

Dead Time Eats the Stability Margin Without Touching the Amplitude

— Nothing Appears on the Gain Plot, and Yet at $L = \mathbf{0.361739}$ Seconds the Phase Margin Falls from $\mathbf{101.5370}$ Degrees to Zero — A First-Order Lag Alone Can Never Destabilise Anything — [Paper 350]

Yuuki YAMAGISHI (山岸 勇輝)

Yamagishi Complexity Torus Lab, Japan

September 2026

— *From thinking to syncing.* —

Abstract

A plain delay enters a control loop. This paper asks what the delay takes away—the answer is that it takes nothing from the amplitude and everything from the phase. No new mathematical theorem and no new law is claimed. First, the magnitude of a dead time e^{-Ls} is exactly 1 at every frequency (Section 2). Second, this is the core of the paper. So the gain crossover frequency does not depend on L , and nothing appears on the gain plot (Section 2). Third, the phase lags by ωL radians, growing without bound with frequency (Section 3). Fourth, in a loop with $\omega_c = 4.898979$, $L = 0.1$ s costs 28.0691 degrees and $L = 0.3$ s costs 84.2073 (Section 3). Fifth, at $L = \mathbf{0.361739}$ s the phase margin reaches zero. And the gain is still 1.0000000000 (Section 3). Sixth, the separator is whether an element changes magnitude or only phase (Section 5).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed—the Nyquist criterion, Bode plots, phase margin, and the transfer function e^{-Ls} are all standard. We do not build control theory—the proof of Nyquist’s theorem and the general theory of multivariable systems are not entered. They are mentioned and no more. We design no compensator—Smith predictors and model predictive control are not entered. Only what a delay takes away is looked at. We do not treat discrete systems—sampling delay and zero-order hold are not the subject. We claim no approximation—rationalisation by Padé approximants is not entered; e^{-Ls} is used as it stands. We do not treat implementation—the physical causes of delay in particular equipment are not the subject. Relation to earlier papers: Paper 313 showed the corpus using “synchrony” in four senses—there too phase was the tool that decided. Here phase alone moves while amplitude does not. Paper 343 showed the size of the hole is not in the stress concentration—there size does not enter the formula; here the delay does not enter the magnitude. The shape in which not entering decides the outcome. Paper 333 showed a constant ratio to follow from cancellation and not from measurement—there what was visible was not evidence. Here what is invisible creates the danger. Paper 300 showed whether two things share a root is decidable—the phase of a first-order lag and the phase of a dead time have distinct roots by that test. What is added is confirming numerically that the magnitude of a dead time is exactly 1, showing that phase margin falls in strict proportion to L , solving for the L at which the margin vanishes and checking both sides of it, and putting the separator on magnitude against phase.

1. Notation and the Question

Setting: Take a loop transfer function

$$G(s) = \frac{K}{s+1} e^{-Ls},$$

with K the gain and L the dead time in seconds.

Observation 1.1

A dead time only delays the signal by L seconds. It neither magnifies nor diminishes— $|e^{-j\omega L}| = 1$ holds exactly at every frequency.

Observation 1.2

The phase, meanwhile, lags by ωL radians. Proportional to frequency, so at high frequency it lags without limit—quite unlike the phase of a first-order lag, which saturates at 90 degrees.

Observation 1.3

The way of measuring is fixed first. Take $K = 5$ and measure phase margin at the gain crossover ω_c , where $|G| = 1$ —phase margin = 180 degrees plus the phase at ω_c .

Question: What does the delay take away?

2. Nothing Appears on the Gain Plot

Calculation 2.1: Find ω_c where $|G(j\omega_c)| = 1$ (computed in this paper).

Quantity	Value
K	5.0
Gain crossover $\omega_c = \sqrt{K^2 - 1}$	4.898979 rad/s
$K G(j\omega_c) $	1.0000000000

Observation 2.2

This is the core of the paper. L does not appear once in that ω_c —since $|e^{-j\omega L}| = 1$, a dead time moves no magnitude at all. However large L grows, the gain plot keeps exactly the same shape.

Observation 2.3

So watching amplitude alone, the delay is invisible. With only a magnitude response on the instrument, a loop with $L = 0$ and one with $L = 1$ cannot be told apart—though one is stable and the other is not.

Observation 2.4

And gain margin, as an index, loses its meaning. The frequency at which the phase reaches -180 degrees comes down as L grows—the answer to “how much more gain can I add” changes with the delay.

Observation 2.5

The shape of Paper 343. Paper 343 wrote that the size of the hole is not in the stress concentration—here the dead time is not in the magnitude. In both, not entering is what decides.

3. Only the Phase Is Eaten, and in Proportion

Calculation 3.1: Phase margin against L at $\omega_c = 4.898979$ (computed in this paper).

L [s]	Phase from the delay [deg]	Phase margin [deg]	Stable
0.00	0.0000	101.5370	stable
0.10	28.0691	73.4679	stable
0.20	56.1382	45.3988	stable
0.30	84.2073	17.3297	stable
0.50	140.3454	−38.8085	unstable
1.00	280.6908	−179.1539	unstable

Observation 3.2

The phase lag is strictly proportional to L . 28.0691 degrees at $L = 0.1$, exactly twice that, 56.1382, at $L = 0.2$ —because it is the single expression $\omega_c L$.

Calculation 3.3: Solve for the L at which the margin vanishes (computed in this paper).

Quantity	Value
Critical dead time L_{crit}	0.361739 s
Phase margin at $0.99L_{\text{crit}}$	+1.0154 deg
Phase margin at $1.01L_{\text{crit}}$	−1.0154 deg
$K G(j\omega_c) $ throughout	1.0000000000

Observation 3.4

The sign flips across the critical point. One per cent apart divides stable from unstable—and throughout, the gain has not moved from 1.0000000000.

Observation 3.5

0.361739 seconds is short by everyday standards. Shorter than a blink—a communications round trip, a sensor average, or one computation cycle reaches this easily.

Observation 3.6

For comparison, take a first-order lag alone. The phase of $1/(s + 1)$ is $-\arctan \omega$, which at $\omega = 10^6$ is 89.9999 degrees—it does not pass 90. However much gain is added, one first-order lag can never reach -180 .

Observation 3.7

Here the two “lags” part. The phase of a first-order lag is bounded above; the phase of a dead time is not—by the criterion of Paper 300 they have distinct roots, and one word “lag” names two different things.

4. Why It Does Not Show in the Amplitude

Observation 4.1

A dead time takes no energy at all. What went in comes out unchanged L seconds later—an attenuating element lowers the magnitude; an element that merely waits does not.

Observation 4.2

And still it destabilises. Feedback subtracts what comes back, so if the return is late, it is subtracted at the wrong moment—shift the phase by 180 degrees and subtraction becomes addition.

Observation 4.3

So the danger is in the timing, not in the size. The magnitude of the signal is right and only its arrival is wrong—an instrument that measures size cannot, in principle, find it.

Observation 4.4

It pairs with Paper 333. Paper 333 wrote that a constant ratio follows from cancellation and is not a measurement—there what was visible was not evidence; here what is invisible creates the danger. In both, what is seen and what acts are different.

Observation 4.5

To be stated honestly—a dead time shows plainly on a Nyquist plot. It winds about the origin in the complex plane—what hides it is the gain plot alone; the phase plot and the Nyquist plot both show it. What is claimed is “do not watch amplitude alone”, not “it cannot be found”.

5. The Separator: Does It Change Magnitude, or Only Phase?

Observation 5.1

The separator——whether an element changes the size of a signal or only its timing.

	Magnitude	Phase	Bound
First-order lag $1/(s + 1)$	falls	lags	saturates at 90 deg
Dead time e^{-Ls}	stays 1	lags	unbounded

Observation 5.2

The right two columns divide the character of the danger. A first-order lag eats phase and lowers gain at the same time, so it stops itself——a dead time eats only phase, so it does not stop.

Observation 5.3

The separator can be moved. If the delay cannot be reduced, lower the gain and bring ω_c down——the phase lag is $\omega_c L$, so halving ω_c halves the lag. Give up responding quickly and the delay becomes tolerable.

Observation 5.4

This is the practical trade. Speed and tolerance of delay sit on two sides of one ω_c ——one cannot take only one of them.

Observation 5.5

To be stated honestly——the 101.5370 degrees and the 0.361739 seconds depend on $K = 5$ and a first-order plant. Another loop gives other values——what does not move is only that the magnitude is 1 and that the phase goes as ωL .

Observation 5.6

One last thing—a dead time is an element that does nothing. It neither amplifies nor attenuates nor reshapes; it waits—that a thing which does nothing can break a system is because feedback uses “when”.

6. The Census and the Close

1. $|e^{-j\omega L}| = 1$ (Observation 1.1): exactly, at every frequency.
2. $\omega_c = 4.898979$ rad/s regardless of L (Calculation 2.1): invisible on the gain plot.
3. $K|G(j\omega_c)| = 1.0000000000$ (Calculation 2.1): whatever L is.
4. 28.0691 deg at $L = 0.1$, twice that at 0.2 (Calculation 3.1): proportional to $\omega_c L$.
5. Margin down to 17.3297 deg at $L = 0.3$ (Calculation 3.1): from 101.5370.
6. $L_{\text{crit}} = 0.361739$ s (Calculation 3.3): shorter than a blink.
7. ± 1.0154 deg at one per cent either side (Calculation 3.3): the sign flips.
8. A first-order lag gives 89.9999 deg at $\omega = 10^6$ (Observation 3.6): never past 90.
9. Plain on a Nyquist plot (Observation 4.5): only the gain plot hides it.
10. Halving ω_c halves the lag (Observation 5.3): speed traded for tolerance.

Closing: A dead time e^{-Ls} is an element that only delays a signal by L seconds— $|e^{-j\omega L}| = 1$ holds exactly at every frequency, so it moves no magnitude at all. Hence L does not appear once in the gain crossover—with $K = 5$, $\omega_c = 4.898979$ rad/s and $K|G(j\omega_c)| = 1.0000000000$, and however large L grows the gain plot keeps exactly the same shape. With only a magnitude response, a loop with $L = 0$ and one with $L = 1$ cannot be told apart — though one is stable and the other is not. What is eaten is the phase alone—the lag is $\omega_c L$: 28.0691 degrees at $L = 0.1$ s, exactly twice that at 0.2 s, 84.2073 at 0.3 s. The phase margin falls from 101.5370 degrees and reaches zero at $L = 0.361739$ s. One per cent short of that it is +1.0154 degrees and one per cent beyond it is −1.0154 — and throughout, the gain has not moved from 1.0000000000. A first-order lag alone does not behave this way—the phase of $1/(s + 1)$ is $-\arctan \omega$, which at $\omega = 10^6$ is 89.9999 degrees and never passes 90. One phase is bounded above and the other is not — by the criterion of Paper 300 they have distinct roots, and one word “lag” names two different things. Why it does not show in the amplitude—a dead time takes no energy; what went in comes out unchanged. It destabilises because feedback subtracts what comes back, and a late return is subtracted at the wrong moment: shift the phase by 180 degrees and subtraction becomes addition. The danger is in the timing and not in the size, and an instrument that measures size cannot in principle find it. One thing separates them—whether an element changes the size of a signal or only its timing. A first-order lag eats phase and lowers gain together, so it stops itself; a dead time eats only phase, so it does not stop. And the separator can be moved—if the delay cannot

be reduced, lower the gain and bring ω_c down: the lag is $\omega_c L$, so halving ω_c halves it. Give up responding quickly and the delay becomes tolerable. To be said honestly—a dead time shows plainly on a Nyquist plot; only the gain plot hides it, and what is claimed is “do not watch amplitude alone”. The 101.5370 degrees and 0.361739 seconds also depend on $K = 5$; what does not move is only that the magnitude is 1 and that the phase goes as ωL . One last thing—a dead time is an element that does nothing. It neither amplifies nor attenuates nor reshapes; it waits. That a thing which does nothing can break a system is because feedback uses “when”.

References

1. Nyquist, H. (1932). Regeneration Theory. *Bell System Technical Journal*, 11(1).
2. Bode, H. W. (1945). *Network Analysis and Feedback Amplifier Design*. New York: Van Nostrand.
3. Smith, O. J. M. (1957). Closer Control of Loops with Dead Time. *Chemical Engineering Progress*, 53(5).
4. Åström, K. J., Murray, R. M. (2008). *Feedback Systems: An Introduction for Scientists and Engineers*. Princeton: Princeton University Press.
5. Michiels, W., Niculescu, S.-I. (2007). *Stability and Stabilization of Time-Delay Systems*. Philadelphia: SIAM.
6. Yamagishi, Y. (2026). *The Three Hundredth Census: Whether Two Things Share a Root Is Decidable* [Paper 300]. Zenodo. 10.5281/zenodo.22171914
7. Yamagishi, Y. (2026). *This Corpus Has Used “Synchrony” in Four Senses, and Only the Fifth Has a Threshold* [Paper 313]. Zenodo. 10.5281/zenodo.22174953
8. Yamagishi, Y. (2026). *That the Ratio of Osmotic Pressure to Freezing-Point Depression Is Constant Follows from an Assumption, Not from a Measurement* [Paper 333]. Zenodo. 10.5281/zenodo.22681740
9. Yamagishi, Y. (2026). *The Size of the Hole Is Not in the Stress Concentration* [Paper 343]. Zenodo. 10.5281/zenodo.22682218

— *From thinking to syncing.* —

Yuuki YAMAGISHI (山岸 勇輝)

© 2026 Yamagishi Complexity Torus Lab, Japan · Licensed under Creative Commons Attribution 4.0 International (CC BY 4.0)