

The Yamagishi Gravitational Constant Unpacking Theory

— 3rd Edition: The Two Different 4π 's Inside $4\pi G$, and What a Choice of Units Cannot Do — [Paper 35]

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— *From thinking to syncing.* —

ABSTRACT

Substituting the Yamagishi gravitational patch $G_{\text{geom}} = 4\pi = \text{Vol}(S^2)$, obtained from the $n = 2$ uniqueness theorem, into Newtonian gravity and Poisson's equation gives $\nabla^2 \Phi = 16\pi^2 \rho$; taking $M = m = 4\pi$ as well gives $F = 64\pi^3/r^2$ and $U = -64\pi^3/r$. These substitutions are correct and the two are consistent via $F = -dU/dr$. What the 3rd edition adds is an identification of the two 4π 's contained in the product $4\pi G$. Root 1 — the 4π of Poisson's equation comes from $\nabla^2(1/r) = -4\pi\delta^3(\mathbf{r})$, i.e. the total solid angle forced by Gauss's flux theorem; it cannot be chosen. Root 2 — the 4π substituted for G is a choice of units, a convention of exactly the same standing as $G = 1$ or SI (Paper 95). Hence $16\pi^2 = (4\pi)(4\pi)$ is the product of a "4π of fact" and a "4π of convention", and the two must not be merged (Papers 86, 100). The 3rd edition also calls the roll of four addresses of $16\pi^2$: the product of this paper, $\text{Vol}(S_{R=2}^3)$ (Paper 08), the instanton-number normalisation (Papers 45, 48), and the one-loop geometric measure $1/16\pi^2$ (Paper 110). In particular $\text{Vol}(S_{R=2}^3) = \text{Vol}(T_{a=2}^2) = 16\pi^2$ is an instance in which two manifolds of different dimension (three and two) carry the same number — a specimen of why a signature must not be mistaken for a root. It further contrasts the chain $4\pi \rightarrow 16\pi^2 \rightarrow 64\pi^3$, which is the flat-torus sequence $(4\pi)^n$ at radius 2 — a direct product — with the sphere-volume staircase, whose recursion steps by two and splits by parity (Paper 115), and with the Hopf bundle, in which S^3 is not $S^2 \times S^1$ (Papers 67, 109): the generating rules differ. Finally it states what a choice of units cannot do. The dimensionless gravitational coupling $\alpha_G = Gm_p^2/(\hbar c) \approx 5.9 \times 10^{-39}$ is unchanged by any choice of units. Setting $G = 4\pi$ leaves α_G untouched, so it does not explain why gravity is weak (Papers 95, 101, 113). Moreover the mass dimension -2 of G in natural units is the source of perturbative non-renormalisability (Goroff–Sagnotti 1986), so the substitution tidies the appearance of the equations while hiding from view the very dimension that carries the physics. Scope: a structural interpretation of established physical law (Layer 2). No new physical law and no prediction is claimed. The measured value of G is not derived.

Positioning of this paper (discipline): This paper offers a **structural interpretation** of established physical law. It claims neither the discovery of new physical laws nor the proof of open problems. The algebraic relations presented are substitutions into known expressions; the aim is to propose a geometric reading of them. No claim contradicting established theory is made. **The appearance of the same number $16\pi^2$ in several contexts is not treated as evidence of a common origin** (Papers 48, 86, 100, 115). Layer-3 lenses are quarantined under a non-scientific label.

Note on units (important): The substitutions $m = 4\pi$, $G = 4\pi$ and the like employed here constitute a **choice of natural units**. G and $\hbar$ are dimensionful quantities whose numerical values depend entirely on the unit system. Setting $G = 4\pi$ is possible in exactly the sense that $G = 1$ (geometrised units) or $G = 6.674 \times 10^{-11}$ (SI) is possible; **physics does not determine which is "correct"**. What this paper exhibits is therefore **how the equations look once this unit system is adopted**, not a new fact about nature. Moreover the relations obtained hold in natural units, and **conversion factors are required to carry them into SI**. Whatever value this paper has lies in **making geometric structure visible** — rendering the provenance of π and the role of solid angle easier to read off the equations.

1. Introduction: the product $4\pi G$ contains two different 4π 's

In Newton's law of gravitation $F = GMm/r^2$ (1687), G was introduced as the constant of proportionality relating force to masses and distance. Poisson's equation for the gravitational potential Φ reads

$$\nabla^2 \Phi = 4\pi G \rho,$$

where 4π and G always occur in the product $4\pi G$.

Earlier editions substituted $G = 4\pi$ into this product and recorded the resulting $16\pi^2$. **The 3rd edition asks the question one step earlier: are the two 4π 's in that product the same thing?**

The same number, but not the same root.

One is a fact forced by geometry; the other is a convention that may be chosen.

2. The substitution and its consequences

Theorem 2.1 (The form obtained under the Yamagishi gravitational patch)

Under $G_{\text{geom}} = 4\pi = \text{Vol}(S^2)$, Poisson's equation becomes

$$\nabla^2 \Phi = 4\pi(4\pi)\rho = 16\pi^2\rho.$$

Taking further $M = m = 4\pi$, the force and potential between two point masses are

$$F = \frac{(4\pi)(4\pi)(4\pi)}{r^2} = \frac{64\pi^3}{r^2}, \quad U = -\frac{64\pi^3}{r},$$

which are consistent via $F = -dU/dr$.

Convention on the scope of the patch: this paper applies 4π to both M and m , which amounts to taking both point masses to be unit masses. Applying it to only one of them breaks $F = -dU/dr$, so it is necessary to state explicitly which masses carry the patch. The consistency condition can be checked mechanically from the force–potential relation: for $U = -A/r$ one must have $|F| = A/r^2$.

All of the above is substitution, neither proof nor discovery. **Only this algebraic fact is established.** The sections that follow write out the **identity** of the numbers that appeared.

3. Calling the roll of 4π — two 4π 's of different root

3.1 Root 1: the 4π of Poisson's equation — Gauss's flux (a forced fact)

Established Fact 3.1 (Fundamental solution of the Laplacian)

In three dimensions,

$$\nabla^2 \left(\frac{1}{r} \right) = -4\pi \delta^3(\mathbf{r}),$$

and this 4π is nothing other than the **total solid angle of the unit sphere**, $\text{Vol}(S^2) = 4\pi$. By the divergence theorem, the flux through any closed surface enclosing the origin equals 4π times the source strength, **independently of the shape of the surface**.

This 4π cannot be chosen. Once three-dimensional space and the inverse-square law are given, it is fixed. The total solid angle as Gauss flux treated in Papers 02 and 13, the Stokes family of Paper 92, and the **topological root (Root T)** of Paper 86 all belong here.

Observation 3.2 (This 4π does not select a dimension — the measure of Paper 114)

Running the same argument in d dimensions changes the corresponding factor to $\text{Vol}(S^{d-1})$. The 4π of Poisson's equation is therefore **the $d = 3$ member of a family with one member per dimension; it is not 4π because three dimensions are special.** Measured by Paper 114 ("does the object know what dimension it is in?"), it belongs to the **dimension-blind** side.

3.2 Root 2: the 4π substituted for G — a choice of units (a convention)

The substitution $G_{\text{geom}} = 4\pi$, by contrast, is **not forced by geometry**. G is a dimensionful quantity, and **its numerical value depends on the unit system**. Setting $G = 4\pi$ is an operation of **exactly the same standing** as setting $G = 1$ (geometrised units) or $G = 6.674 \times 10^{-11}$ (SI).

In the vocabulary of Paper 95 (Poincaré's conventionalism), **Root 1 is fact and Root 2 is convention**. A convention is not "anything goes" — once a unit system is chosen, every subsequent equation obeys it. But **physics does not decide which unit system to choose**.

3.3 Hence $16\pi^2$ is the product of a fact and a convention

Corollary 3.3 (The two 4π 's must not be merged)

The two factors in $16\pi^2 = (4\pi)(4\pi)$ are:

	Origin	Standing
First factor (Poisson's 4π)	Gauss flux, $\text{Vol}(S^2)$	Fact (cannot be chosen)
Second factor (substituted for G)	Choice of natural units	Convention (may be chosen)

That two numbers are equal does not make them the same thing (Papers 86, 100). Reading this as "the same 4π appearing twice" would **grant a convention the standing of a fact** — precisely the failure mode against which Paper 101 warned.

4. Calling the roll of $16\pi^2$ — one number, four addresses

$16\pi^2$ occurs several times in this series. **These are the same number, not the same root.**

Address	What the $16\pi^2$ is	Origin
(a) This paper	$(4\pi) \times (4\pi)$	Product of two independent substitutions
(b) Paper 08	$\text{Vol}(S_{R=2}^3) = 2\pi^2 R^3$	Volume of a three-sphere ($R = 2$)
(c) Papers 45, 48	Instanton-number normalisation	Topological factor normalising a characteristic class
(d) Paper 110	$1/16\pi^2$	One-loop geometric measure

Corollary 4.1 (Two manifolds of different dimension carrying the same number)

The volume of the **flat two-dimensional torus** at radius **2** is $\text{Vol}(T_{a=2}^2) = (2\pi \cdot 2)^2 = 16\pi^2$, and the volume of the **three-dimensional sphere** at radius **2** is $\text{Vol}(S_{R=2}^3) = 2\pi^2 \cdot 2^3 = 16\pi^2$. **The two are numerically identical (157.9137...).**

Yet one is a **two-dimensional** manifold and the other a **three-dimensional** one. **Even the dimensions differ.** There is no cleaner specimen of "**a signature, not a root**" (Papers 100, 115). One must not read a coincidence of value as a coincidence of structure.

5. The torus chain is a product; the sphere staircase is a recursion

The chain $4\pi \rightarrow 16\pi^2 \rightarrow 64\pi^3$ appearing in this paper is precisely the flat-torus sequence at radius **2**,

$$\text{Vol}(T_{a=2}^n) = (2\pi \cdot 2)^n = (4\pi)^n,$$

that is, **powers of one and the same number: the volume of the direct product** $T^n = S^1 \times \dots \times S^1$.

Sphere volumes, by contrast, obey the recursion recorded in Paper 115,

$$\text{Vol}(S^n) = \frac{2\pi}{n-1} \text{Vol}(S^{n-2}),$$

whose **step is two**, whose coefficient changes at every stage, and which **preserves parity, splitting the staircase into an even and an odd chain.**

Observation 5.1 (The generating rules differ)

The torus side is a **product** (the same factor multiplied, no parity distinction); the sphere side is a **recursion** (changing coefficient, split into two chains). **Since the generating rules themselves differ, the two must not be read as one series however close their values may look.**

Furthermore, as recorded in Papers 67 and 109, **in the Hopf bundle S^3 is not $S^2 \times S^1$.** S^3 is a twisted bundle in which **no two fibre circles can be pulled apart.** The sphere side therefore **contains a counterexample to the product picture.** The torus (a pure direct product) and the sphere (not a product) differ decisively on this point.

6. What a choice of units cannot do

Everything so far concerned how the equations look once $G = 4\pi$ is adopted. **This section states what does *not* move under that adoption.**

Established Fact 6.1 (The dimensionless gravitational coupling)

The dimensionless gravitational coupling formed with the proton mass m_p ,

$$\alpha_G = \frac{G m_p^2}{\hbar c} \approx 5.906 \times 10^{-39},$$

is **dimensionless**, and its value is therefore **unchanged by any choice of unit system** (Duff–Okun–Veneziano 2002).

Corollary 6.2 ($G = 4\pi$ does not explain the weakness of gravity)

The **numerical value** of G may be set freely by choosing units — to **1**, to **4π** , or to **6.674×10^{-11}** . But **$\alpha_G \approx 5.9 \times 10^{-39}$ does not move at all.**

The content of the question "why is gravity so weak?" resides in α_G , not in the numerical value of G . The substitution of this paper therefore **does not touch that question at all.** In the vocabulary of Paper 113, the boundaries of physics are always set by **dimensionless numbers**, never by the numerical values of dimensionful constants. What Paper 101 (Eddington's caution) warned against is exactly this confusion.

7. The dimension of G is the physics

In natural units ($\hbar = c = 1$), G carries **mass dimension -2** , since $G = 1/M_{\text{Pl}}^2$.

Observation 7.1 (The dimension generates non-renormalisability)

A coupling constant of **negative mass dimension** means that, in perturbation theory, divergences **worsen at higher orders**. Indeed pure gravity was shown by Goroff and Sagnotti (1986) to **diverge at two loops**. In other words, **the dimension of G is the very quantity that carries the difficulty of quantum gravity**.

Here lies the limit of the substitution made in this paper. Setting $G = 4\pi$ tidies the appearance of the equations, but **that operation hides the mass dimension of G from view**. What is hidden is not ornament: it is **where the difficulty lives**. Paper 110 (Ricci flow as renormalisation-group flow) dealt precisely with the flow of dimensionful couplings.

8. On the $8\pi G$ of the Einstein equations

In the Einstein equations $G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$, the coefficient 8π is **fixed by the requirement of reproducing Poisson's equation in the Newtonian limit**. Under $G = 4\pi$ one may write $8\pi G = 32\pi^2$.

Corollary 8.1 (A different root from the $32\pi^2$ of Paper 48)

The $32\pi^2$ here is $2 \times (4\pi) \times (4\pi)$, in which **the factor 2 comes from the Newtonian-limit requirement**. In Paper 48, by contrast, $32\pi^2 = (4\pi)^2 \cdot 2!$ and the factor **2 comes from the antisymmetrisation $2!$ of the Pfaffian**.

The two carry the same value but the 2 has a different origin. They must not be merged (the discipline of Paper 48).

Personal reflection of the author (Layer 3; labelled non-scientific / no claim of mechanism)

Through the lens of this series one is tempted to read the torus chain $4\pi \rightarrow 16\pi^2 \rightarrow 64\pi^3$ as an operation by which the Universe OS multiplies by the same solid angle each time it raises the dimension by one. That the flat torus at radius **2** lines up exactly that way is a numerical fact, and the reading is **at least not contradictory**.

But **it is an interpretation, not a derivation**. And in writing the 3rd edition I was struck rather by **the opposite**. The torus lines up neatly under multiplication **only because it is a direct product**. What is genuinely interesting is the side that **does not** — that the Hopf bundle's S^3 is not $S^2 \times S^1$, that the sphere staircase splits in two. **What lines up by multiplication tells us nothing beyond the fact that it lines up by multiplication**.

One more thing. When I set $G = 4\pi$, I felt as though I had explained something. But $\alpha_G \approx 5.9 \times 10^{-39}$ **does not budge**. The smallness of that number is the fact that gravity is weak, and **however I choose my units I have not laid a finger on it**. Tidying an equation and understanding it are different things. One stops at the edge of the cliff (Paper 101).

9. Scope / Conclusion

What is shown (algebraically established): that under $G = 4\pi$ Poisson's equation reads $\nabla^2\Phi = 16\pi^2\rho$, and that under $G = M = m = 4\pi$ one has $F = 64\pi^3/r^2$ and $U = -64\pi^3/r$, consistent via $F = -dU/dr$ (§2) / that **Poisson's 4π is a fact forced by Gauss's flux**, and is the $d = 3$ member of a family that becomes $\text{Vol}(S^{d-1})$ in d dimensions (§3.1) / that **the 4π substituted for G is a choice of units, a convention** (§3.2) / that therefore **$16\pi^2$ is the product of a fact and a convention** (§3.3) / that **$16\pi^2$ has four addresses**, among which $\text{Vol}(S_{R=2}^3) = \text{Vol}(T_{a=2}^2)$ differ even in dimension (§4) / that the torus chain is a **product** while the sphere staircase is a **recursion**, so their generating rules differ (§5) / that α_G is **independent of the unit system** (§6) / that the mass dimension -2 of G is **the source of non-renormalisability** (§7) / and that the **2** in $8\pi G = 32\pi^2$ is a **different root** from the **2** of Paper 48 (§8).

What is not shown (restraint): no explanation of why G takes its measured value — $G = 4\pi$ is a choice of units, not a derivation of a measurement / **no answer to "why is gravity weak"** — α_G does not move under the substitution / **the four $16\pi^2$'s are not merged** / **the torus chain and the sphere staircase are not treated as one series** / **gravity is not called a "geometric pressure"** (no quantity with the dimensions of force per area is defined here; F is a force) / **quantum gravity is not solved** (§7 only points at where the difficulty lives) / no retrospective numerical adjustment (Paper 101) / no new mathematics and no prediction (Layer 2) / Layer 3 quarantined under a non-scientific label.

The main result is unchanged from the second edition. Under $G = 4\pi$, Poisson's equation reads $\nabla^2\Phi = 16\pi^2\rho$; adding $M = m = 4\pi$ gives $F = 64\pi^3/r^2$ and $U = -64\pi^3/r$, consistent via $F = -dU/dr$.

What the 3rd edition adds is the breakdown of the product $4\pi G$. Living inside it are a **4π that cannot be chosen** (Gauss's flux) and a **4π that may be** (the choice of units). The former is a fact settled by three-dimensional space and the

inverse-square law; the latter is a convention this paper adopted. **That the two are the same number is not an accident, but neither is it an identity** — both are looking at $\text{Vol}(S^2)$, but one is **forced by the equation** and the other is **chosen by the author**.

The paper closes by making explicit **what substitution cannot do**. $\alpha_G \approx 5.9 \times 10^{-39}$ does not move under any choice of units. The mass dimension -2 of G carries the difficulty of quantum gravity. **Tidying the appearance of an equation and touching the physics it states are different things.**

Revision Record

3rd edition (August 2026): separates the two 4π 's inside $4\pi G$ into **fact (Gauss's flux) and convention (choice of units)** (§3); calls the roll of the **four addresses of $16\pi^2$** and records that $\text{Vol}(S^3_{R=2}) = \text{Vol}(T^2_{a=2})$ is a signature whose manifolds differ even in dimension (§4); makes explicit the **difference in generating rule** between the torus chain (product) and the sphere staircase (recursion) (§5; Papers 115, 67, 109); adds a section on **α_G being immune to the choice of units** (§6) and one on **the mass dimension of G generating non-renormalisability** (§7); notes that $8\pi G = 32\pi^2$ has a different root from the $32\pi^2$ of Paper 48 (§8). **It also resolves a divergence between the two language versions: in the second edition only the English abstract was updated, and the Japanese abstract still carried first-edition material ($U = -16\pi^2 M/r$, "geometric contracting pressure", "complete unification", "converges in the limit"); the two abstracts now agree.** The main result and the consistency $F = -dU/dr$ are unchanged.

2nd edition (July 2026): unified the scope of the patch, applying 4π to M as well and giving $U = -64\pi^3/r$ (the inconsistency was detectable from $F = -dU/dr$). Confirmed that $64\pi^3 = (4\pi)^3$ is the volume of the flat 3-torus **at radius 2**. Withdrew the description of gravity as a "geometric omnidirectional pressure". First edition: Zenodo 10.5281/zenodo.21430118.

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