

A Mirage Needs Temperature Rising with Height, Not Falling

— Bending Light Around the Curve of the Earth Takes an Inversion of **+0.131351** K/m, Which Is **20.2078** Times the Standard Lapse Rate and in the Opposite Direction — [Paper 349]

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— *From thinking to syncing.* —

Abstract

Air changes density with temperature, and light bends as it passes through. This paper asks what temperature gradient makes light follow the curve of the Earth—the answer is **20.2078** times the standard atmosphere, and in the opposite direction. No new mathematical theorem and no new law is claimed. First, what is needed is **+0.131351** K/m: an inversion, with temperature rising with height (Section 3). Second, this is the core of the paper. The standard atmosphere falls at -0.006500 K/m, so the direction is reversed (Section 3). Third, temperature is not the only thing moving the refractive index. The fall of pressure with height matters more, with a scale height of **8434.4** m (Section 2). Fourth, light bends a little even in a standard atmosphere. The refraction coefficient is **0.1671**, inside the usual figure of about **0.13** (Section 2). Fifth, at $+1$ K/m the radius of curvature falls to **0.1600** of the Earth's (Section 3). Sixth, the separator is whether temperature rises or falls with height (Section 5).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed—that the refractive index of air goes with density, hydrostatic equilibrium, the refraction coefficient, and temperature inversions are all standard. We do not build meteorology—how an inversion forms, radiative cooling, and sea-surface temperature contrasts are not entered. They are mentioned and no more. We do not trace rays—only the radius of curvature under a uniform gradient is looked at; real mirages occur where the gradient itself varies with height. We do not compute the image—the appearance of superior and inferior mirages and the number of inverted images are not entered. We do not treat humidity—dry air is assumed; the effect of water vapour is not the subject. We treat no observation record—mirages at particular times and places are not the subject. Relation to earlier papers: Paper 325 showed the size of the drop is not in the angle of the rainbow—there size does not enter; here the temperature gradient decides everything. Both count what enters a formula. Paper 328 showed aberration to be independent of distance—there and here alike, atmosphere or motion moves the position of an image. Paper 340 showed the shorter path is not the one a navigator steers—the visibility of the horizon treated there is exactly where this refraction coefficient acts. Paper 300 showed whether two things share a root is decidable—the pressure contribution and the temperature contribution here have distinct roots by that test. What is added is separating the pressure and temperature contributions in numbers, showing with its sign that the required gradient is an inversion, computing the standard refraction coefficient and checking it against the accepted value, and putting the separator on whether temperature rises or falls.

1. Notation and the Question

Setting: For air, $n - 1$ is proportional to density. For dry air at 15 °C and one atmosphere, $n - 1 = 2.732600 \times 10^{-4}$. Density goes as P/T , so

$$\frac{d(n - 1)}{dh} = (n - 1) \left[\frac{1}{P} \frac{dP}{dh} - \frac{1}{T} \frac{dT}{dh} \right].$$

Observation 1.1

There are two terms on the right. One for pressure falling with height, one for temperature changing with height—both move the refractive index, and they differ in size.

Observation 1.2

A vertical gradient in refractive index bends a ray. The radius of curvature is $R = 1/|dn/dh|$ —when that R equals the Earth's radius, light runs along the ground.

Observation 1.3

The way of measuring is fixed first. $T = 288.15$ K, dry air, hydrostatic equilibrium, and dT/dh written positive when rising——without fixing the sign first, the direction of the answer cannot be read.

Question: What temperature gradient makes light follow the curve of the Earth?

2. What Moves the Refractive Index Is Mostly Pressure

Calculation 2.1: Separate the two terms (computed in this paper).

Term	Expression	Value
Pressure scale height	$H = R_s T / g$	8434.4 m
Pressure term	$-1/H$	-1.185617×10^{-4} /m
Temperature term (standard)	$-(dT/dh)/T$	$+2.255770 \times 10^{-5}$ /m

Observation 2.2

The pressure term is 5.2559 times the temperature term. Under the standard lapse rate——most of what bends light is pressure, not temperature.

Observation 2.3

And the two carry opposite signs. Pressure falls with height and lowers n ; the standard temperature also falls, which raises n ——they oppose, and pressure still wins.

Calculation 2.4: Ray curvature and the refraction coefficient in a standard atmosphere (computed in this paper).

Quantity	Value
Ray radius of curvature, standard atmosphere	38118.4 km
Radius of the Earth	6371.0 km
Refraction coefficient k	0.1671

Observation 2.5

A check—the coefficient 0.1671 falls in the same decade as the value of about 0.13 used in surveying. Computed at first with the temperature term alone it came to 0.0393, nearly a decade out—because the pressure term had been dropped. Without this check the paper would have carried a wrong number.

Observation 2.6

$k = 0.1671$ means light bends slightly downward with nothing done at all. So the horizon lies a little further away than pure geometry gives—the sea horizon treated in Paper 340 carries this effect as it stands.

3. The Core: What Is Needed Is an Inversion

Calculation 3.1: Solve for the dT/dh that makes the radius of curvature equal the Earth's (computed in this paper).

Quantity	Value
Required dT/dh	+0.131351 K/m
Standard atmosphere dT/dh	−0.006500 K/m
Ratio of magnitudes	20.2078
Direction	reversed

Observation 3.2

This is the core of the paper. The sign is positive——temperature must rise with height. The standard atmosphere falls, so the direction is reversed.

Observation 3.3

+0.131351 K/m means it grows 0.13 degrees warmer for every metre of height. At eye height, 1.7 m, that is 0.2233 degrees——small as a number, and 20.2078 times as steep as the standard atmosphere.

Calculation 3.4: Radius of curvature against gradient (computed in this paper).

dT/dh [K/m]	Radius of curvature [km]	Ratio to Earth's radius
−0.006500 (standard)	38118.4	5.9831
0.000000 (isothermal)	30865.9	4.8448
+0.050000	12529.1	1.9666
+0.100000	7859.7	1.2337
+0.131351	6371.0	1.0000
+0.300000	3155.6	0.4953
+1.000000	1019.7	0.1600

Observation 3.5

Even isothermal air gives 4.8448 times the Earth's radius. With temperature held exactly constant, pressure alone bends light——the temperature gradient does not start from zero; it adds to or subtracts from a bending already there.

Observation 3.6

At +1 K/m the radius falls to 0.1600 of the Earth's. Light then bends more sharply than the Earth—things below the horizon rise into view: a superior mirage.

Observation 3.7

Reverse it — hot ground, temperature falling steeply — and light bends upward. A large negative dT/dh flips the sign of dn/dh —that is the inferior mirage, the sky reflected in the road. The same formula, on the other side of the sign.

4. One Formula Makes Two Kinds of Mirage

Observation 4.1

Within one formula, the sign of dT/dh alone divides the phenomena.

	dT/dh	Direction of bending	What is seen
Superior mirage	strongly positive	downward, strongly	things below the horizon rise
Standard atmosphere	−0.0065	downward, weakly	a slightly distant horizon
Inferior mirage	strongly negative	upward	a road that looks wet

Observation 4.2

These are not three phenomena. One dn/dh with different values and signs—by the criterion of Paper 300 they share a root, and what divides them is the sign of one number.

Observation 4.3

The pressure term and the temperature term, however, have distinct roots. One comes from hydrostatic equilibrium, the other from the distribution of heat——move one and the other stays. One gradient, two independent entrances.

Observation 4.4

To be stated honestly——a uniform gradient is assumed here. Real mirages occur in thin layers where the gradient itself changes sharply with height——that is why inverted images appear; a uniform gradient distorts without inverting. These numbers indicate how steep is steep enough to follow the Earth, not how the image is formed.

Observation 4.5

Where this stands relative to Paper 325. Paper 325 wrote that the size of the drop is not in the angle of the rainbow——here it runs the other way: one quantity, the temperature gradient, decides everything. The case where it does not enter and the case where nothing else does.

5. The Separator: Does Temperature Rise, or Fall?

Observation 5.1

The separator——whether temperature rises or falls with height.

	Sign	Gradient of n	What happens
Inversion	$dT/dh > 0$	strongly negative	light follows the Earth
Ordinary lapse	$dT/dh < 0$	weakly negative	a somewhat distant horizon

Observation 5.2

The separator lies neither in the air nor in the light. The same air, the same light, the same formula——what makes the difference is only which side is warmer.

Observation 5.3

The separator can be moved. Warm air over a cold sea makes an inversion; hot pavement makes the reverse——on one day, a superior mirage at sea and a shimmering road inland.

Observation 5.4

The steepness required is large by everyday standards. 0.131351 K/m means 1.3135 degrees warmer only ten metres up——such a layer can be held steady only over water or snow. That is why mirages cluster in particular places.

Observation 5.5

To be stated honestly——one single criterion was used, “follows the Earth”. Real mirages appear well short of that—— $+0.131351 \text{ K/m}$ is not a threshold for mirages but the particular value at which light runs parallel to the ground.

Observation 5.6

One last thing——even a standard atmosphere bends light by $k = 0.1671$. The horizon we see every day is already an image lifted by refraction——it is not that mirages are special; it is that there is no day without refraction.

6. The Census and the Close

1. The required gradient is $+0.131351 \text{ K/m}$ (Calculation 3.1): **temperature rising.**
2. 20.2078 times the standard, reversed (Calculation 3.1): **standard is -0.006500 K/m .**
3. Pressure scale height 8434.4 m (Calculation 2.1): **pressure term 5.2559 times temperature.**

4. Standard refraction coefficient $k = 0.1671$ (Calculation 2.4): same decade as about 0.13.
5. Temperature alone gives 0.0393 (Observation 2.5): this is how it first went wrong.
6. Isothermal still gives 4.8448 (Calculation 3.4): pressure alone bends light.
7. 0.1600 at +1 K/m (Calculation 3.4): sharper than the Earth.
8. Flip the sign and it is a road mirage (Observation 3.7): the other side of one formula.
9. Pressure and temperature terms have distinct roots (Observation 4.3): two independent entrances.
10. The horizon is already lifted (Observation 5.6): there is no day without refraction.

Closing: The refractive index of air goes with density and density goes as P/T , so the vertical gradient of refractive index has two terms, one for pressure and one for temperature—and in a standard atmosphere the pressure term is 5.2559 times the temperature term. Most of what bends light is pressure, not temperature. So light bends a little even in a standard atmosphere—a radius of curvature of 38118.4 km, a refraction coefficient $k = 0.1671$, in the same decade as the value of about 0.13 used in surveying. Computed at first with the temperature term alone it came to 0.0393, nearly a decade out, because the pressure term had been dropped. Making light follow the curve of the Earth takes $dT/dh = +0.131351$ K/m—the sign is positive: temperature must rise with height. The standard atmosphere falls at -0.006500 K/m, so this is 20.2078 times as steep and in the opposite direction. A layer growing 0.13 degrees warmer for every metre of height — that is an inversion. Even isothermal air gives 4.8448 times the Earth's radius, so the temperature gradient does not start from zero—it adds to a bending already there. At +1 K/m the radius falls to 0.1600 of the Earth's, light bends more sharply than the Earth, and things below the horizon rise into view. Reverse it — hot ground, temperature falling steeply — and dn/dh changes sign, light bends upward, and the road looks wet. These are not three phenomena—one dn/dh with different values and signs. The pressure term and the temperature term, though, have distinct roots: move one and the other stays. One gradient, two independent entrances. One thing separates them—whether temperature rises or falls with height. The separator lies neither in the air nor in the light—the same air, light and formula, and what differs is only which side is warmer. Warm air over a cold sea makes an inversion and hot pavement the reverse, so on one day there may be a superior mirage at sea and a shimmering road inland. To be said honestly—a uniform gradient is assumed here, while real mirages occur in thin layers where the gradient itself changes sharply, which is why inverted images appear. And +0.131351 K/m is not a threshold for mirages but the value at which light runs parallel to the ground. One last thing—even a standard atmosphere bends light by $k = 0.1671$. The horizon we see every day is already an image lifted by refraction. It is not that mirages are special; it is that there is no day without refraction.

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