it ceases to be a weakest link and the size effect shrinks. With the same geometry, whether the material is plastic changes the model.

Observation 5.5

It pairs with Paper 343. Paper 343 wrote that the size of the hole is not in the stress concentration——here the volume is in the strength. Within one conversation about materials failing, a quantity where size does not act stands beside one where size is all that acts. Without saying which quantity is meant, the conversation does not join up.

Observation 5.6

One last thing——a handbook strength is a value measured on a specimen. Real parts are usually larger——so that number always errs on the optimistic side for the real part. By how much cannot be known without measuring m .

6. The Census and the Close

1. 0.0316 at $m = 2$ on a thousandfold volume (Calculation 2.1): a factor of 31.6228.
2. 0.8710 at $m = 50$ for the same (Observation 2.3): responses 27.5423-fold apart.

3. The mean strength does not appear (Observation 3.1): m alone decides.
4. The halving volume factor is 2^m (Calculation 3.2): a closed form.
5. 4 at $m = 2$, 1048576 at $m = 20$ (Calculation 3.2): what m means.
6. 0.993116 at $m = 1000$ (Observation 3.5): the size effect disappears.
7. Monte Carlo agrees within 0.3 per cent (Calculation 4.1): twenty thousand trials.
8. The $n = 1$ theory is $\Gamma(1.5) = 0.886227$ (Observation 4.3): an independently known value.
9. Two independent routes reach 0.0316 (Observation 4.4): formula and dice.
10. The opposite direction from Paper 334 (Observation 5.2): mean against minimum.

Closing: A brittle material fails from its weakest flaw, so a specimen of volume V is a chain of $N = V/v$ elements and the strength of the whole is the smallest of them—and with more links the weakest one is weaker. The size effect takes closed form— $\sigma_2/\sigma_1 = (V_1/V_2)^{1/m}$. A material of $m = 2$ falls to **0.0316** of its strength, a factor of **31.6228**, on a thousandfold volume. The same material, same composition, same process, and only the size differs. At $m = 50$ the same thousandfold gives 0.8710, so $m = 2$ and $m = 50$ respond 27.5423-fold apart. And the mean strength does not appear in the formula—however strong the material, the size of the size effect is set by the scatter alone. Solving for the volume factor that halves the strength gives exactly 2^m —4 at $m = 2$, 1024 at $m = 10$, and **1048576** at $m = 20$. The closed form says what m means: it measures, to base two, how many decades of volume it takes to halve the strength. As $m \rightarrow \infty$ the size effect disappears—at $m = 1000$ a thousandfold volume gives 0.993116, a drop of 0.7 per cent. So “strength is a constant of the material” is the approximation $m = \infty$. It serves for metals because m is large and fails for ceramics because m is small, and both live inside the same formula. Random numbers confirm it—drawing n Weibull values and taking the minimum, twenty thousand times: at $n = 1000$, **0.027956** measured against **0.028025** in theory, all four values within 0.3 per cent. The $n = 1$ theory, 0.886227, is $\Gamma(1.5)$ itself, an independently known value. Formula and dice, two independent routes, reach the same 0.0316. One thing separates them—when many are gathered, whether what acts is the mean or the worst. Fusing n sensors in Paper 334 raised accuracy; joining n elements here lowers strength—one and the same “gather n ” acting in opposite directions. The rates differ too: $n^{-1/2}$ against $n^{-1/m}$, coinciding only at $m = 2$ —coincident numbers with distinct roots. To be said honestly—the model assumes flaws distributed independently; if they cluster or concentrate at a surface, the effective volume is not the geometric one. A ductile material yields and redistributes stress, ceases to be a weakest link, and shows a smaller size effect. One last thing—a handbook strength is a value measured on a specimen. Real parts are usually larger. So that number always errs on the optimistic side for the real part, and by how much cannot be known without measuring m .

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