

# One and the Same $\log_2$ Counts a Ratio in One Law and Items in the Other

— Fitts’s Law Stays at **4.3219** When  $D$  and  $W$  Are Scaled Together — Hick’s Law Turns **32** Times the Choices  
into Only **2.7333** Times the Time — [Paper 345]

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— *From thinking to syncing.* —

## Abstract

Fitts’s law and Hick’s law both say that time goes as  $\log_2$ . This paper asks whether they are saying the same thing — the answer is that what is inside the  $\log_2$  differs: a dimensionless ratio in one, a count in the other. No new mathematical theorem and no new law is claimed. First, Fitts’s  $ID = \log_2(2D/W)$  does not move when  $D$  and  $W$  are scaled together (Section 2). Second, this is the core of the paper. Because what is inside the  $\log_2$  is dimensionless, neither the units nor the size of the screen enters (Section 3). Third, Hick’s  $\log_2(n + 1)$  has no unit to move in the first place (Section 3). Fourth, so 32 times the choices gives only **2.7333** times the time (Section 4). Fifth, doubling  $D$  alone raises  $ID$  by exactly 1; doubling  $W$  alone lowers it by exactly 1 (Section 2). Sixth, the separator is a dimensionless ratio against a count (Section 5).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed——Fitts’s law, Hick’s law, and Shannon’s definition of information are all standard. We do not build psychology——the mechanisms of motor control and the internal processes of decision are not entered. They are mentioned and no more. We do not adjudicate the information-theoretic reading——whether either law measures “information” is disputed, and this paper does not enter that dispute. Only the form of the formulas is looked at. We predict no constants——the  $a$  and  $b$  of Hick’s law are set here to 0.2 and 0.15 seconds as an illustration, not as measurements. Only the shape of the ratio is claimed. We perform no experiment——no subjects and no apparatus; only the algebraic properties of the formulas. We give no design guidance——how to make an interface usable is not the subject. Relation to earlier papers: Paper 316 showed the same integer returning from three different counts——there different routes reached one answer. Here one symbol,  $\log_2$ , covers two different things being counted. A pair running the other way. Paper 296 showed the hierarchy of languages set by the type of memory and not its amount——there too the separator was “amount or type”. Here it is “ratio or count”. Paper 306 showed how many “base” units there are to be a promise——Hick’s side has no unit and Fitts’s has one that cancels. The presence or absence of a unit is itself what divides them. Paper 300 showed whether two things share a root is decidable——the two laws have distinct roots by that test. What is added is exhibiting pairs that return the same ID to show scale invariance in numbers, confirming that moving  $D$  or  $W$  alone shifts ID by exactly  $\pm 1$ , giving the time ratios for 32-fold and 128-fold choices, and putting the separator on a dimensionless ratio against a count.

# 1. Notation and the Question

Setting: Set the two laws side by side.

| Law   | Formula                          | What it predicts                                    |
|-------|----------------------------------|-----------------------------------------------------|
| Fitts | $T = a + bID, ID = \log_2(2D/W)$ | time to reach a target of width $W$ at distance $D$ |
| Hick  | $T = a + b\log_2(n + 1)$         | time to choose one of $n$ options                   |

## Observation 1.1

Both carry a  $\log_2$  and both are explained as “information”. Time proportional to a number of bits, in the same words——they look as though they come from one root.

### Observation 1.2

But what is inside the  $\log_2$  differs. Fitts has  $2D/W$  and Hick has  $n + 1$ —a ratio of two lengths against a count of things.

### Observation 1.3

The way of measuring is fixed first. Hick's constants are set to  $a = 0.2$  and  $b = 0.15$  seconds—illustrative values, not measurements; only the shape of the ratio is claimed.

Question: Under one and the same  $\log_2$ , is the same thing being counted?

## 2. Scale $D$ and $W$ Together and ID Does Not Move

Calculation 2.1: Compute ID for several pairs of distance and width (computed in this paper).

| $D$  | $W$ | $2D/W$ | ID = $\log_2(2D/W)$ |
|------|-----|--------|---------------------|
| 100  | 10  | 20     | 4.3219              |
| 400  | 40  | 20     | 4.3219              |
| 1000 | 100 | 20     | 4.3219              |
| 200  | 10  | 40     | 5.3219              |
| 100  | 5   | 40     | 5.3219              |

### Observation 2.2

The first three rows differ tenfold in distance and share one ID. 100 and 1000, both 4.3219—because the ratio  $2D/W$  is the same.

### Observation 2.3

The last two moved different quantities and share one ID. Double the  $D$  or halve the  $W$  and both give 5.3219—the formula does not distinguish going further from aiming smaller.

### Observation 2.4

Double  $D$  alone and ID rises by exactly 1. From 4.3219 to 5.3219—it is a  $\log_2$ , so a doubling is exactly one bit.

### Observation 2.5

Double  $W$  alone and it falls by exactly 1. From 5.3219 to 4.3219—opposite sign, same magnitude.

## 3. The Core: Fitts's $\log_2$ Holds a Dimensionless Quantity and Hick's Holds a Count

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### Observation 3.1

This is the core of the paper.  $2D/W$  is a length divided by a length, and so is dimensionless—measured in millimetres, in inches or in pixels, it is the same number.

### Observation 3.2

So Fitts's law carries no units. Enlarge the screen and, if target and distance grow together, the time does not change—which is why  $100/10$  and  $1000/100$  gave the same 4.3219 in Section 2.

### Observation 3.3

Hick's  $n + 1$  could not have had a unit. A count is dimensionless too, but for a different reason—in Fitts the unit cancelled in a division; in Hick there was never one.

Observation 3.4

The difference shows up as a difference in what can be moved. In Fitts,  $D$  and  $W$  move independently, so infinitely many pairs give the same ID—in Hick there is only  $n$  to move, and no other arrangement gives the same time.

Observation 3.5

In the language of Paper 306, one has a promise and the other has none. A unit of length may be chosen by convention, millimetre or inch—and Fitts’s ID is independent of that convention. Hick has no convention to choose.

Observation 3.6

Apply the criterion of Paper 300.

| Criterion                                        | Result                                                 |
|--------------------------------------------------|--------------------------------------------------------|
| Is there a correspondence preserving an identity | No. None can be drawn between a ratio and a count      |
| Does the same quantity appear                    | No. $2D/W$ against $n + 1$                             |
| Do they keep agreeing as variables move          | No. Scale $D$ and $W$ together and only one is unmoved |

Observation 3.7

All three point to distinct roots. The same  $\log_2$  appears in both because each handles a quantity that grows by a fixed amount when doubled—that coincidence is a property of the logarithm, not a shared feature of the phenomena.

4. Thirty-Two Times the Choices Gives Only 2.7333 Times the Time

Calculation 4.1: Times from Hick’s law as choices increase (computed in this paper,  $a = 0.2$ ,  $b = 0.15$  s).

| Choices $n$ | Time [s]      | Factor over $n = 1$ |
|-------------|---------------|---------------------|
| 1           | 0.3500        | 1.0000              |
| 2           | 0.4377        | 1.2507              |
| 4           | 0.5483        | 1.5665              |
| 8           | 0.6755        | 1.9300              |
| <b>32</b>   | <b>0.9567</b> | <b>2.7333</b>       |
| 64          | 1.1034        | 3.1524              |
| 128         | 1.2517        | <b>3.5762</b>       |

#### Observation 4.2

Thirty-two times the choices gives only 2.7333 times the time. A hundred and twenty-eight times gives 3.5762——going from 32 to 128, a fourfold increase, costs only 1.3084.

#### Observation 4.3

This is how a logarithm acts. Each doubling of the choices adds one fixed  $b$ ——the price of a longer menu is smaller than one expects.

#### Observation 4.4

But on Fitts's side the same “doubling” means something else. Doubling  $D$  and halving  $W$  both make one bit——Hick has no way to make one bit but by doubling  $n$ .

#### Observation 4.5

To be stated honestly——the 2.7333 depends on setting  $a = 0.2$  and  $b = 0.15$ . A larger  $a$  brings the factor toward 1; with  $a = 0$ , thirty-two times the choices gives  $\log_2(33)/\log_2(2) = 5.0444$ ——the factor itself moves with the constants. What does not move is the shape: time grows only logarithmically.

Observation 4.6

The same caution applies on Fitts’s side. $ID$  does not depend on the constants, but the time  $T = a + bID$  does——what this paper calls independent of constants is  $ID$ , not the time.

5. The Separator: A Dimensionless Ratio, or a Count?

Observation 5.1

The separator——whether what sits inside the  $\log_2$  is a ratio of two quantities or a count of things.

|       | Inside the $\log_2$ | Units              | Other arrangements giving the same value |
|-------|---------------------|--------------------|------------------------------------------|
| Fitts | $2D/W$ (a ratio)    | cancel in division | infinitely many                          |
| Hick  | $n + 1$ (a count)   | never present      | none                                     |

Observation 5.2

The last column is what divides practice.With Fitts one may enlarge the target or bring it closer——with Hick there is nothing to do but reduce the choices. The very freedom of design differs.

Observation 5.3

And  $n$  can only be an integer. $2D/W$  moves continuously; a number of choices cannot be 3.7——one continuous, one discrete. Inside the same  $\log_2$ , the domains differ.

Observation 5.4

It pairs with Paper 316.Paper 316 wrote that the same integer returns from three different counts——there the routes differed and the answer agreed. Here the symbol agrees and the content differs. This is the case that warns against reading one symbol as one root.

### Observation 5.5

To be stated honestly—whether either law measures “information” is disputed. This paper does not enter that dispute—what is claimed is that even granting both are information, the things counted are different. It holds independently of how the dispute is settled.

### Observation 5.6

One last thing—the symbol  $\log_2$  was making the two look like one thing. The same function appears because each handles a quantity that grows by a fixed amount when doubled—sharing that property and measuring the same thing are different matters.

## 6. The Census and the Close

1.  $100/10$  and  $1000/100$  both give  $ID = 4.3219$  (Calculation 2.1): **tenfold in distance.**
2. Double  $D$  or halve  $W$ , both give  $5.3219$  (Observation 2.3): **the formula does not distinguish.**
3. Double  $D$  alone,  $ID$  rises by 1 (Observation 2.4): **a doubling is one bit.**
4. Double  $W$  alone, it falls by 1 (Observation 2.5): **opposite sign, same size.**
5.  $2D/W$  is dimensionless (Observation 3.1): **the same in any units.**
6. Hick has no unit to cancel (Observation 3.3): **dimensionless for another reason.**
7. Distinct roots on all three criteria (Observation 3.6): **no correspondence between ratio and count.**
8. **32 times the choices, 2.7333 times the time** (Calculation 4.1): **128 times gives 3.5762.**
9. **Infinitely many pairs for Fitts, none for Hick** (Observation 5.1): **the freedom differs.**
10.  $n$  cannot be 3.7 (Observation 5.3): **continuous against discrete.**

Closing: Fitts’s law and Hick’s law both say that time goes as  $\log_2$  and both are explained as “information”—but what is inside the  $\log_2$  differs:  $2D/W$  against  $n + 1$ , a ratio of two lengths against a count of things. Fitts’s content is dimensionless—a length divided by a length, so the same in millimetres, in inches or in pixels. Hence  $D = 100$ ,  $W = 10$  and  $D = 1000$ ,  $W = 100$  give the same **4.3219** though the distances differ tenfold. Doubling  $D$  or halving  $W$  both give 5.3219, so the formula does not distinguish going further from aiming smaller. Double  $D$  alone and  $ID$  rises by exactly 1; double  $W$  alone and it falls by exactly 1. Hick’s  $n + 1$  could not have carried a unit—a count is dimensionless too, but for a different reason: in Fitts the unit cancelled in a division, and in Hick there was never one. So Fitts admits infinitely many pairs giving one  $ID$ , and Hick admits no other arrangement giving one

time. And the logarithm acts gently on both——32 times the choices gives **2.7333** times the time, 128 times gives **3.5762**. The price of a longer menu is smaller than one expects. Though this factor depends on setting  $a = 0.2$  and  $b = 0.15$ : with  $a = 0$ , thirty-two times gives 5.0444. What does not move is the shape, that time grows only logarithmically. All three criteria of Paper 300 point to distinct roots——no correspondence can be drawn between a ratio and a count, no quantity is shared, and scaling  $D$  and  $W$  together leaves only one of them unmoved. One thing separates them——whether what sits inside the  $\log_2$  is a ratio of two quantities or a count of things. And what divides practice is whether other arrangements give the same value——with Fitts one may enlarge the target or bring it closer; with Hick there is nothing to do but reduce the choices. Further,  $2D/W$  moves continuously while a number of choices cannot be 3.7——inside the same  $\log_2$ , the domains differ. To be said honestly——whether either law measures “information” is disputed, and this paper does not enter that dispute. What is claimed is that even granting both are information, the things counted are different, and that holds however the dispute is settled. One last thing——the symbol  $\log_2$  was making the two look like one thing. The same function appears because each handles a quantity that grows by a fixed amount when doubled. Sharing that property and measuring the same thing are different matters.

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